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# MICROCRACKING IN CONCRETE UNDER COMPRESSION: ITS GRADIENT MECHANISMS AND REFLECTION IN MACRO

I.Blechman
National Building Research Institute
Technion – Israel Institute of Technology, Haifa, Israel

## Abstract

The problem of microcracking in concrete under compression is analyzed in terms of lateral gradient strains and it is shown that very strong gradient mechanisms appear and create local transverse gradient strains and cause microcracking due to differences in Poisson's ratio between the concrete's components or to the differences in their elastic moduli. High strength concrete, with the elastic moduli of the matrix and aggregate close, behaves as a crystalon, a brittle solid built from randomly oriented crystals. It is shown that in crystalon a population of laterally tensioned crystals is created due to differences in Poisson's ratio of a single crystal along its three axes. A model of gradient strains between these crystals is obtained, including crystals acting as "pistons". The gradient models explain the appearance of microcracks, their stochasticity and why, instead of growing into macrocracks, they are stable, in good accordance with a vast number of experiments. The Poisson gradient mechanisms are universal, affecting any brittle material such as concrete, rock, ceramics - and do not need initial microcracking to initiate and realize the process of their degeneration. The models of gradient mechanisms are descriptive and based on measurable parameters. They suffice to exhaust the bearing

capacity of concrete under increasing compression without recourse to shear stresses.

On the macrolevel the nonlinear behavior of a concrete under compression is described by the central function: $\sigma = \varepsilon EG$, where the Gaussian – $G$ expresses the stochastic character of the heterogen's survival and comprises two parameters: the limiting strain of elasticity (atrophy threshold) – $\varepsilon_a$, and the scattering factor – $d$. The gradient models were linked with the above macroparameters and it was shown that the strength of a brittle solid under compression is a function of its resistance to microrupture and of the gradient factor.

**Definitions** A brittle heterogeneous solid will be referred to as a heterogen. A matrogen is a heterogen which consists of a continuous matrix and of particles (aggregate) that 'float' in the matrix without mutual contact. Concrete is an artificial matrogen with the granulometry given and kept under very close control. Another kind of heterogen and a case of special interest is a crystalon, which is built of randomly oriented crystals.

**Nomenclature**

Heterogen – brittle heterogeneous solid.
SSc - the curve of stress-strain relationship.
$\varepsilon$ – longitudinal strain;
$\varepsilon_2$ – lateral strain induced by load;
$\epsilon, \epsilon^*$ – transverse strains and their gradients, resp.;
$\sigma, \sigma_2$ – longitudinal and lateral stress, resp.;
$E$ – elastic modulus;
$\nu$ – Poisson's ratio;
$\delta$ – gradient factor;
$a, m$ – indices of the components.

## 1 Rupture of brittle solid

Traditionally the strength of concrete was considered with the failure-inducing peak load and with the development of a major crack. Today this definition is obsolete. With more precise specimen-testing techniques, the descending branch in the stress-strain curve (SSc) is well recognized, and an ever-growing volume of findings regarding its features and design applications is available, Berg (1961), Shah et al (1978), van Mier (1986). The upshot is that, in spite of the inherent brittleness of concrete and rock their limiting stress is not the endpoint of the loading curve; although it still represents the peak response of the specimen, it is no longer associated with total failure. In other words, it can no longer be identified with the moment of rapid collapse of the material following the onset and development of a major crack.

And so the need has arisen to bring out the "cushioned, soft" effects whereby the resistance of the material reduces gradually as the stress level increases. Thus the strength problem is a part of a general one – that of describing the state changes in the material which accompany the increase in the strains and stresses. It is now clear enough that destruction in uniaxially compressed concrete is a result of local transverse tension, revealed in microcracking, O.Berg (1950), Slate (1981),(1986), Delibes (1987), NMBA Report (1983).

