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A FRACTAL STUDY OF THE SIZE EFFECT OF CONCRETE FRACTURE ENERGY

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Abstract

The three-dimensional crack structure of concrete, which is divergent, discontinuous and complex, is quantitatively described with fractal theory, and the calculation formula of total crack surface area deduced. In accordance with the definition of fracture energy by TC50-FMC, a relation is built between fracture energy and the parameters of micro crack structure in micro crack zone, and hence the size effect problem of fracture energy solved theoretically. The relation was also proved by experiments. Results show that: concrete strength increases with the increase of fracture surface energy γ_s; concrete strength increases and the significance of the size effect of fracture energy G_f decreases with the decrease of the structure parameter D_c of the micro crack zone; the structure of the micro crack zone changes with varying specimen size, so does the value of D_c; thickness of specimen also causes the size effect of G_f.

1 Theoretical proof

1.1 Relation between fracture energy and ligament height

The different fracture toughness of various part of concrete is caused by large amount of discontinuous pores and micro cracks widely and randomly dispersed, as well as heterogeneity of internal structure of the material. Stress intensity factors of different parts also vary even under the same applied force. The part with lower fracture toughness or higher stress intensity factor always begins to crack first. Therefore, micro cracks caused by external force are also discontinuous and randomly distributed. Like the main macro crack, micro cracks are extremely tortuous and branching. Such a complex and irregular micro crack system can be simulated with a fractal model.

Suppose the micro crack dispersion in micro crack zone is a dendritic structure as shown in Fig. 1a, and assume the length of each broken line segment of the generator (Fig. 1b) to be a unit length, then, according to the following formula:

$$D=\lg N/\lg(1/r),$$

the fractal dimension of crack structure can be calculated with:

$$D=\lg 3/\lg(2\cos(\beta/4)).$$

Therefore the fractal dimension of micro crack structure is greater than 1. Though the real micro crack structure is much more complex than the dendritic structure model, it can be qualitatively described with such a model.

Fig. 2 shows the fracture process of the micro and macro cracks of several groups of specimens with different height of ligament. When external force P_h rises to a certain degree, micro crack zone begins to converge and link up, leading to the starting of fracture, and from then on

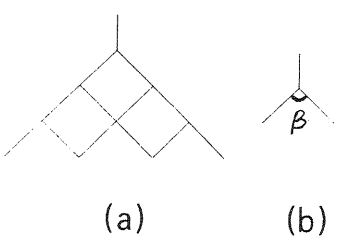

Fig. 1 Dendritic structure of micro cracks in micro crack zone

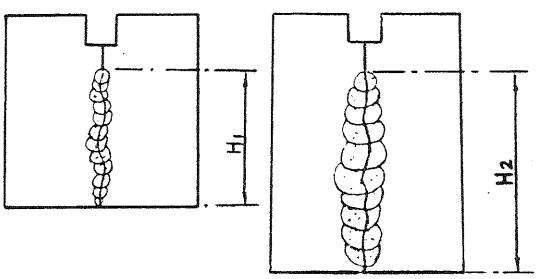

Fig. 2 Sketch map of micro and main cracking in specimens with different ligament height

the sub-critical propagation process of fracture begins. If the main crack has developed to a certain level, it will turn into a destablizing propagation. During the whole fracture process (starting of fracture, sub-critical propagation and destablizing propagation), the micro crack zone always exists at the tip of the crack as a forerunner. Fractal theory is used next to reinvestigate the nonlinear difference of the real fracture surface area caused by the micro crack zone in the specimens of different height shown in Fig. 2.

If a series of dishes are used to cover the fracture surface which corresponds to the fractal structure, the relation among the necessary number of dishes(N(r)), the radius of the dishes(r) and the fractal dimension of fracture surface(D) can be expressed as:

$$N(r) = C\, r^{-D} \tag{1}$$

where C is a constant.

If a dish with a constant radius r_0 is used to cover a set of fracture surfaces with various lengths R, what about the relation between N and R? No matter how irregular a section is, it can be regarded as a part of spherical surface with a radius of R_o if the magnification times at observing are low enough. When the measuring yardstick, e.g. the radius of the dish, is replaced by r, the corresponding radius r is:

$$R = R_o r_o / r,$$

then $r = R_o r_o / R$,

and the relation between N and R can be expressed as:

$$N = C(R_o r_o)^{-D} R^D = C' R^D, \tag{2}$$

where C' is also a constant, which equals to $C(R_o r_o)^{-D}$. If $R=R_o$ and $r=r_o$, then Eq. (2) is equated to Eq. (1).

Similarly, if a constant yardstick r_o is used to measure the fracture surface of specimens (Fig. 2) along the track of the crack in crack expanding direction, the number N is:

$$N = C' H^{Dc},$$

where the fractal dimension Dc is not of the macro fracture surface, but of the total crack structure formed during the whole fracture process.

