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FRACTURE BEHAVIOR OF CONCRETE TO MIXED LOADING

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Abstract
In the first part of this paper, the crack extension in the center notched circular disk subjected to a diametral compression is numerically examined. The crack initiation from the notch tip is estimated by assuming that crack growth occurs in the direction along which the drop of total potential of tractions per unit length of crack growth takes a maximum value. It was observed that the result of numerical analysis reproduced fairly well an experimantal observation. In the second part, the maximum potential drop concept is applied to the crack initiation and propagation analysis for an unsymmetrically notched beam subjected to the three-point-bent. The obtained results are reasonable.

1 Introduction

Crack initiation and propagation in concrete is very complicated, since various sizes of aggregate and shapes of void are scattered. So with brittle materials. Behavior of crack in these materials subjected to mixed-mode loading has been examined by a number of investigators after Erdogan and Sih(1963). On the mixed-mode fracture behavior of concrete Jenq and Shah(1988) presented results of an extensive investigation in which they proposed two parameters model. With the model they predicted well the fracture behavior of concrete. About the same time Shetty, Rosenfield and Duckworth(1987) presented results of experimental investigation on the mixed-mode fracture of soda glass. They stated that none of exsisting mixed-mode fracture theories was completely adequate to explain their test results.

In the first part of this paper, a concept of the maximum drop of potential of surface tractions at the crack initiation is developed, and applied to the test results by Shetty et al. In the next part the concept is extended to analyze crack initiation and propagation in an unsymmetrical concrete beam with two notches on one side edge. Numerical analyses were conducted with the FEM.

2 Crack initiation from notch tip in Brazilian test

Center notched circular disks subjected to the diametral compression were used by Awaji and Sato(1978) to study the mixed-mode fracture toughness of brittle materials. After their investigation Atkinson, Smelser and Sanchez(1982) formulated an explicit expression of the stress intensity factors for the notched circular disk. Yarema et al.(1984) and Shetty et al.(1987) conducted a number of tests to investigate the direction of crack extension as well as the fracture criterion for brittle materials (Yarema used sintered carbide and Shetty used soda glass). Yarema stated that the maximum hoop stress theory gives the best agreement with test results. On the contraty, test results of Shetty were smaller than estimations by both theories of the maximum hoop stress and the minimum strain energy density. Shetty expressed that the mixed-mode fracture theories formulated only in terms of stress intensity factors did not well account for the center notched disk test results.

It should be noted that the material ahead of crack tip in the center notched disk is directly affected with the diametral compression load action, unless the size of the specimen is large and the length of the notch is adequately short. It is considered that Shetty's opinion is not irrelevant to the abovementioned ill nature of the center notched test specimen. Alternative for the center notched disk test is required to overcome the unadaptability.

Figure 1 shows the states 0 and 1 of a cracked elastic body just before and after the crack growth takes place. The surface tractions X_{ni} are assigned on the surface S_F, and displacements u_i are

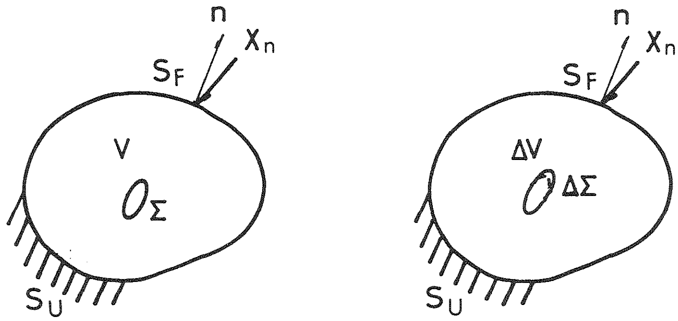

Fig. 1 Crack configuration before and after crack extension

assigned on the surface S_U. $\Delta\Sigma$ is an increase of the surface area and ΔV is a decrease of the volume due to the crack extension. The total potential energies of the states 0 and 1 are expressed as