F.Slate et al (1986) drew the following conclusion: "A tensile (or tensile-shear) mechanism is the most relevant crack mechanism controlling failure of concrete in uniaxial compression. This failure occurs in a direction perpendicular to applied load for all the concretes tested. Normal strength concretes develop highly irregular failure surfaces including a large amount of bond failure. Medium strength concretes develop a similar mechanism, but at higher strain. The failure mode of high strength concretes is typical of nearly homogenical material. Failure occurs suddenly in a vertical , nearly flat plane, passing through the aggregate and the mortar." This fact is the basis of the approach, developed by Blechman (1988),(1992), where the nonlinearity of concrete behavior on the ascending branch of SSc is explained by microcracking. But theoretically, in a continuous elastic solid under uniaxial compression transverse tensile stresses cannot be induced. An attempt to lay the responsibility on the existing oblique cracks is limited by the simple fact that they do not change under loading, but stay "dormant" up to stresses near the peak point of SSc. Instead of opening the initial microcracks, new microcracks appear, but they also are stable and do not grow, Slate (1981,1986), NMBA Report (1983). It should be noted that besides concrete and rock, other brittle materials like ceramic and cast iron, fail in the same manner. If so, we should come to the conclusion that the mechanisms of failure of brittle solids are very general and independent of the individual features of concrete, rock and others.

## 2 Specificity of heterogens

### 2.1 General features

- Heterogens are isotropic in macro, since their behavior and strength are independent of the direction of loading.
- Heterogens are heterogeneous and anisotropic in micro, since they are built of randomly oriented and randomly combined components, whose properties are different when taken in the direction of the load.
- The internal order in artificial heterogens like concrete and ceramics is of a heavily restricted stochastic structure.
- Local parameters in it fluctuate only between given limits and mean

parameters are kept in line with the technical requirements by rigorous quality control during the production process.
- Their intrinsic elastic modulus $-E$, measured under low-cyclic loading, is constant, as long as the integrity of heterogen is retained.
- Under short-term uniaxial load and normal temperature, a heterogen has no plastic strains, which can smooth out the influence of local gradients.

## 2.2 Microcracking – fundamentals

- The failure of a heterogen under compression is always preceded by microcracking, Berg (1950),(1961) Slate (1981,1987).
- Microcracks induced in a heterogen during loading are local and stable.
- Their plane is parallel to the direction of the maximal compressive stress. It has also been known for a long time as the Kaiser effect, namely that under repeated load the microcracks are usually detected by acoustic emission, when the previously applied stress state is exceeded, Li and Nordlund (1993).
- Microcracking is the reason for the nonlinearity of the stress-strain curve at the ascending branch under short-term loading in both uniaxial and triaxial compression. (Under short-term loading the influence of creep can be neglected).
- It is clear now that there is no plasticity in the non-linear stage of loading, except for triaxial compression with high lateral stress.
- Accumulation of dormant microcracks gradually causes the heterogen to degenerate internally. This process is intrinsical and therefore is called "atrophy" and not damage, which can be a result of external mechanical action.
- Failure sets in when the limiting atrophy is reached, which is the moment when the increment in loading energy absorbed by the heterogen equals the loss of energy due to its degeneration, Blechman (1992).

**Definition of heterogen** For the proposed aim the following main features of the material, considered at the ascending branch of SSc, define a brittle heterogen: (a) it is isotropic in macro, (b) it is heterogeneous in micro, (c) its integrity is retained in the longitudinal direction of loading, (d) there are essential differences in Poisson's ratio and/or in the elastic moduli of its components, (e) its intrinsic elastic modulus is constant in macro, and (f) plasticity is absent under uniaxial compression.

## 3 Concrete as matrogen

The following factors can induce gradient strains and stresses in a brittle solid in general and in concrete in particular under compression:

- Differences in Poisson's ratio and elastic moduli of the components.
- Differences in Poisson's ratio in a single crystal along its main axes.
- Residual stresses and local variations in density.
- Local tension around pores and flaws, Zaitsev (1991).
- Local shear due to gradients in the shear modulus of the components.

Only the gradients induced by differences in Poisson's ratio are considered here.