In order to calculate exactly the real length (H_r) of the fracture surface

from the height of ligament H, the least yardstick r_o should be used which equals to 2b (Hornbogen, 1989). Here 2b represents twice of the atomic space length.

$$H_r = N\, r_o = C'\, r_o\, H^{Dc} = C''\, H^{Dc},$$

where $C''=C'r_o=2C'b$. Then A_r, the real area of fracture surface, can be calculated from the obtained H_r :

$$A_r = BH_r = C''\, H^{Dc}B,$$

where B is the thickness of specimen.

Assume the surface energy of concrete to be γ_s, then the total energy W consumed during the whole fracture process can be expressed as follows:

$$W = 2\gamma_s A_r = 2C''H^{Dc}B\gamma_s \ .$$

According to the definition of A (area of fracture section) and G_f (fracture energy) by TC50-FMC, then:

$$A=BH$$

$$G_f = W/A = 2\gamma_s C''H^{Dc-1} \qquad (3)$$

It can be seen from Eq. (3) that G_f is influenced by the following factors:

1. Surface energy of concrete γ_s: fracture energy increases with the increase of surface energy.
2. Fractal dimension D_c of concrete micro crack zone: G_f increases as D_c increases. The value of D_c is related to the size of the micro crack zone, the density and tortuosity of cracks, the size and shape of specimen and the pattern of loading.
3. Ligament height H of specimen: G_f will be greater if H is higher, but the relation between them is not linear.
4. C'': It is a factor relating to the least yardstick used to measure the area of fracture surface. The calculated value of G_f is a function of the least yardstick.

1.2 Size effect of G_f

The relation between G_f and H, as shown in Eq. (3), indicates the size effect of G_f. It can be changed into a differential form:

$$dG_f/dH = 2\gamma_s c''(Dc-1)H^{Dc}/H^2 \quad (4)$$

If the material is an ideal elastic and brittle body, the fracture surface would be a plane, that is, Dc=1 and $dG_f/dH=0$. It means that G_f of such a material does not relate to H and has no size effect. But in cement-based materials there is always a micro crack zone at the tip of the main crack. Therefore, $1 < D_c < 2$ and $dG_f/dH > 0$, showing that G_f is an increasing function of H. D_c increases with the size of micro crack zone, the density and tortuosity of micro cracks in the zone. This will lead to a more significant size effect.

If D_c is of a great value, H^{Dc}/H^2 would be very small because $1 < D_c < 2$, so that dG_f/dH would be small too. It means that if the size of specimen is great enough, the size effect of G_f can be ignored.

From Eq. (3) the size effect caused by the thickness B of specimen can also be deduced. The relation between B and G_f is similar to that of H and G_f.

D_c can be regarded as an index of the significance of the size effect of G_f. If D_c is known, G_f can be well calculated.

1.3 Fractal dimension D_c of micro crack structure

The logarithmic form of Eq. (3) is

$$\lg G_f = \lg(2\gamma_s C'') + (D_c - 1)\lg H \quad (5)$$

so D_c-1 can be obtained by regressing in $\lg G_f$-lgH coordinate and D_c got. The intercept S of straight line can be expressed as:

$$S=(G_f)_{H=0}=\lg(2\gamma_s C'') . \quad (6)$$

Since C'' is a constant, γ_s can be evaluated if S has been obtained. The higher the intercept is, the greater the surface energy. Wittmann (1991) has recommended a method to calculate the surface energy of concrete material by expolation:

$$\gamma_s=g(a)_{H=0} , \quad (7)$$

in which g(a) is a distribution function of G_f along the whole height of fracture ligament. It can be seen that the methods to evaluate γ_s by Eq. (6) and Eq. (7) are similar.

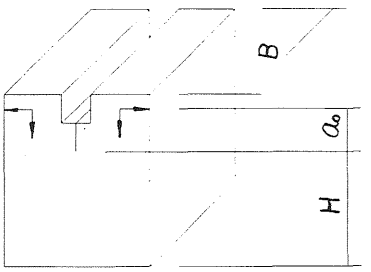

Fig. 3 Sketch map of specimen parameters

2 Experimental test

2.1 Size effect according to the height H of ligament

2.1.1 Specimens and concrete mixture
Fig. 3 shows the geometric parameters and loading pattern of the fracture specimen. Concrete mix proportions are listed in Table 1.

Table 1. Mix proportion and compressive strength of concrete

No.	Materials (kg/m^3)							28d Compressive strength (MPa)
	W/C	Enhancer	Cement	Water	Sand	Gravel	NF*	
1	0.45	----	450	203	575	1222	----	44.9
2	0.30	----	550	165	541	1264	55	66.8
3	0.30	55	495	165	541	1264	55	74.4

*NF is a brand of superplasticizer.

