10th International Conference on Fracture Mechanics of Concrete and Concrete Structures
FraMCoS-X
G. Pijaudier-Cabot, P. Grassl and C. La Borderie (Eds)
https://doi.org/10.21012/FC10.244962

FRACTURE MECHANICS BASED DESIGN OF REINFORCED CONCRETE BEAMS-AN ANALYTICAL STUDY

SRI KALYANA RAMA J[*#], RAM SAGAR M[*] AND RAMACHANRA MURTHY A[†]

[*] Department of Civil Engineering
*Birla Institute of Technology and Science, Pilani-Hyderabad Campus, Hyderabad, 500078, India
[#] Vignana Bharathi Institute of Technology, Hyderabad, India
e-mail: kalyan@hyderabad.bits-pilani.ac.in, h20171430038@hyderabad.bits-pilani.ac.in

[†]Fatigue and Fracture Testing Laboratory
Structural Engineering Research Centre (CSIR-SERC), Chennai, 600113, India,
e-mail: murthyarc@serc.res.in

Key words: Size independent fracture energy, relevance vector machine, support vector machine, minimum reinforcement

Abstract: For a developing country like India, construction sector will be a governing factor in deciding the growth of the nation marching towards 2040. Growing infrastructure demands utilization of huge quantities of concrete. Concrete being quasi-brittle in nature is weak in tension which has got a tendency to crack when exposed to external loading. Steel is used as a reinforcement to enhance concrete tensile strength. Traditional and conventional methodologies of design of concrete structures are based on limit states of collapse and serviceability. The existing design principles from various standards across the world overestimates the steel for increasing grades of concrete which indeed will escalate the cost of construction. Its high time that these conventional design principles needed to be revised. Fracture mechanics is one such methods which can be a deciding factor for the design of concrete structures. This can be achieved using concrete mix characteristic length and structural ductility index. The present study deals with the design of reinforced concrete beam as per existing Indian standards for both gravity and earthquake loads and compare the same with fracture mechanics-based design approach. Size independent fracture energy, tensile strength of concrete, modulus of elasticity of concrete, for various grades of concrete are used as material parameters. These parameters are obtained from the experimental investigations on concrete available in the literature. A standard prediction model is proposed based on the strong physical principles associated with the concept of fracture mechanics. It was observed that there is a 10% to 30% reduction in steel requirement for concrete flexural members using fracture mechanics-based design compared to existing Indian standard IS 456:2000 for gravity loads.

1 INTRODUCTION

Fracture mechanics is the study of propagation of cracks. There are many theories proposed which exhibit the failure pattern in materials. Some of the prominent theories are:

1.1 Linear Elastic Fracture Mechanics (LEFM)

Griffith's Theory: This theory is only applicable to materials which exhibit Linear elastic behaviour during loading and fail suddenly once it reaches its ultimate load carrying capacity. Brittle failure is significant in LEFM. Hence it is only applicable to the brittle materials only. There is no residual strength to carry further load for the brittle materials as shown in **Figure *1***

1.2 Elastic-plastic Fracture mechanics (EPFM)

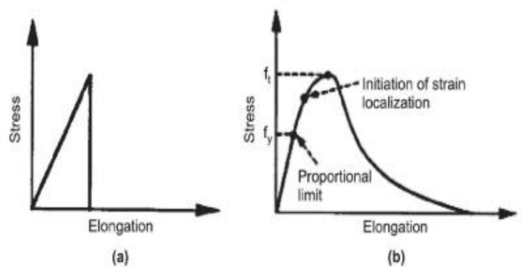

Figure 1: Stress -crack relation for (a) brittle materials (b) Quasi brittle materials.

Irwin's theory: This theory is a modification to the Griffith's theory which suggests a small plastic zone ahead of the crack tip. Blunting of crack tip takes place in the plastic zone.

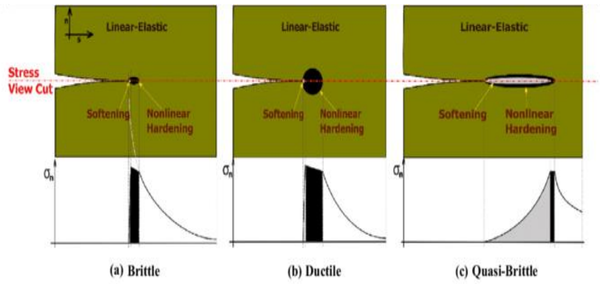

Figure 2: Classification of fracture mechanics-(a) LEFM,(b) EPFM,(c) NLFM (Bazant and Planas, 1998)

1.3 Non-Linear Fracture Mechanics (NLFM):

LEFM and EPFM are not applicable to quasi brittle materials like concrete. Quasi brittle materials are those which exhibit some strain-hardening before it reaches the ultimate tensile strength and thereafter, tensile softening. With the inherent nature of concrete due to its non-homogenous nature there exists flaws in the concrete which are mainly due to less binding, improper vibration, compaction, type of mixing, Interfacial transition zone (ITZ), size of aggregate, W/C ratio, type of admixture. With the increase in age of concrete the durability will show its adverse effects increasing the existing flaw in concrete. This is due to the fact that the concrete being heterogeneous in nature will expose to external loading due to environmental changes along with the service loads. This would increase the size of the flaw and a visible crack on the surface of concrete will be formed. There has to be some concept of energy in order to overcome these problems. But the current code provisions did not consider any parameters which take care of these flaws.