$$\Pi^0 = \int_V U^0(\epsilon_{ij})\, dV - \int_{S_F} X^0_{ni} u^0_i \, dS \tag{1}$$

$$\Pi^1 = \int_{V-\Delta V} U^1(\epsilon_{ij})\, dV - \int_{S_F+\Delta\Sigma} X^1_{ni} u^1_i \, dS \tag{2}$$

where $U(\epsilon_{ij})$ is the strain energy density. X_{ni} are constant during evolution of the state. Contribution of incremental displacements on S_F to the potential energy of state 1 should be taken into account, since the length of crack growth in the Brazilian test specimen of brittle materials is not infinitesimal, and the diametral compression is not remote from the notch or crack tip. The effect of cohesive forces should be considered on the surface $\Delta\Sigma$ from the first of the above reasons. Then, the difference of the potential energies can be expressed as

$$\Delta\Pi = \frac{1}{2}\sum_{\Delta\Sigma}(X^1_{ni})_{\Delta\Sigma}\, \Delta u_i - \int_{S_F} X^0_{ni} \Delta u_i \, dS \tag{3}$$

X^1_{ni} in the first term of the right side are the cohesive forces on $\Delta\Sigma$. From the linear theory, $\Delta\Pi$ can be expressed by ΔA, the difference of the potentials of surface tractions, as follows.

$$\Delta\Pi = \Delta A = -\int_{S_F+\Delta\Sigma} X^1_{ni} u^1_i \, dS + \int_{S_F} X^0_{ni} u^0_i \, dS \tag{4}$$

Employing a quarter of side length of element in the front of notch tip to the length of crack initiation, the cohesive forces $(X^1_{ni})_{\Delta\Sigma}$ can be estimated by equivalent nodal forces at the notch tip node at the state 0. The magnitude of the forces depends on crack initiation angle. The virtual crack extension technique by Hellen(1975) was used to detect the probable initiation direction.

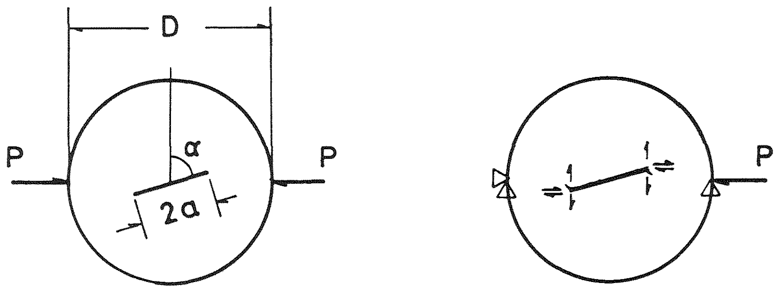

Fig. 2 Geometry and loading condition of the notched disk

Figure 2 shows geometry and loading condition of the calculation. Side length of quarter point singular triangular elements used at the notch tip was 1.6×10^{-2}D, hence the length of virtual crack initiation was 4×10^{-3}D. Crack initiation angles θ for typical notch orientations α are shown in Table 1. Numerical values in the Table are potential drops normalized by the maximum one for each orientation α . The directions of the maximum potential drop are plotted in Figure 3. The proposed method gives a fairly good estimation for the crack initiation direction in brittle materials.

Table 1 Normalized potential drop of tractions versus crack initiation angle in center notched disk subjected to mixed-mode loading (α = notch orientation angle, θ = crack initiation angle)

	α °			
-θ °	80.0	75.0	70.0	62.5
32.5	0.989290			
35.0	0.995647			
37.5	1.0			
40.0	0.999315	0.993944		
42.5	0.994865	0.997550		
45.0		1.0	0.996590	
47.5		0.998565	0.999427	
50.0		0.995834	1.0	0.996770
52.5			0.999341	0.991487
55.0				1.0
57.5				0.975803
60.0				0.963677