### 3.1 Poisson's gradient in matrogen

To simplify the analysis we will cut from concrete a rectangular part of two layers representing the 'matrix' and 'aggregate' (fig.1), indexed below by $'m'$ and $'a'$, where Poisson's ratio $\nu_a > \nu_m$, and write for it the following equations: a) of continuity between the components, b) of equality in the increment of the gradient lateral forces – $dF$ (transverse compression and tension) and c) of equality of their longitudinal increment.

$$d\epsilon_a^{ps} - d\epsilon_a^* = d\epsilon_m^{ps} + d\epsilon_m^*, \tag{1}$$

$$dF_a = dF_m, \tag{2}$$

$$\frac{1}{k_a} d\boldsymbol{\varepsilon}_a = \frac{1}{k_m} d\boldsymbol{\varepsilon}_m = d\boldsymbol{\varepsilon}, \tag{3}$$

where: $\epsilon^{ps}$ – free Poisson extension; $\epsilon^*$ – transverse gradient strain; $d\boldsymbol{\varepsilon}$ – increments in longitudinal strains; $k_a, k_m$ – coefficients of nonequality in the increments of the components, related to the average increment in the concrete.

The increment of the gradient forces can be expressed by the parameters of the layers, with their elastic moduli in the lateral and longitudinal directions equal, because the components are taken as isotropic in macro.

$$dF_a = h_a E_a d\epsilon_a^*, \tag{4}$$

$$dF_m = h_m E_m d\epsilon_m^*, \tag{5}$$

here: $E$ – modulus of elasticity, $h_a, h_m$ – thicknesses of layers.
As follows from (2),(4) and(5)

$$d\epsilon_a^* = \frac{h_m E_m}{h_a E_a} d\epsilon_m^*. \tag{6}$$

The factor:

$$\rho_o = \frac{h_m E_m}{h_a E_a},$$

expresses the relationship between the stiffnesses of the two layers.

In an artificial matrogen, like concrete, the granulometry, i.e. the compositon of aggregate of different sizes, is kept under very close control. At the same time the disposition of the large aggregate particles is random enough for assuming the same average value for the stiffness ratio of the components – $\rho_o$ at every cross-section, as follows

$$\rho_o = \frac{k_m v_m^{2/3} E_m}{k_a v_a^{2/3} E_a}.$$

when $v_a + v_m = 1$. Here $v_m$ and $v_a$ – are the volume fractions of matrix and aggregate in the heterogen, respectively. The matrix comprises the hardened cement paste, sand and voids. The increments in free Poisson extension of the layers are:

$$d\epsilon_a^{ps} = \nu_a d\varepsilon_a, \tag{7}$$

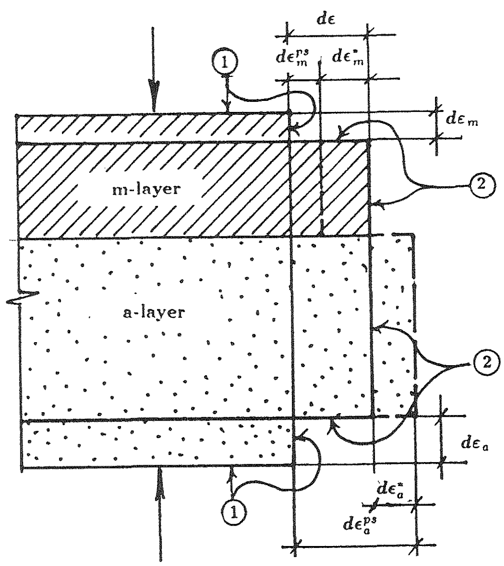


Fig. 1. Gradient strains in matrogen

1, 2 – states of matrogen before and after strain loading;
$d\epsilon_m, d\epsilon_a$ – longitudinal strains of matrix components;
$d\epsilon_m^{ps}, d\epsilon_a^{ps}$ – free Posson extension of matrix and aggregate;
$d\epsilon_m^*$ – gradient strain of lateral tension in matrix;
$d\epsilon_a^*$ – gradient strain of lateral compresssion in aggregate;
$d\epsilon$ – full extension of matrogen.