1.4 Fracture energy

This is the total energy absorbed by the material per unit area of crack. This is a material property.There are two types :1) Size dependent or local or specific fracture energy (G_f) and 2) Size independent or global or true fracture energy (G_F). The size dependent fracture energy (G_f) is variable with respect to size of the specimen. Methods like boundary effect method, Simplified boundary effect method and p-delta method can be used to correct/convert the size dependent to size independent fracture energy.

1.5 Characteristic length (l_{ch})

This is the material property coined by Hillerborg in the year 1976 which relates young's modulus(E), fracture energy (G_F) and tensile strength (f_t) of concrete. Shah et.al (1995) found that the value of l_{ch} is 0.2-0.3 times the length of fracture process zone. It is given by,

$$l_{ch} = \frac{EG_F}{f_t^2} \tag{1}$$

1.6 Minimum reinforcement in Reinforced Concrete beams

According to IS 456:2000, minimum tensile reinforcement is given by,

$$\frac{A_{st}}{bd} = \frac{0.85}{f_y} \tag{2}$$

Where b and d are width and depth of the cross section and f_y is the grade of steel. This equation is independent of grade of concrete. The actual expression for mínimum reinforcement was derived from the expression as given by N.Subramaniam [23],

$$\frac{A_{st}}{bd} = \frac{0.17\sqrt{f_{ck}}}{f_y} \quad (3)$$

Equation 3 gives the same result as in IS 456 when f_{ck} is 25 N/mm^2. Hence it was concluded that the IS code has standardized M25 irrespective of the grade used practically. This is not always posible especially for higher grades. There is a need to incorpórate additional parameters that would have a significant impact on the tensile reinforcement of concrete.

Considering these drawbacks, the present study would link the characteristic compressive strength of concrete i.e. f_{ck} to the G_F, fibre content and f_t so that the fracture energy and tensile strength would be utilised and the minimum reinforcement can be reduced.

Machine learning techniques Support Vector Machine (SVM) and Relevance Vector Machine (RVM) are adopted for the present study to come up with a standard expression that would accurately predict the percentage of fibers in concrete based on the size independent fracture energy and characteristic compressive strength.

2. MACHINE LEARNING TECHNIQUES

Machine learning algorithms are used to learn from the data without depending on the actual rules-based programming. The algorithms used in the present study are explained below.

2.1 Support Vector Machine-Regression (SVM-R):

This algorithm is used for building both the classification and regression models. It mainly deals with introducing a hyperplane which divides or classifies the dataset as shown in the figure. The support vectors are the nearest data points of the dataset to the hyperplane. These are also critical in deciding the position of the hyperplane. The distance between the support vectors and the assumed hyperplane is called as the margin distance. This hyperplane location is finalized based on the maximization of the margin between the classification entities.

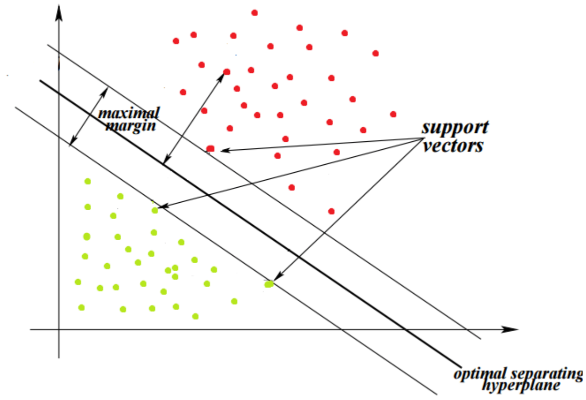

Figure 3: Support vector machine (**Hacker earth)**

In a 2D space, when the features are randomly oriented (in a mixed manner) as shown in the figure, It is quite difficult to find a Hyperplane. In these cases, the 2D space is converted to a 3D space using the kernel tricks. Kernels are the functions which are used to move entities from a 2D space to 3D space which indeed makes easy to locate the hyperplane.

