

# THERMOELASTIC MULTISCALE ANALYSIS OF CONCRETE PAVEMENTS SUBJECTED TO HAIL SHOWERS

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**Abstract.** Climate change is likely to increase the occurrence of extreme weather events. Thus, concrete pavements will likely be subjected more frequently to hail showers within their service life. Their influence on the structural behavior of the pavements is studied by means of thermoelastic multiscale analysis. Firstly, the stiffness and the thermal expansion coefficient of concrete are homogenized by means of a multiscale thermoporoelastic model, based on knowledge of the microstructural composition and the properties of the microstructural constituents. The quantified thermoelastic properties serve as input for macroscopic structural analysis of a concrete pavement subjected to thermomechanical loading. It delivers the temperature fields and macroscopic stress and strain states of the concrete. Finally, top-down scale transition is used to quantify the average microstresses of the cement paste and the aggregates.

## 1 INTRODUCTION

Concrete pavements are generally subjected to temperature changes. They include regular daily and yearly temperature fluctuations as well as extreme loading scenarios, such as hail showers. This contribution provides an overview of the application of a recent thermoelastic multiscale model [1–5] on structural analysis of concrete pavements.

The following multiscale representation of concrete pavements is used, see Fig. 1. At the structural scale, the concrete plate is subjected to thermomechanical loading. At the next smaller scale of observation, concrete is represented as a matrix-inclusion composite. It consists of a cement paste matrix and aggregate

inclusions. The microstructure of the cement paste consists of a hydrate foam matrix, hosting unhydrated clinker inclusions. The hydrate foam is further resolved as a matrix-inclusion composite, consisting of a hydrate gel matrix and gel pore inclusions. Finally, at the smallest scale of observation resolved in this study, the hydrate gel consists of nano-sized gel pores, embedded as inclusions in the matrix of hydration products.

Multiscale structural analysis refers to the combination of multiscale material models with structural simulations. This is performed as follows:

- 1) Bottom-up homogenization of the macroscopic properties of concrete, on the ba-

sis of its microstructural constituents and their properties.

- 2) Macroscopic structural simulations based on standard methods.
- 3) Top-down quantification of microscopic stresses of concrete constituents.

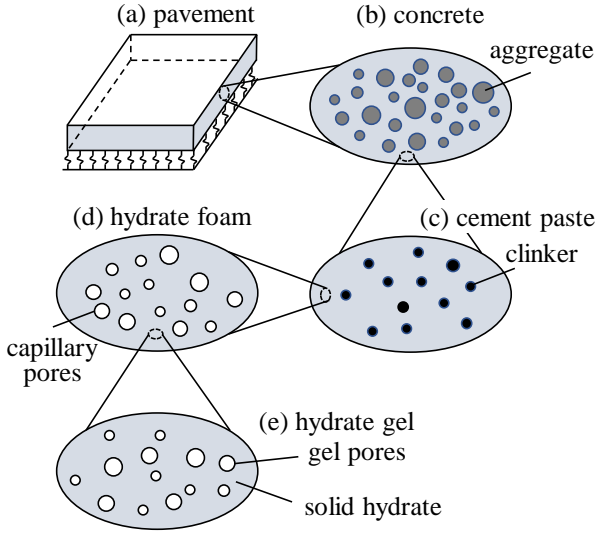


Figure 1: Multiscale representation of a concrete pavement.

## 2 MULTISCALE THERMOPOROELASTIC MODEL OF CONCRETE

Homogenization of the thermoelastic properties of concrete, resolved into four matrix-inclusion composites, see Fig. 1, is carried out with the help of the Mori-Tanaka scheme [6]. Each representative volume element (RVE), occupying the volume  $V_{RVE}$ , consists of a matrix phase with the volume  $V_m$  and an inclusion phase with the volume  $V_i$ . Both material phases,  $k \in [m, i]$ , exhibit a specific elastic stiffness  $\mathbb{C}_k$  and a specific eigenstress  $\boldsymbol{\sigma}_k^e$

$$\forall \underline{x} \in V_k : \begin{cases} \mathbb{C}(\underline{x}) = \mathbb{C}_k \\ \boldsymbol{\sigma}^e(\underline{x}) = \boldsymbol{\sigma}_k^e \end{cases}, \quad (1)$$

with a known volume fraction defined as  $f_k = V_k/V_{RVE}$ .

The volume fractions of concrete constituents are functions of the maturity of the material, quantified by the hydration degree, and

of the initial composition, accounted for by the aggregate-to-cement mass ratio and the initial water-to-cement mass ratio [1]. The eigenstress  $\boldsymbol{\sigma}_k^e$  of the solid phase is proportional to the thermal eigenstrain  $\boldsymbol{\varepsilon}_k^e$ . The latter is proportional to the product of the thermal expansion coefficient  $\alpha_k$  and the temperature change  $\Delta T$ :

$$\boldsymbol{\sigma}_k^e = -\mathbb{C}_k : \boldsymbol{\varepsilon}_k^e = -\mathbb{C}_k : \alpha_k \Delta T \mathbf{1}, \quad (2)$$

where  $\mathbf{1}$  represents for the second-order identity tensor. The eigenstress  $\boldsymbol{\sigma}_k^e$  of the pore phase is equal to the average effective pore pressure, i.e.