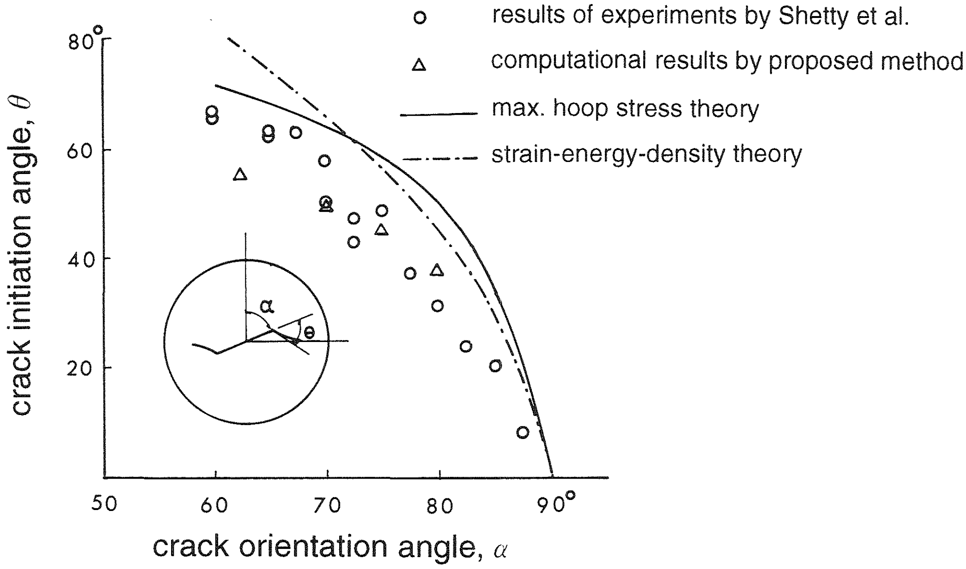

Fig. 3 Crack initiation angle in center notched disk subjected to mixed-mode loading

3 Crack growth in multiple notched concrete beam

In this section the numerical technique for the prediction of crack initiation given in the previous section is extended to predict the crack initiation and propagation in a three point bent concrete beam with 2 notches on one side edge.

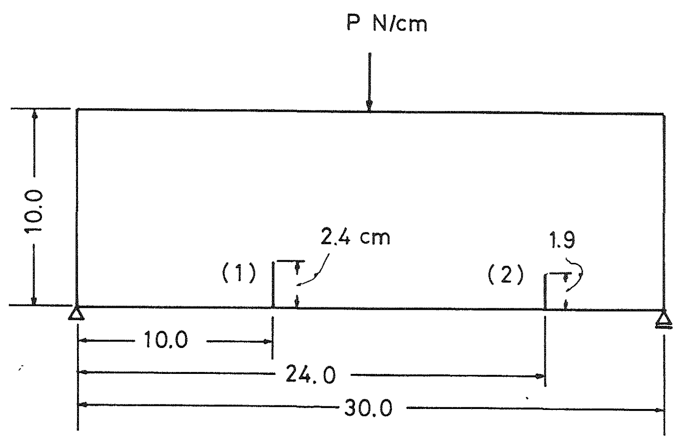

Fig. 4 Beam with two notches on one side edge

Figure 4 shows the beam analyzed. Loading mode for both notches is the mixed-mode. It is self evident that the fracture in the beam will occur from notch-1. The whole on cracking, however, is not always evident. For example, direction of cracking, load-crack growth relation, and load-deflection relation in a beam with multiple notches are not clear. Following assumptions were adopted to study the above unkown problems.

1. Fracture criterion that two of the present authors presented in the FRaMCoS 1(1993) is applicable to the crack propagation as well as crack initiation. The criterion was obtained from experimental results for the center notched concrete disk. It is given as

$$\left(\frac{K_{II}}{mK_{IC}}\right)^2 + a\frac{K_I}{K_{IC}} + b\left(\frac{K_I}{K_{IC}}\right)^3 = 1 \qquad (5)$$

where $m = (K_{IIC}/K_{IC}) = 2.2,\ a = 0.4,\ b = 0.6$.