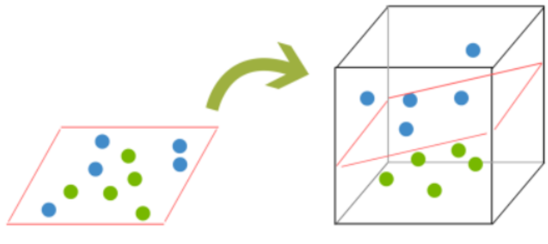

Figure 4: Explanation of Kernel

Support Vector Machines is very accurate and works efficiently with limited data sets but the training time would be high for large data sets and less effective when the data contains more noises.

2.2 Relevance Vector Machine (RVM)

This was introduced by Tipping in the year 2001. This algorithm is an addition of the Bayesian treatment to SVM (Support vector machine). In RVM the function used is same as that of the SVM, i.e.

$$y(x; w) = \sum_{i=1}^{N} w_i K(x, x_i) + w_o \qquad (4)$$

Where, $K(x, x_i)$ is the kernel function

$w_i = w_1, w_2, \ldots\ldots w_n$ are the weights

w_o is the bias

In the present study based on several trial and errors radial kerner (Gaussian kernel) is adopted. The expression for the same is given by,

$$K(x, x_i) = e^{-\left\{\frac{(x_i - x)^T - (x_i - x)}{2\sigma^2}\right\}} \qquad (5)$$

The advantage of RVM is that the prediction is sparser than that of the SVM prediction and the number of non-zero weights are increased.

The Generalized flowchart of the of RVM process is given below

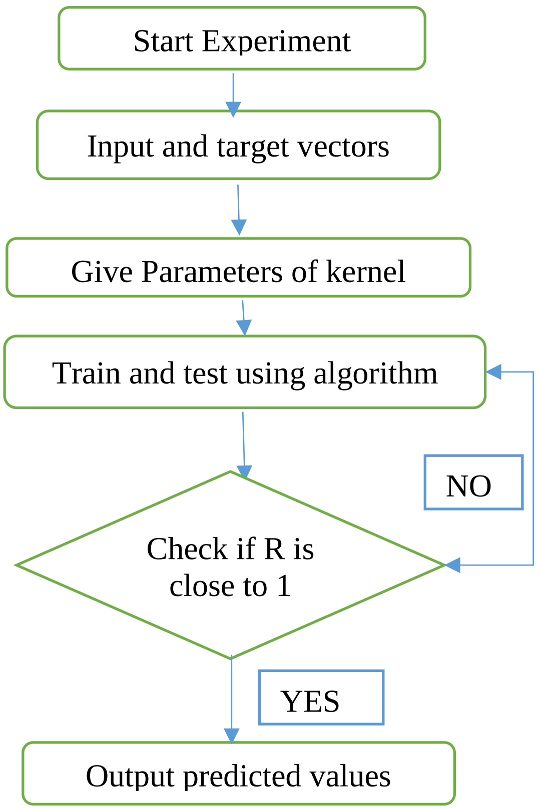

Figure 5:RVM Flowchart

Muralidhara et.al [1] states that the fracture energy which is size dependent becomes size independent (i.e. tends to become constant) for large specimen. Murthy A. R et. al [2] has explained the derivation of bilinear softening curve using inverse analysis for various range of mixes. They have performed Inverse analysis using non-linear crack hinge model to derive the closest solution of the stress-crack width relations. The boundary effect method by Karihaloo was used to correct the size dependent fracture energy i.e. for the conversion of size dependent to the size independent fracture energy assuming a distribution which is bilinear stating that the fracture energy (local) varies at the end due to back face stress variation effect. Matallah et.al [5] found the size independent fracture energy of the specimens at early ages (i.e. specimens which are less than 24hours old) using inverse analysis. Using the P-CMOD curves from TPB test, Inverse analysis was implemented using finite element method. Inverse analysis of non-linearity (Levenberg-marquardt algorithm) is used and a damage softening model was also used to evaluate the softening properties. Cifuentes et.al [6] in his paper, has found the size independent fracture energy using two prominent methods which indirectly take into consideration of the size effect. The two methods are the boundary effect method/local fracture energy method and the curtailment method. These methods are used because the RILEM (work of fracture) method was incapable of giving size independent fracture energy. Hence the result from work of fracture method was corrected using the above-mentioned methods. It was found that both the methods were giving almost the same values of size independent fracture energy. Hence any of these two methods can be used to calculate the true fracture energy. Azzawi and B L Karihaloo [7] have found the size independent fracture energy and stress-crack relation of CARDIFRC Mix II which is a type of self-compacting UHPFRC. The tensile strength is also found using the cracked hinge model. The local fracture energy is found and corrected with the back-boundary effects to obtain the size independent fracture energy.