$$\boldsymbol{\sigma}_k^e = -p_k \mathbf{1}. \quad (3)$$

$p_k$  accounts for the fluid pressure and the surface tension. It is defined as the integral of the effective pore pressure  $p(r)$  over the pore sizes [1]:

$$p_k = \int_0^\infty p(r) \phi_k^{pdf}(r) dr, \quad (4)$$

where  $\phi_k^{pdf}(r)$  denotes the pore size probability distribution function of pore phase  $k$ . This function can be quantified from adsorption isotherms of cementitious materials [1]. The elastic stiffness  $\mathbb{C}_k$  is considered to be isotropic. It can be determined by means of the bulk and the shear modulus [1].

The elastic behavior of the homogenized composite follows the generalized Hooke's law, i.e.

$$\boldsymbol{\Sigma}_{\text{hom}} = \mathbb{C}_{\text{hom}} : \mathbf{E}_{\text{hom}} + \boldsymbol{\Sigma}_{\text{hom}}^e, \quad (5)$$

where  $\boldsymbol{\Sigma}_{\text{hom}}$  and  $\mathbf{E}_{\text{hom}}$  denote the macroscopic stress and strain, respectively. The homogenized stiffness reads as

$$\mathbb{C}_{\text{hom}} = f_m \mathbb{C}_m : \mathbb{A}_m + f_i \mathbb{C}_i : \mathbb{A}_i, \quad (6)$$

and the homogenized eigenstress as

$$\boldsymbol{\Sigma}_{\text{hom}}^e = f_m \boldsymbol{\sigma}_m^e : \mathbb{A}_m + f_i \boldsymbol{\sigma}_i^e : \mathbb{A}_i, \quad (7)$$

where  $\mathbb{A}_k$  represents the strain concentration tensor of phase  $k$ , estimated by means of the Mori-Tanaka scheme [1].

The thermal expansion coefficient of the homogenized matrix-inclusion composite is quantified by considering the RVE to deform freely,

such that the macroscopic stress  $\Sigma_{\text{hom}}$  is equal to zero. In that way, the macroscopic strain  $\mathbf{E}_{\text{hom}}$  in Eq. (5) becomes equal to the macroscopic thermal eigenstrain  $\mathbf{E}_{\text{hom}}^e$ , which depends on the homogenized thermal expansion coefficient  $\alpha_{\text{hom}}$  and on the temperature change  $\Delta T$ , i.e.

$$-\mathbb{C}_{\text{hom}}^{-1} : \Sigma_{\text{hom}}^e = \mathbf{E}_{\text{hom}} = \mathbf{E}_{\text{hom}}^e = \alpha_{\text{hom}} \Delta T \mathbf{1}. \quad (8)$$

The sought expression for  $\alpha_{\text{hom}}$  is obtained by substituting Eqs. (6) and (7) into Eq. (8) and dividing the results by the temperature change  $\Delta T$ .

Bottom-up homogenization of the elastic stiffness, following Eq. (6), and of the thermal expansion coefficient, following Eq. (8), is carried out in four subsequent steps, starting from the nanoscopic scale of hydrate gel up to the macroscopic scale of concrete, see Fig. 1. By utilizing the established model, the nano-sized hydration products are found to release water when heated up and to take up water when cooled down. This water uptake/release at the nanoscopic scale results in the redistribution of water within the pores of partially-saturated cementitious materials and, furthermore, in a strong dependence of the macroscopic thermal expansion coefficient of cementitious materials on their internal relative humidity [1].

### 3 MULTISCALE STRUCTURAL ANALYSIS OF A CONCRETE PAVEMENT

The homogenized elastic stiffness and thermal expansion coefficient of concrete serve as input for macroscopic thermoelastic analysis of a concrete pavement, see Fig. 2. As for the thermal boundary condition, the pavement plate is subjected to a uniform initial temperature  $T_{\text{ref}}$ . This temperature remains constant at the bottom surface while the top surface is subjected to a time-dependent temperature history  $T_{\text{top}}(t)$ , representing temperature fluctuations resulting from solar heating and a hail shower. The lateral surfaces are considered as thermally-insulated, which leads to one-dimensional heat conduction along the thickness of the plate. As for

the mechanical boundary condition, the pavement plate rests on an elastic Winkler foundation. The plate is subjected to dead load and to the temperature change resulting from the transient heat conduction.

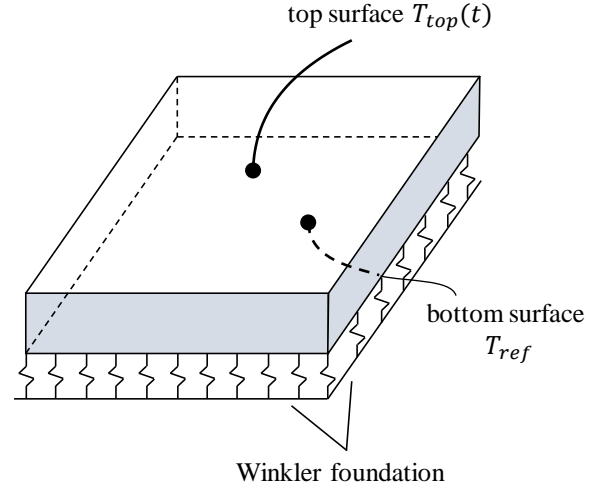


Figure 2: One-dimensional heat conduction along the thickness of the pavement plate, resting on an elastic Winkler foundation.