2. Cohesive forces distribute on the plane of crack growth in front of the present crack tip. The resultant of the cohesive forces equilibrate with the resultant of nodal forces.
3. The crack initiation criterion is affected by the cohesive forces. In other words the fracture toughnesses depend upon the cohesive forces.
4. Finite elements adjacent to the present crack tip are in the FPZ(fracture process zone), and the Young's modulus of material in the FPZ is 1/3 of that in intact zone.

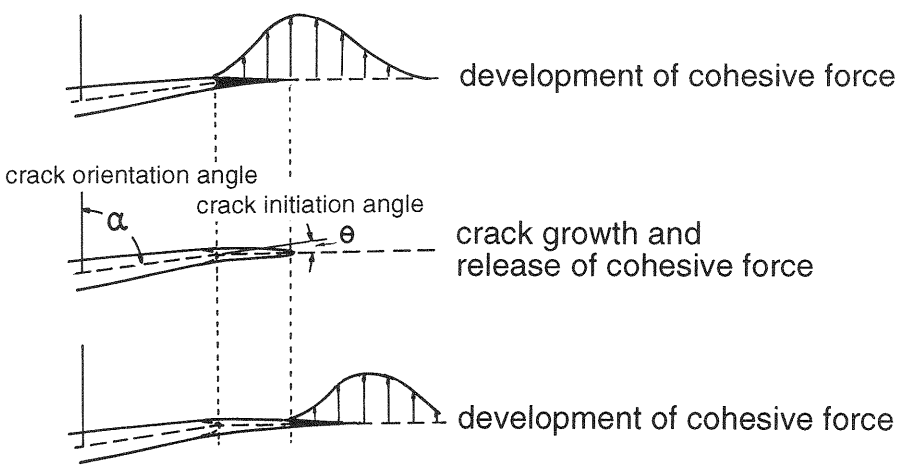

Fig. 5 Crack growth and change of cohesive force along the crack line

Figure 5 shows the schematic configuration of crack tip before and after the crack growth, and development and release of the cohesive forces along the crack line. Stress intensity factors at virtual crack tip due to the cohesive forces at the instant of their release can be obtained employing the virtual crack extension method. Let them call cohesive factors, and designate as $\bar{K}_I$ and $\bar{K}_{II}$. Then, the assumption-3 can be numerically treated only by adding the cohesive factors to the fracture toughnesses in the fracture criterion Eqn (5). Material constants for the crack propagation analysis were assumed to be E(Young's modulus of intact material) $= 32\,\mathrm{GPa/m^2}$, ν (Poisson's ratio) $= 0.2$, $K_{IC} = 0.5\,\mathrm{MPam^{1/2}}$, and $K_{IIC} = 1.1\,\mathrm{MPam^{1/2}}$.

Table 2 Results of crack propagation analysis

Step	Load	Def.	Crack tip position				Crack ext.angle	Stress intensity factor		Cohesive factor	
	P (N/cm)	$v_c \cdot 10^{-3}$ (cm)	x_1	y_1 (cm)	x_2	y_2	θ (°)	$(K_I)_1$ (MPam $^{1/2}$)	$(K_{II})_1$	$\bar{K}_I$ (MPam $^{1/2}$)	$\bar{K}_{II}$
0	298.2	1.396	10.00	2.4	24.0	1.9	0.0	0.499	0.07	0.103	0.0133
1	321.1	1.591	10.07	2.753			-11.0	0.603	0.0263	0.135	0.0143
2	307.3	1.621	10.15	3.103			-13.0	0.635	0.0205	0.148	0.0152
3	285.1	1.616	10.24	3.453			-14.0	0.648	0.0188	0.157	0.0167
4	262.8	1.612	10.33	3.851			-15.0	0.657	0.0163	0.166	0.0182
5	241.7	1.620	10.43	4.147			-16.0	0.666	0.0134		

The results of the analysis are listed in Table 2. The load–deflection curve at point C and position of crack tip are shown in Figures 6 and 7. The results seem to be reasonable. First, K_I values are remarkably higher than K_{II} values in each step. It suggests that the fracture propagates predominantly in mode I. Second, relation of failure angle to ratio of K_{II}/K_I is expected to be consistent with the experimental measurement for large specimen by Jenq(1988). Third, the values of cohesive factor $\bar{K}_I$ varies between 1/5 to 1/3 of fracture toughness K_{IC}. The factor $\bar{K}_I$ has not a few influence on resistivity to cracking at the early stage of fracturing.