Trivedi et.al. [8] found the size independent fracture energy of specimens which are geometrically similar with length to depth ratio constant and notch to depth ratios as varying in analytical way. The non-linear behaviour of the concrete specimen was modelled using its softening behaviour. The obtained P-CMOD curves from the analysis were compared with the values from the of P-CMOD (experimental) curves which followed the RILEM standards. The experimental testing was done using the Three point bending test of the notched specimens. The numerical results and the experimental results are found to be in good agreement. Cifuentes and B L Karihaloo [9] have found the size independent fracture energy for the high and normal strength SCC using the wedge splitting test. A comparative study of high and normal strength vibrated concrete and high and normal strength self-compacting concrete was done. It was found that the SCC had less size independent fracture energy and marginally less ductility (due to high fines). This is due to Higher Young's modulus and matrix stiffness and lower tensile strength for SCC. Merta and Tschegg [10] determined the concrete's fracture energy by adding the natural fibres of elephant grass, wheat straw and hemp to the concrete. The addition was 0.19% (by weight) in each case of fibres. The fracture energy was found to increase with the addition of fibres. The most predominant change was in the case of hemp fibres which increased the fracture energy by 70% (as compared to plane concrete). The addition of the other two fibres was not as appreciable as hemp fibres. This was found to be due to the fibre's high tensile strength. The split tensile strength was found to be marginally less than the plain concrete.

Pajak et.al [11] observed the influence of fibre type and percentage of fibres in flexural behaviour of Self-Compacting Concrete (SCC) in his paper. For this work, he considered hooked and straight steel fibres with 0.5,1,1.5% by volume of concrete in SCC. The results were compared with Normally vibrated concrete. It was observed that the pre peak and post peak responses were enhanced using fibres in SCC. The study also suggested a formulae to describe the fracture energy and the flexure tensile strength of SCC. The Fracture energy seemed to be higher for the hooked end fibres than the steel fibres.

Cifuentes et,al. [12] dealt with the impact of the polypropylene fibres on the fracture mechanism of low, normal and high strength FRC. Three-point bend tests and simplified boundary effect method were used to find the size independent fracture energy. Brittleness number was used to analyse the ductility of different mixes when the fibres are used. The results show that fibres have a greater impact in low strength concrete as the bridging effect is greater. The fibre effect also reduces the boundary effect on the specimen more predominantly at the tail end of the P-CMOD curve. In normal and low strength concrete, the failure was due to pull-out whereas for high strength concrete failure was by rupture. Hence thinner and longer fibres (High ductile) are to be used for low and normal concrete whereas thicker and stronger fibres are to be used for high-strength concrete. The effect of steel fibres and water to cement ratio on steel-fibre reinforced concrete (SFRC) was evaluated in the paper by Koksal et.al [13]. The optimization technique was used to maximize the fracture energy. Response surface method was used for the experimental design. It was observed that both the water/cement ratio and matrix strength influenced the fibres behaviour. The above-mentioned properties influenced a lot; Hence, the optimized values of these properties/parameters have to be used for the preliminary design of the mix. Using these values would enhance the fracture energy and even cuts down the cost.

On the availability of different fibres and its unknown effect on fracture properties of concrete, Bencardino et.al [14] studied the effect of steel and polypropylene fibres by varying the fibres content on concrete with 1 and 2%. The cube tests and RILEM standard Three point beam tests were conducted on different notched specimens to infer their compressive strength, flexural strength and fracture energy. It was observed that the fibres contribute extensively to enhance the properties of concrete. The peak load under flexure using steel fibres has enhanced 2-3 times than that of the Normal concrete at 1 & 2% volume fibres respectively. The peak load was decreased at about 40% than normal concrete for 2% polypropylene fibres. The steel fibres increased the fracture energy higher than that of the polypropylene fibres due to the high modulus of steel fibres. Pajak et.al [16] used crimped fibres which are crimped throughout the length and hooked at the both ends. The change in mechanical and fracture properties were investigated with these fibres and compared with the conventional hooked end fibres. The compressive and three-point bend tests were conducted on the specimens with 0.32,0.45, and 0.57% as the fibre percentages. It was deduced that the new fibre did not affect the fracture energy and flexure strength as compared to conventional fibres but the compressive strength was increased by 30% that of plain concrete. It was observed that the crimped fibres delayed the crack formations. The comparison between the steel fibre high strength concrete and plain high strength concrete was investigated by Kazemi [17]. According to the RILEM standards, Three point bend tests were conducted and the fracture properties were compared using both work of fracture (WOF) method and size effect method(SEM). It was observed that with increase in fibres the rate of increase in WOF method was more than that of the size effect method and this accuracy is valid for low steel fibre fractions. The ductility and characteristic length was also increased with increase in fibres. Hence the high strength concrete's ductility can be increased with addition of fibres. Kim et.al [18] investigated the fracture properties of SFRC (steel fibre reinforced concrete). The double hooked steel fibre volume fraction in concrete were 0.5%, 0.75%, 1%. The notch length was changed from 0 to 45mm with an increment of 15mm.Using the three-point bend test (TPBT), the fracture energy was calculated. The correction using the P-Delta method is also used to obtain size independent fracture energy. It was observed that the steel fibre resists the crack growth and hence fracture energy increases. The increase in fracture energy was more for less notch lengths.