The heat conduction problem is analyzed first. The obtained history of the temperature field is subsequently used as input for thermoelastic structural analysis of the pavement plate. This provides access to the macroscopic stress and strain states of concrete inside the pavement plate.

Thermomechanical multiscale analysis continues with top-down quantification of the microstresses in the cement paste matrix and the aggregate inclusions. The microstrains of the cement paste and of the aggregates are determined, with the help of the macrostrain  $\mathbf{E}_{\text{con}}$  and the temperature change  $\Delta T$  of the concrete [2]

$$\begin{aligned} \boldsymbol{\varepsilon}_k = & [\mathbb{I} + \mathbb{P} : (\mathbb{C}_k - \mathbb{C}_{cp})] : [\mathbf{E}_{\infty} + \mathbb{P} \\ & : (\mathbb{C}_k : \boldsymbol{\varepsilon}_k^e - \mathbb{C}_{cp} : \boldsymbol{\varepsilon}_{cp}^e)], \quad k \in [cp, agg], \end{aligned} \quad (9)$$

with  $\varepsilon_k^e$  following Eq. (2) and  $\mathbf{E}_\infty$  reading as [6]

$$\mathbf{E}_\infty = \left\{ f_{cp} \mathbb{I} + f_{agg} \left[ \mathbb{I} + \mathbb{P} : (\mathbb{C}_{agg} - \mathbb{C}_{cp}) \right]^{-1} \right\}^{-1} \\ : \left\{ \mathbf{E}_{con} + f_{agg} \left[ \mathbb{I} + \mathbb{P} : (\mathbb{C}_{agg} - \mathbb{C}_{cp}) \right]^{-1} \right. \\ \left. : \mathbb{P} : (-\mathbb{C}_{agg} : \varepsilon_{agg}^e + \mathbb{C}_{cp} : \varepsilon_{cp}^e) \right\}, \quad (10)$$

where  $\mathbb{P}$  and  $\mathbb{I}$  stand for the fourth-order Hill tensor and the symmetric fourth-order identity tensor, respectively. The microstresses of the cement paste and aggregates follow from the generalized Hooke's law as

$$\sigma_k = \mathbb{C}_k : (\varepsilon_k - \varepsilon_k^e), \quad k \in [cp, agg]. \quad (11)$$

#### 4 CONCLUSIONS

A multiscale thermoporoelastic model of concrete was combined with classical structural analysis of concrete pavements. This included two types of scale transition:

- Bottom-up homogenization provided quantitative access to the elastic stiffness and the thermal expansion coefficient of concrete on the basis of the properties of its microstructural constituents. The homogenized properties served as input for macroscopic structural analysis.
- Top-down scale transition was used to quantify the microscopic stresses of the cement paste and the aggregates. It was carried out in the context of postprocessing of macroscopic structural simulations. The microscopic stresses allowed for the assessment of the risk of microcracking.

The presented combination of multiscale material models and structural simulations resulted in multiscale structural analysis. It provided valuable insight into the structural performance.

#### REFERENCES

- [1] H. Wang, C. Hellmich, Y. Yuan, H. Mang, and B. Pichler. May reversible water uptake/release by hydrates explain the thermal expansion of cement paste? – Arguments from an inverse multiscale analysis. *Cement and Concrete Research*, 113:13–26, 2018.
- [2] H. Wang, E. Binder, H. Mang, Y. Yuan, and B. Pichler. Multiscale structural analysis inspired by exceptional load cases concerning the immersed tunnel of the Hong Kong-Zhuhai-Macao Bridge. *Underground Space*, 3(4):252–267, 2018.
- [3] H. Wang, H. Mang, Y. Yuan, and B. Pichler. Microporomechanical modeling of thermal expansion of cement pastes. In *Poromechanics 2017 - Proceedings of the 6th Biot Conference on Poromechanics*, pages 746–753, Paris, France, 2017.
- [4] H. Wang, H. Mang, Y. Yuan, and B. Pichler. Multiscale quantification of thermal expansion of concrete and thermal stresses of concrete structures. In *EURO-C 2018, Computational Modelling of Concrete and Concrete Structures*, page 257, Bad Hofgastein, Austria, 2018.
- [5] H. Wang, R. Höller, M. Aminbaghai, C. Hellmich, Y. Yuan, H. Mang, and B. Pichler. Concrete pavements subjected to hail showers: a semi-analytical thermoelastic multiscale analysis. *Engineering Structures*, under review, 2019.
- [6] Y. Benveniste. A new approach to the application of Mori-Tanaka's theory in composite materials. *Mechanics of Materials*, 6(2):147–157, 1987.