2.1.2 Obtainment of different height of ligament
Two methods were used to obtain different H:

1. With a constant W($W=H+a_0$) of 18.45 cm, original crack length a_0 was set at the value of 3.95 cm, 6.65 cm, 8.45 cm, and 10.3 cm for each group of specimens, and the corresponding H was 14.5 cm, 11.8 cm, 10.0 cm and 8.15 cm respectively. The concrete mixture is as No. 2 in Table 1.
2. With a constant a_0 of 3.95 cm, H was set at the value of 14.55 cm, 12 cm, 10.0 cm, 8.0 cm, 6.0 cm and 4.0 cm. The concrete mixtures are listed in Table 1. as No. 1, 2 and 3.

2.2 Experimental determination

P-COD curves at different H are shown in Fig. 4. Fig. 5 shows the G_f~H relation regressed in lgG_f-lgH coordinate. Some parameters of the two kinds of specimens with different H are listed in Table 2, where K is the slope, S the intercept, r the correlation coefficient, and D_c the fractal dimension.

Table 2. Parameters of two kinds of specimens with different H

Type of specimens	K	S	r	Dc
1. set W, change a_o	0.4406	1.9534	0.9787	1.4406
2. set a_o, change W	0.4950	1.9089	0.9950	1.4950

In Table 2, D_c of Type 1 is smaller than that of Type 2. It indicates that the size effect of G_f of the latter is much larger. The parameters listed in Table 2 also show that the size effect relates significantly to the shape of specimens.

2.3 Size effect of G_f of Type 2

Concrete mixtures are listed in Table 1 as No. 1, 2 and 3. The relation between fracture toughness K_{IC} of the concrete with different W/C and height of ligament H is shown in Fig. 6, indicating that the lower the W/C, the greater the compressive strength and K_{IC} of concrete. With the same W/C, the concrete with strength enhancer has higher compressive strength and fracture toughness K_{IC} . Fig. 7 shows the regressed relation between

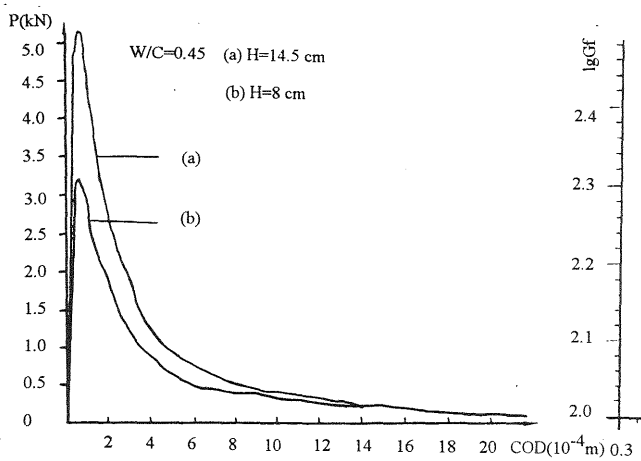

Fig. 4 P-COD curves at different H

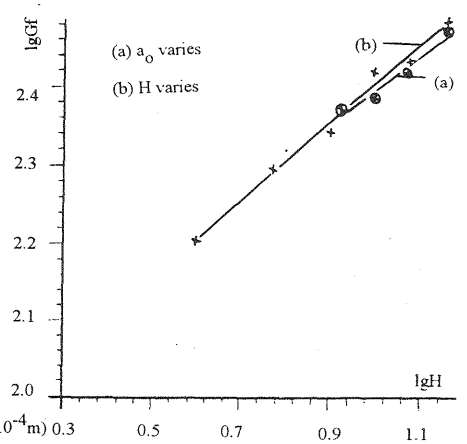

Fig. 5 lgGf versus lgH of concrete with different shapes

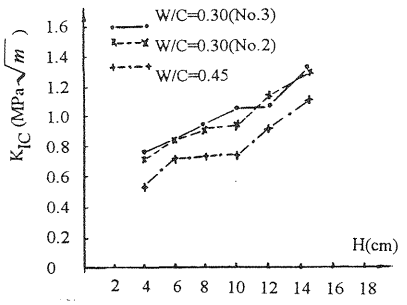

Fig. 6 K_{IC} of concretes with different W/C and ligament height H

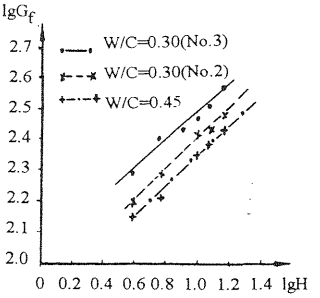

Fig. 7 Regressed lgGf~lgH of three kinds of concrete

lgGf and lgH of three kinds of concrete. The parameters such as K (slope), S (intercept), r (correlation coefficient) and Dc (fractal dimension) of the three straight lines in Fig. 7 are listed in Table 3.