$$d\epsilon_m^{ps} = \nu_m d\varepsilon_m. \tag{8}$$

Substituting the above equations in (1) yields:

$$d\epsilon_m^* = \frac{k_a\nu_a - k_m\nu_m}{1+\rho_o} d\varepsilon. \tag{9}$$

Taking $\rho_m = 1/(1+\rho_o)$ and integrating (9) from $\varepsilon = 0$ to $\varepsilon$ we obtain the equation for the gradient strain of local transverse tension $-\epsilon_m^*$ induced in the matrix:

$$\epsilon_m^* = \rho_m(k_a\nu_a - k_m\nu_m)\varepsilon. \tag{10}$$

The expression

$$\delta_\nu = (k_a\nu_a - k_m\nu_m)\rho_m, \tag{11}$$

can be defined as the gradient factor in tthe matrogen. Then the gradient tensile strain will be

$$\epsilon_m^* = \delta_\nu\varepsilon. \tag{12}$$

Let us attempt a rough estimation of the gradient factor in a concrete with $\nu_m = 0.14$ and $\nu_a = 0.24$. For $E_a = 40,000$ MPa, $E_m = 15,000$ MPa, $v_a = v_m$, $k_a = 1.05, k_m = 0.95$ we have $\rho_0 = 0.375$ and $\rho_m = 0.73$. Then the gradient factor will be: $\delta_\nu = 0.73 * (0.24 * 0.95 - 0.14 * 1.05) = 0.06$.

### 3.2 Poisson gradient – alternative approach

In considering the gradient strain we can try another approach, based on the apparent value of Poisson's ratio of the matrogen – $\nu_o$, measured in tests. Then the average gradient strain – $\epsilon^*$ induced by Poisson's ratio between the matrix and aggregate can be estimated as

$$\varepsilon_m^* = (\nu_o - \nu_m)\varepsilon. \tag{13}$$

The condition of continuity is "built in" in (13), as the apparent value of Poisson's ratio was taken. Under this approach the gradient factor is defined as

$$\delta_\nu = \nu_o - \nu_m. \tag{14}$$

Since Poisson's ratio for concrete is about 0.20 and, as given above, $\nu_m = 0.14$, the second estimation yields similar result to the above $\delta_\nu = 0.20 - 0.14 = 0.06$.

### 3.3 Importance

The importance of the Poisson gradient factor lies, in the first place, in the possibility of explaining the process of microcracking and predicting the critical loading strains.

But due to stochasticity of the microcracking process the gradient models cannot by simply used to explain the behavior and strength of concrete. In the section "Macrolevel" the solution of this problem is given.

On the other hand these models can be used for revealing the resistance of the heterogen to microrupture. For example, if $\varepsilon_a = 1.7 * 10^{-3}$ and $\delta = 0.06$ then the critical lateral gradient strain of microrupture can be predicted using eq.(12) as $1 * 10^{-4}$, within the well known limits of $0.5 - 1.5 * 10^{-4}$ millistrain for concrete tensile strain at failure.

## 4 High strength concrete as crystalon

### 4.1 Acrons

As defined above, a crystalon is a heterogen built from randomly oriented crystals. Usually Poisson's ratios related to the main axes of a single crystal are very distinct. Therefore in a crystalon under compression gradient strains appear and two populatons of laterally tensioned and laterally compressed crystals are created. As the origin of the crystalon's atrophy, the laterally tensioned population is of highest importance. Being an antithesis to the laterally compressed particles they can be called acrons, the suffix "on" being a common part of the particle nomenclature.

### 4.2 Elastic gradients in acron

In contrast with the matrogen whose particles and matrix are homogeneous in macro irrespective of the direction of the load, the acron strains in a crystalon depend on its angle to the direction of the main stress.

To estimate the gradients a simple model of a symmetrical acron is taken, (fig.2). Its main axes $a$ and $b$ are at the angle $\alpha$ to the direction of the main compressive strain $\varepsilon_1$. The axis $c$ is horizontal and at right angles to $a$ and $b$ and to the axis '1'. The elastic moduli and Poisson ratios of the acron are: $E_a, \nu_a, E_b, \nu_b, E_c, \nu_c$, when $E_c = E_b$, and $\nu_c = \nu_b$.