Karihaloo [21] proposed a relation between the structural ductility inidex and percentage of tensile reinforcement using size independent fracure energy and mix characteristic. As the tensile strength of the material increases the characteristic length (l_{ch}) decreases. As the characteristic length decreases ductility of the mix decreases. Based on this, the structural ductility index(β) (which is a measure of ductility) was defined as the,

$$\boldsymbol{\beta} = \frac{\boldsymbol{l_{ch}}}{\boldsymbol{W}} \quad (6)$$

For the concrete with the same mix and same reinforcement ratio, the ductility decreases as the characteristic size (W) increases. For the concrete with same mix and different reinforcement ratio (ρ), the structural ductility index is modified as under,

$$\boldsymbol{\beta}^* = \frac{\sqrt{\boldsymbol{\beta}}\boldsymbol{f_t}}{\boldsymbol{f_{yk}\rho}} \quad (7)$$

Where f_{yk} is the yield strength of reinforcing steel. Based on minimum tensile reinforcement clause of different countries code provisions, he found that the minimum reinforcement ratio was increasing as the compressive strength of mix (f_{ck}) increases. Large scale experimental investigations were carried out on wide range on specimens and found that by adding fibres to the mix, the characteristic length increases and hence, the ductility and fracture energy too. So, he proposed that the reinforcement of the lower mix can be used in the higher mix provided that the fibres are added to higher mix. In this way the minimum reinforcement can be decreased for the higher grades of concrete. But in his study, he did not propose any standard expression to calculate the minimum reinforcement.

The present study proposes a relation between the characteristic compressive strength of concrete i.e. f_{ck} to the G_F, fibre content and f_t so that based on the percentage of fibers to be added into the mix the minimum reinforcement can be kept constant for increasing grades of concrete. The proposed equation for percentage of fibers is obtained based on the 96 datapoints chosen from the existing literature. Support Vector Machine and Relevance Vector Machine are adopted for the proposed prediction of fiber content.

3 METHODOLOGY

 In IS 456:2000, Astmin (Minimum tensile reinforcement) is defined as the term which does not depend on the grade of concrete (see Eq.7). Hence a comparative study of IS 456:2000 with the international codes is done to know the variance of code and the values are tabulated.

 Based on the derivation by N. Subramaniam [23], which relates to IS 456, the compressive strength is aimed to relate with the tensile strength, Fibre content and fracture energy.

 Based on Data by Karihaloo [21], it is found that the same reinforcement can be provided for any two grades of concrete by varying the fibre content without compromising the ductility. This fibre content has to be predicted using machine learning algorithms.

 A wide range of data has been collected from the existing literature which gives the dataset about the various mixes and its mechanical and fracture properties.

 Then the prediction models are developed to predict these fibre content values using the RVM and SVM-regression algorithms. The coding is done using the R programming.

 The type of kernels to be used in the algorithms, and their respective parameters play a major role in determining the close estimates of original/actual values. Hence, an iterative process has to be done in order to choose the right kernel and its parameter values.

 In the present study, the RBF (Radial basis function) kernel or gaussian kernel is used and the sigma value is chosen to be 0.9 for fibre data and 0.3 for non-fibre data.

3.1 Process to code using R programming

R programming language was adopted in the entire study for the analysis of data.

- Import the data set. Make sure that the imported file is in the same folder as that of the R code.
- Split the data into the training set and testing set in the required ratio. In present study it is 70:30.
- Now, the data is normalized between 0 to 1 in order to reduce the errors.
- The formula used for normalization is

$$x_i^n = \frac{x_i^a - x_i^{min}}{x_i^{max} - x_i^{min}} \qquad (8)$$

Where,

x_i^n *and* x_i^a are the i[th] components of the input vectors after and before normalization respectively.

x_i^{max} and x_i^{min} are the maximum and minimum values of the input vector before normalization.

- After Normalizing the dataset, the required regressor syntax/function is used and then the model will be predicted. In this project RVM and SVM regressors are used.
- After the regression is completed, again the normalized values are denormalized using the below formula,

$$x_i^a = x_i^n\left(x_i^{max} - x_i^{min}\right) + x_i^{min} \qquad (9)$$

- The accuracy function is used to find the accuracy of the model.