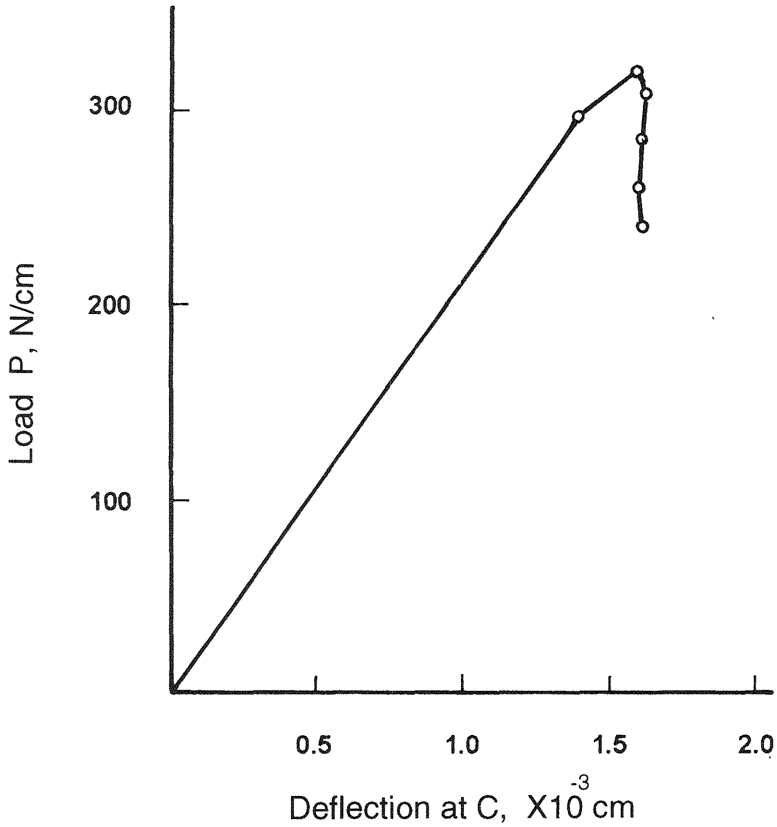

Fig. 6 Load-deflection curve at point C

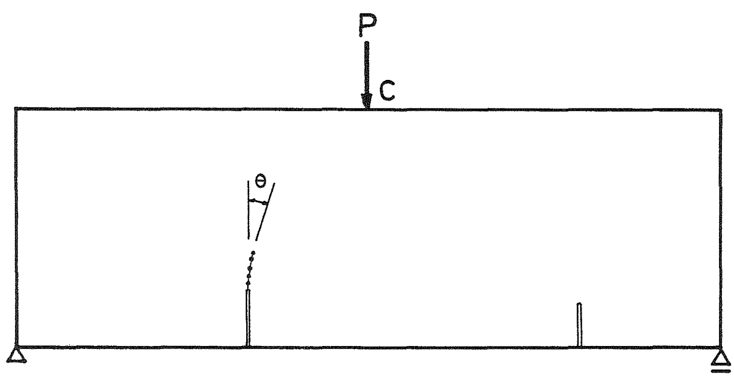

Fig. 7 Advance of crack tip position

4 Conclusion

1. The maximum drop of the potential of surface tractions including cohesive forces can be used to predict the angle of crack initiation in the brittle material subjected to the mixed-mode loading.
2. The maximum potential drop concept is usable for analyzing the crack propagation in concrete.
3. Cohesive force acts as an inconstant resistivity to cracking at the early stage of fracturing.

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