The shear-induced gradients are not taken here into consideration since these gradients are zero when $\alpha = 0$ and the Poisson gradients are maximal. Shear gradients have their maximum at $\alpha = 45$, but then the Poisson gradients are minimal and the contribution of shear gradients is negligible.

The bulk of the crystalon around the acron is taken in whole with its average parameters $E_o, \nu_o$ in all directions. By definition of the acron $\nu_a < \nu_o < \nu_b$. The element is under longitudinal strain $\varepsilon_1$ and lateral strains $\varepsilon_2 = \varepsilon_3$, with $\varepsilon_2 = \omega\varepsilon_1$.

According to fig.2 the infinitesimal strains for axes $a$ and $b$ are:

$$d\varepsilon_a = (\cos^2\alpha + \omega\sin^2\alpha)d\varepsilon_1.$$

$$d\varepsilon_b = (\sin^2\alpha + \omega\cos^2\alpha)d\varepsilon_1.$$

and

$$d\varepsilon_c = d\varepsilon_3.$$

Denoting

$$f_a = \cos^2\alpha + \omega\sin^2\alpha.$$

$$f_b = \sin^2\alpha + \omega\cos^2\alpha.$$

We have

$$d\varepsilon_a = f_a d\varepsilon_1. \tag{15}$$

$$d\varepsilon_b = f_b d\varepsilon_1. \tag{16}$$

$$d\varepsilon_c = d\varepsilon_3. \tag{17}$$

The increment of lateral strain – $d\epsilon_a$ for a free single crystal is:

$$d\epsilon_a = \nu_a d\varepsilon_a - d\varepsilon_b + \nu_c d\varepsilon_c. \tag{18}$$

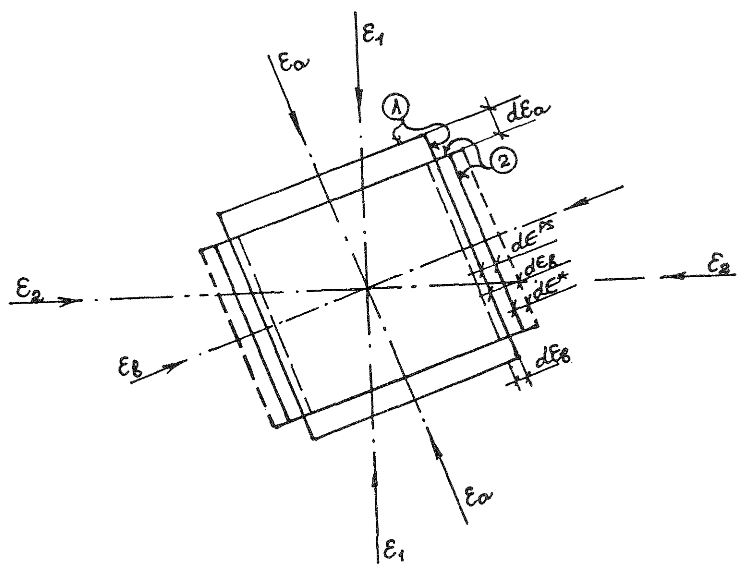


Fig. 2. Gradient strains in acron

1, 2 – states of acron before and after strain loading;
$\varepsilon_a, \varepsilon_b$ – infinitesimal longitudinal strains in 'a' and 'b' direction;
$\epsilon^{ps}$ – free Poisson extension of acron;
$\epsilon^*$ – gradient strain of tension in acron;
$\epsilon$ – effective extension of acron.