4 RESULTS AND OBSERVATIONS

Comparison of the minimum reinforcement for various grades of concrete from M20 to M50 (‘M’ representing mix) are shown in Table 1. It is observed that there is no change in the minimum reinforcement with the increase in grade of concrete as per IS 456:2000 and it increases based on the expression by [23]. The minimum reinforcement values for various codes of practice as shown in Table 2 depicts that for a given grade of concrete because of the non-dependency of the grade of concrete IS 456 results in least value of reinforcement comprared to other codes. This clearly shows that the existing codal provisions of IS 456 require a change in a way that the minimum reinforcement will be used as per the needs of the industry considering the economal aspect of the construction.

To address the same, RVM and SVM tools are adopted to predict a standard equation that would estimate the percentage of the fibers to be added into the mix to maintain the same mínimum reinforcement for increasing grade of concrete

Table 1: Comparison of Ast/bd ratio values given in IS 456:2000 and N. Subramanian

$\left(\frac{A_{st}}{bd}\right)$	20	30	40	50
$\frac{0.85}{fy}$	0.00204	0.00204	0.00204	0.00204
$0.17 * \frac{(fck)^{0.5}}{fy}$	0.00183	0.00224	0.00259	0.00289

Table 2: Comparision of Ast/bd ratio values for a given mix with different codal provisions for a concrete of grade 40 and Steel of grade Fe 415

Code of Practice	Min. Tensile reinforcement	Structural ductility index
IS456	0.002048	6.828
ACI318	0.003414	4.097
CSA A23.3	0.002743	5.098
Eurocode 2	0.002193	6.378
NZS 3101	0.003414	4.097

4.1 Concrete data with fibres

- The Test Data Vs predicted data (percentage of fibres) is plotted using SVM-regression algorithm as shown in Fig. 6.
- The co-relation value (R) is 0.83 and the mean error is 0.235.
- Support Vectors= 45

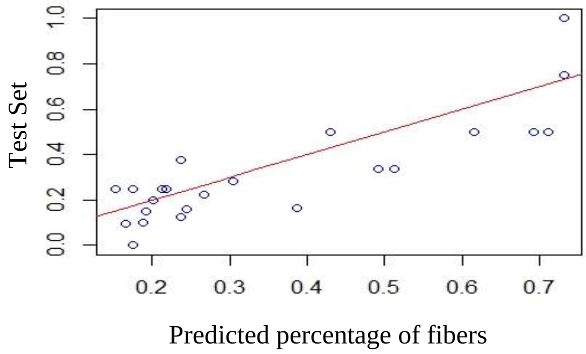

Figure 6: SVM-Regression plot for fibres prediction

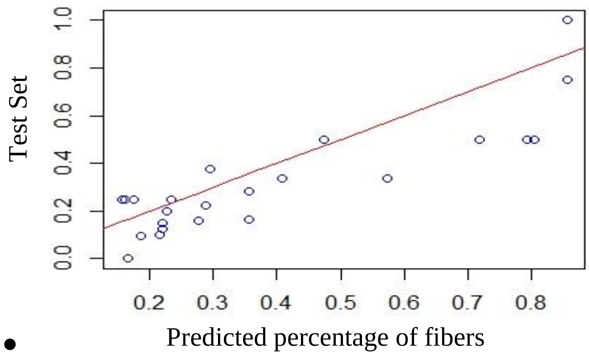

Figure 7: RVM Regression plot for fibres prediction

- The test data Vs predicted data (percentage of fibres) is plotted using the RVM regression algorithm as shown in Fig. 7.
- The correlation value (R) is 0.861 and the mean error is 0.19 for the sigma value of 0.9 for RBF kernel.
- Relevance Vectors= 4
- Based on these results, it can be said that the RVM algorithm has performed better as compared to SVM algorithm.
- The predicted fibre content along with the minimum Steel reinforcement (of the base value of concrete grade) can be used in the higher-grade concrete beams to maintain same ductility as that of the lower grade concrete beams.

4.2 Concrete data without fibres

- The Test Data Vs Predicted G_F is plotted using the RVM regression algorithm as shown in Fig. 8.
- The correlation value (R) is 0.87 and the mean error is 0.008 for a sigma value of 0.3 using an RBF kernel.
- Relevance vectors=5

- The Test data Vs Predicted G_F plot using the SVM regression algorithm as shown in Fig. 9.
- The correlation value (R) is 0.73 and the mean error is 0.009.
- Support Vectors =115