Replacing the single crystal by the bulk material we can find the lateral deformation of this "bulk" crystal $d\epsilon_o$ as

$$d\epsilon_o = \nu_o d\varepsilon_a - d\varepsilon_b + \nu_o d\varepsilon_c. \tag{19}$$

The strain gradient of tension between the bulk and the single crystal - $d\epsilon^*$ is

$$d\epsilon^* = d\epsilon_o - d\epsilon_a. \tag{20}$$

It should be noted that the condition of continuity is preferred in these equations, because the actual values of $\nu_o$ and $E_o$ in eqs.(18) express the real interaction of the crystalon's components. Substituting eqs.(15)-(17) in (20) we obtain

$$d\epsilon^* = (\nu_o - \nu_a)d\varepsilon_a - (\nu_c - \nu_o)d\varepsilon_c. \tag{21}$$

The above differences can be denoted as

$$\delta_a = \nu_o - \nu_a,$$

$$\delta_c = \nu_c - \nu_o.$$

Now we can rewrite (21) as

$$d\epsilon^* = (f_a\delta_a - \omega\delta_c)d\varepsilon_1. \tag{22}$$

The expression in brackets is the acron's gradient factor – $\delta_{acr}$

$$\delta_{acr} = f_a\delta_a - \omega\delta_c. \tag{23}$$

and then (22) can be rewritten as

$$d\epsilon^* = \delta_{acr} d\varepsilon_1. \tag{24}$$

In (23) $\delta_{acr}$ is independent of $\varepsilon_1$. Then integrating (24) in the limits $\{0 - \varepsilon_1\}$ we find the local lateral gradient strain as

$$\epsilon^* = \delta_{acr}\varepsilon_1. \tag{25}$$

This gradient strain between the acron and the bulk is maximal when $\sigma_2 = 0$ and $\alpha = 0$. Then

$$\epsilon^* = \varepsilon_1(\nu_o - \nu_a), \tag{26}$$

which corresponds to eq.(14) for a matrogen. The submicrocrack-s found by Darwin et al (1987) seems to be induced by acron-type mechanisms.

### 4.3 Poisson pistons

Olaffson and Peng (1976) described some microcracking mechanisms in a monocrystalline solid – Tennessee marble, tested in uniaxial and also in triaxial compression under high lateral pressure. The induced microcracks (not the lamellae) under all kinds of loading were found oriented close to the direction of maximum compression, demonstrating the action of local lateral tension. Since the nonlinear stage of degeneration (atrophy) of a crystalon determines its bearing capacity, it is important to check the microcracks not at the peak point or even in the after-peak stage, but, first of all, during the nonlinear stage of the ascending branch of the stress-strain curve. In the center of picture A of fig.6 in their paper a clear vertical microcrack splits a grain at this stage (as can be seen from the graph in this figure), its location apparently determined by twin lamellae from the contacting grains. These cracks were frequently observed and their nucleation mechanism was called type I.

The second frequently observed mechanism is of laterally expanding grains, which clearly ruptured their surroundings by vertical microcracks. They act as pistons, and so we will call them. At the elastic stage single crystals will behave as pistons, when their axis of maximum Poisson ratio $\nu_p$ is nearly laterally oriented. In this case the gradient factor, which can be called the piston gradient, can be approximated as

$$\delta_{pst} = \nu_p - \nu_o. \tag{27}$$

For example for $\nu_p = 0.3$ and $\nu_o = 0.2$ the $\delta_{pst}$=0.10!

The grain can go into the state of plasticity due to a phenomenon called mechanism IV, when twin gliding creates lamellae and Poisson's ratio of the grain increses up to 0.5. Then the piston gradient of plasticity – $\delta_{pp}$ appears, and it is especially high:

$$\delta_{pp} = 0.5 - \nu_o. \tag{28}$$

For $\nu_o = 0.2$ we have $\delta_{pp} = 0.30$(!!). This very large gradient can explain the strong piston effect in the lateral direction, which induces not one, but a number of cracks in the neighboring grains, as the pictures in the above paper show. The question is whether this effect takes place within the ascending nonlinear part of SSc or at the after-peak stage (descending branch), where it is no longer relevant in the bearing capacity context in its usual meaning. In any case, the population of "pistons" can add a good part to the process of microcracking and degeneration of a crystalon.

### 4.4 Stability

Since the particles of aggregate or the acrons in a compressed heterogen (concrete) are separated by laterally compressed surroundings, development of microcracks in it is restricted and even arrested at the moment of their appearance. Microrupture of a laterally tensioned particle eliminates not only the strain of tension induced by gradients, but even the possibility of the gradients appearing anew at the ends of the microcrack in spite of continuous Poisson extension during the loading. At the same time other particles with a lower gradient factor will reach the critical strain of local microrupture and after appearance of microcracks they also will sink into the stable state and remain dormant almost up to the stage when the limiting atrophy is reached, explaining the phenomenon of locality and stability of microcracks.

## 5 Macrolevel

According to Blechman (1988),(1992) macrodescription of the non-linear behavior of a heterogen under compression is given by the central function (fig.3):

$$\sigma = \varepsilon E G, \tag{29}$$

where the Gaussian – $G$ expresses the stochastic way of the heterogen's survival and comprises two critical parameters: the limiting strain of elasticity (atrophy threshold) – $\varepsilon_a$ and the scattering factor – $d$:

$$G = \exp\left(-0.5\frac{(\varepsilon - \varepsilon_a)^2}{d^2}\right), \tag{30}$$

where in turn

$$d^2 = \varepsilon_p(\varepsilon_p - \varepsilon_a), \tag{31}$$

and $\varepsilon_p$ is the strain of the peak point of SSc.
The atrophy function – $A$ is complementary to the Gaussian

$$A = 1 - G, \tag{32}$$

and the *pdf* of the atrophy is a result of differentiation of the above atrophy function:

$$P_A = \frac{\varepsilon - \varepsilon_a}{d^2} G. \tag{33}$$

All the gradient models given by eqs.(12),(13) and (24) can be generalized as

$$\epsilon^* = \varepsilon \delta. \tag{34}$$

A microcrack appears when the local gradient strain of tension equals the local resistance of concrete to microrupture – $\epsilon^R$, namely when

$$\epsilon^* = \epsilon^R. \tag{35}$$

If $\hat{\varepsilon}$ is the critical strain of longitudinal compression, which induces local microrupture, then the critical micro and macro strains are linked as follows

$$\epsilon^R = \hat{\varepsilon}\delta. \tag{36}$$

Now the two above mentioned longitudinal critical macrostrain of compression $\varepsilon_a$ and $d$ can be obtained as a function of the local resistance of the heterogen to microrupture $\epsilon^R$ and of the gradient $\delta$ :

$$\varepsilon_a = \frac{\epsilon^R_{min}}{\delta}. \tag{37}$$

and

$$d = \frac{\epsilon^R_M - \epsilon^R_{min}}{\delta}. \tag{38}$$

where $\epsilon^R_{min}$ is the minimal limiting strain of local microrupture, and $\epsilon^R_M$ is the strain of the mode (maximum) of the probability density function of the resistance of concrete to microrupture.

Despite the fact that it is the longitudinal compression that creates the gradient strains, the two critical parameters $\varepsilon_a$ and $d$, are affected (in a probabilistic way) by the critical lateral strains due to the gradient effect.