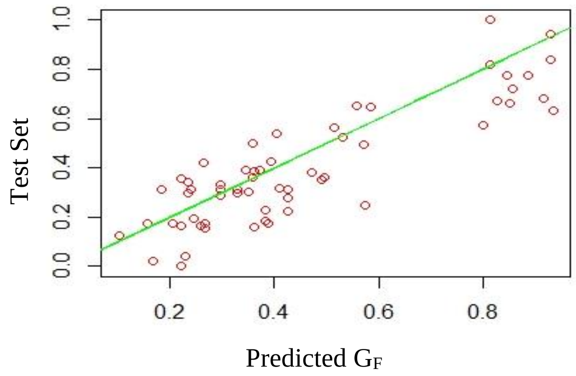

Figure 8: RVM regression plot for Fracture energy prediction

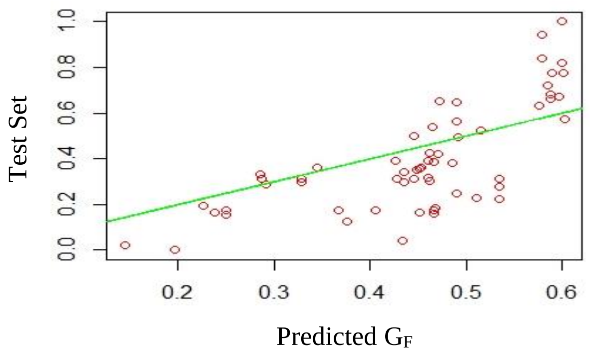

Figure 9: SVM Regression plot for Fracture energy prediction

- The above prediction results of concrete datasets without the fibres clearly show that RVM is far better than SVM in regression analysis for large datasets.
- A comparison is done to check the accuracy of the RVM with Artificial Neural Network (ANN) for the 164 datasets chosen from Nikbin et.al (26) to compare the accuracy of various machine learning techniques in predicting the Fracture energy. The same has been tabulated below.

Table 3: Comparison of ANN and RVM

% Error	ANN [26]	RVM
0-20	62	67
20-40	96	95
40-60	98	96
60-80	99	99
Avg. error	19.8	19

From Table 3, it is observed that RVM has better accuracy in terms of predicting the fracture energy of concrete compared to ANN.

4.3 Example problem demonstrating the proposed methodology with and without fibers

A reinforced concrete beam of M40 grade concrete is subjected to a factored moment of 225kN.m and the cross-sectional dimensions of beam are 300mm X 450mm with an effective cover of 50mm. 28-days compressive and tensile strengths of concrete are found to be 52.8 MPa and 3.85 MPa. Modulus of elasticity and size independent fracture energy are given as 32645 MPa and 115 N/m. Structural ductility index of concrete is 0.75. Area of steel is to be calculated using IS 456:2000 and fracture mechanics approach.

As per IS 456:200 Cl: G-1.1, area of steel is calculated using equation given below

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}}\right) \qquad (10)$$

Substituting the given values will give A_{st} = 1855.67 mm^2.

As per fracture mechanics approach using equations (1), (6) and (7) A_{st} = 1350 mm^2.

There is a reduction of 27.2% of A_{st} using fracture mechanics approach compared to IS 456:2000.

Without the presence of fibers even though there is a reduction in A_{st}, structural ductility will also decrease which makes the beams vulnerable under various loading conditions Therefore the present study suggests the usage of fibers in required percetages depending upon on the various fracture energy values for various grades maintaining the similar reinforcement for higher grades. A fiber percentage of 0.5 can be adopted for the above example maintaining the same reinforcement of

1350 mm^2 and the ductility can be increased to 1.81.

5 CONCLUSIONS

From the extensive analysis of large data sets of concrete with and without fibers using machine learning techniques the following are the conclusions for the present study.

- SVM and RVM have been successfully used in the present study to predict the accuracy of fracture energy and percentage of fibers.
- RVM is accurate in predicting the fracture energy values of concrete without fibers especially for large data sets compared to SVM and ANN.
- Even though SVM gave a reasonably good correlation value close to 1, RVM has proven to be closer to the chosen data sets in predicting the percentage of fibers in concrete with fibers.
- For a given grade of concrete and steel with similar cross-sectional dimensions the area of steel is reduced by 10 to 30% using fracture mechanics approach compared to IS 456:2000.
- Area of steel can be constant for higher grades of concrete with the addition of suitable percentage of fibers depending on the fracture energy of concrete.

6 REFERENCES

[1] Muralidhara, S., Prasad, B. R., & Singh, R. K. (2013). Fracture energy (Size independent) from fracture energy release rate in plain concrete beams. *Engineering Fracture Mechanics*, *98*, 284-295.

[2] Murthy, A. R., Karihaloo, B. L., Iyer, N. R., & Prasad, B. R. (2013). Bilinear tension softening diagrams of concrete mixes corresponding to their size-independent specific fracture energy. *Construction and Building Materials*, *47*, 1160-1166.

[3] Muralidhara, S., Prasad, B. R., Karihaloo, B. L., & Singh, R. K. (2011). Size-independent fracture energy in plain concrete beams using tri-linear model. *Construction and Building Materials*, *25*(7), 3051-3058.

[4] Vydra, V., Trtík, K., & Vodák, F. (2012). Fracture energy (Size independent) of concrete. *Construction and Building Materials*, *26*(1), 357-361.

[5] Matallah, M., Farah, M., Grondin, F., Loukili, A., & Rozière, E. (2013). Size-independent fracture energy of concrete at very early ages by inverse analysis. *Engineering Fracture Mechanics*, *109*, 1-16.

[6] Cifuentes, H., Alcalde, M., & Medina, F. (2013). Measuring the Size-Independent Fracture Energy of Concrete. *Strain*, *49*(1), 54-59.

[7] Al-Azzawi, B. S., & Karihaloo, B. L. (2017). Mechanical and fracture properties of a self-compacting version of CARDIFRC Mix II. *Sādhanā*, *42*(5), 795-803.

[8] Trivedi, N., Singh, R. K., & Chattopadhyay, J. (2015). Fracture energy (Size independent) evaluation for plain cement concrete. *Fatigue & Fracture of Engineering Materials & Structures*, *38*(7), 789-798.

[9] Cifuentes, H., & Karihaloo, B. L. (2013). Determination of size-independent specific fracture energy of normal-and high-strength self-compacting concrete from wedge splitting tests. *Construction and Building Materials*, *48*, 548-553.

[10] Merta, I., & Tschegg, E. K. (2013). Fracture energy of natural fibre reinforced concrete. *Construction and Building Materials*, *40*, 991-997.

[11] Pająk, M., & Ponikiewski, T. (2013). Flexural behavior of self-compacting concrete reinforced with different types of steel fibers. *Construction and Building materials*, *47*, 397-408.

[12] Cifuentes, H., García, F., Maeso, O., & Medina, F. (2013). Influence of the properties of polypropylene fibres on the

fracture behaviour of low-, normal-and high-strength FRC. *Construction and Building Materials*, *45*, 130-137.

[13] Köksal, F., Şahin, Y., Gencel, O., & Yiğit, İ. (2013). Fracture energy-based optimisation of steel fibre reinforced concretes. *Engineering Fracture Mechanics*, *107*, 29-37.

[14] Bencardino, F., Rizzuti, L., Spadea, G., & Swamy, R. N. (2010). Experimental evaluation of fiber reinforced concrete fracture properties. *Composites Part B: Engineering*, *41*(1), 17-24.

[15] Akcay, B., & Tasdemir, M. A. (2012). Mechanical behaviour and fibre dispersion of hybrid steel fibre reinforced self-compacting concrete. *Construction and Building Materials*, *28*(1), 287-293.

[16] Pająk, M., & Ponikiewski, T. (2017). Investigation on concrete reinforced with two types of hooked fibers under flexure. *Procedia engineering*, *193*, 128-135.

[17] Kazemi, M. T., Golsorkhtabar, H., Beygi, M. H. A., & Gholamitabar, M. (2017). Fracture properties of steel fiber reinforced high strength concrete using work of fracture and size effect methods. *Construction and Building Materials*, *142*, 482-489.

[18] Kim, W. J., Kwak, M. S., & Lee, J. C. (2010). Fracture properties of high-strength steel fiber concrete. In *Proceedings of the Korea Concrete Institute Conference*. Korea Concrete Institute.

[19] Kizilkanat, A. B. (2016). Experimental evaluation of mechanical properties and fracture behavior of carbon fiber reinforced high strength concrete. *Periodica Polytechnica Civil Engineering*, *60*(2), 289-296.

[20] Ghasemi, M., Ghasemi, M. R., & Mousavi, S. R. (2018). Investigating the effects of maximum aggregate size on self-compacting steel fiber reinforced concrete fracture parameters. *Construction and Building Materials*, *162*, 674-682.

[21] Karihaloo, B. L. (2015). A new approach to the design of RC structures based on concrete mix characteristic length. *International Journal of Fracture*, *191*(1-2), 147-165.

[22] Stephen, S. J., Raphael, B., Gettu, R., & Jose, S. (2019). Determination of the tensile constitutive relations of fiber reinforced concrete using inverse analysis. *Construction and Building Materials*, *195*, 405-414.

[23] Subramanian, N. (2010). Limiting reinforcement ratios for RC flexural members. *Indian Concrete Journal*, *84*(9), 71.

[24] Murthy, A. R. C., Palani, G. S., & Iyer, N. R. (2009). State-of-the-art review on fracture analysis of concrete structural components. *Sadhana*, *34*(2), 345-367.

[25] Tipping, M. E. (2001). Sparse Bayesian learning and the relevance vector machine. *Journal of machine learning research*, *1*(Jun), 211-244.

[26] Nikbin, I. M., Rahimi, S., & Allahyari, H. (2017). A new empirical formula for prediction of fracture energy of concrete based on the artificial neural network. *Engineering Fracture Mechanics*, *186*, 466-482.