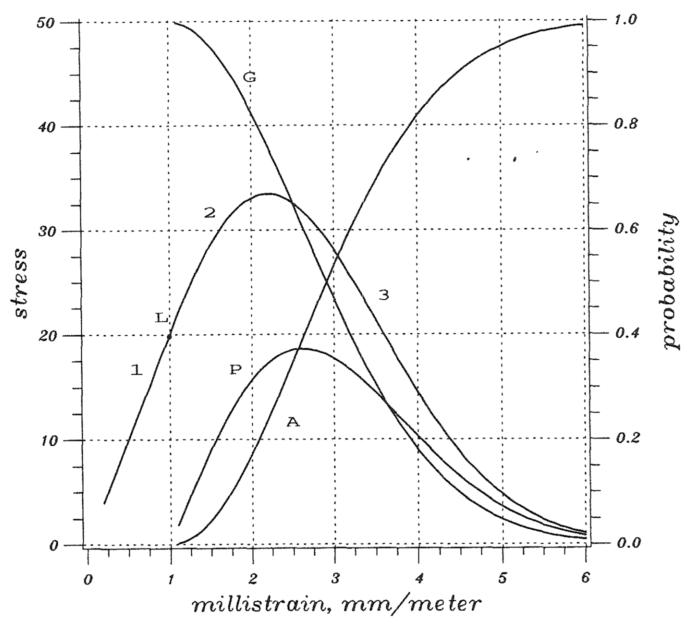


Fig. 3. Characteristic diagram (Char.di) of heterogen

1-2-3 – SSc, stress-strain curve; 1 – linear domain;
2 – nonlinear domain, described by central function;
3 – descending branch; L – microcracking (linear) limit;
G – Gaussian; P – *pdf* of the atrophy; A – cumulative atrophy.

The gradient models give us a new point of view on the linkage between the strengths of concrete in tension and compression. It was found that their ratio reflects the gradient factor in the heterogen.

The decrease in Poisson gradient with increasing strength can explain the faster increase in the compressive strength versus the tensile one. The obtained results confirm an old idea that the properties of a concrete in tension represent its fundamental characteristics.

Corroboration of the role of gradient-strain mechanisms in the behavior of a heterogen under load can be seen in the success of this approach in solving two theoretically interesting and practically important old problems: one – that of bearing capacity of brittle solids and granular materials in triaxial compression, Blechman(1995a); the other – that of modeling the lateral stress release, Blechman (1995b). The solution of the latter shows that the earth's crust is always in the limiting state of its bearing capacity and explains, in good accordance with well known facts, that even a little drop in lateral compression in the crust suffices to induce an earthquake.

## 6 Summary


In the present work, instead of abstracting the brittle solid as a homogeneous material, it is assumed to be heterogeneous from the beginning, together with the paradigm that its failure under compression is always preceded by appearance of microcracks. These microcracks are local, stable (dormant), and uniformly distributed, their plane is parallel to the direction of maximal compressive stress. The intrinsic elastic modulus of a heterogen is constant up to the peak load.

It is shown that gradients in Poisson's ratio and in elastic moduli of the components of the brittle solid can explain the phenomenon of its microcracking in compression. The gradient mechanisms modeled here are: the Poisson mechanisms in a matrogen like concrete; the Poisson gradients of tension in a crystalon, which create "acrons" – laterally tensioned crystals, because of the minimal Poisson's ratio in their lateral direction, when their surroundings are laterally compressed, and also the mechanisms of "pistons" (crystals with maximum Poisson's ratio in the lateral direction), which can go to the state of plasticity and crack the neighbors.

The conclusions which can be drawn from these mechanisms are:

(1) The gradient models can explain the appearance of microcracks and their features (esp. their locality and stability) in a compressed brittle solid, based on its known characteristics.

(2) The mechanisms of Poisson's gradient are universal. They affect every heterogen under compression: rock materials, concrete etc.

(3) The gradient mechanism does not need initial cracks for inducing and realizing the process of microcracking and degeneration.

(4) Gradient mechanisms are stochastic; they do not cause macrocracks, but induce a lot of stable microcracks, in good accordance with a vast number of experiments.
(5) The gradient models are descriptive, based on strightforward mechanics and measurable parameters of the heterogen.
(6) In a compressed crystalon (a brittle solid built from randomly oriented crystals), a population of laterally tensioned crystals, called "acrons", is created. The models of gradient strain in the acrons are given, including the equation of critical strains. Since the acrons are glued in a fully compressed environment, the question of releasable energy in the critical state was checked.
(7) In contrast to acrons, there are crystals or grains, laterally overcompressed due to high positive gradients with their surroundings. They are working as "pistons" and can crack the overlying and underlying neighbors. Their action is especially strong where, due to transition into the state of plasticity, their Poisson's ratio rises up to 0.5.
(8) Gradient mechanisms suffice to cause degeneration in a heterogen and exhaust its bearing capacity under increasing compression, without recourse to shear stresses.
(9) The gradient models show that the strength of a crystalon under uniaxial compression increases when the differences in the mechanical characteristics along their main axes are minimal and when their limiting strain of sliding (plasticity) is maximal.

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