Table 3. Parameters of concrete with different W/C

W/C	Type of concrete	K	S	r	Dc
0.3	reference concrete	0.495	1.9089	0.995	1.495
0.3	concrete with enhancer	0.4532	2.031	0.993	1.4532
0.45	reference concrete	0.5039	1.855	0.984	1.5039

Table 3 shows that if the concrete compressive strength increases, the value of Dc decreases. It also indicates that the strength enhancer causes induction of Dc. Surface energy γ_s is raised with improvement of concrete compressive strength. Dc shows the influence of micro crack zone upon the deterioration of concrete. The greater the Dc, the greater the density and tortuosity of micro crack zone and the more damage of the concrete structure during the whole fracture process.

2.4 Influence of B upon Gf

The concrete mix proportion is listed in Table 1 as No. 1. The thicknesses B of specimens were set at the value of 4.5 cm, 7.2 cm, 10.0 cm and 20.0 cm, and the other size parameters were set as follows: a_o=3.95 cm, W=18.45 cm, H=14.5 cm. The relation between K_{IC} and B is shown in Fig. 8, CTODc (critical crack tip opening displacement) and B in Fig. 9, Gf and B in Fig. 10, and a corresponding regression line obtained in lgGf-lgB coordinate in Fig. 11.

The following conclusions can be drawn from Figs. 8~11:

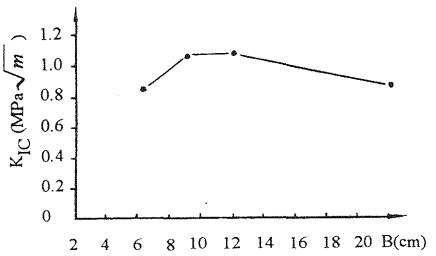

Fig. 8 K_{IC} versus B of concrete

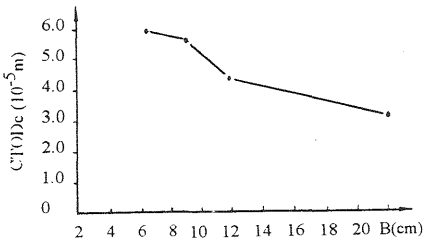

Fig. 9 CTODc versus B of concrete

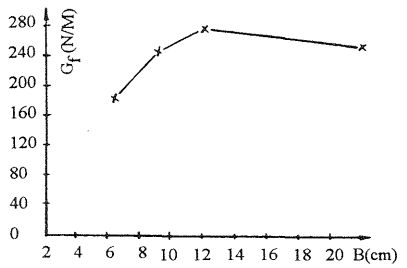

Fig. 10 Gf versus B of concrete

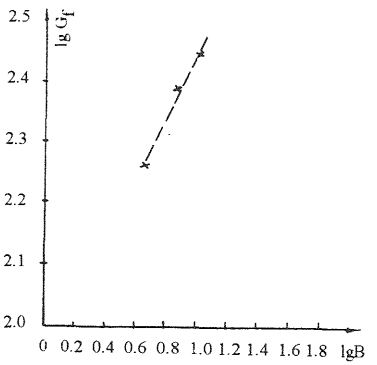

Fig. 11 Regressed lgGf–lgB of concrete

1. The influence of B on K_{IC} is not marked. K_{IC} increases with B at first, but when B is larger than a certain level, K_{IC}-B curve goes down.
2. The value of $CTOD_c$ decreases as B increases. The increasing of B leads to greater brittleness of concrete and smaller value of $CTOD_c$ when the crack is in a destablized propagation.
3. The relationship between G_f and B is similar with the one between K_{IC} and B. But if B > 10 cm, G_f increases with the increase of B. When the thickness of specimen is larger than a certain value, the rupture of concrete is more like the plain strain fracture, and leads to low values of G_f and K_{IC}.
4. In Fig. 9, the points at B=4.5 cm, 7.2 cm and 10.0 cm, are regressed to be a straight line with r=0.993, S=1.918 and Dc=1.536. But the point at B=20 cm is far away from the regression line. It proves that only when B is in a certain range, is Eq. (3) a effective expression for the plain stress fracture of concrete.

3 Conclusions

The basic reason for the size effect of G_f is that there is always a micro crack zone at the tip of the main macro crack during the whole fracture process. At present, the definition of the area of fracture surface by TC50-FMC is obviously too small. The fracture surface area should include the area of the total crack surface produced during the whole fracture process, especially the area of the great amount of micro cracks. The structure of the micro crack zone should include the size of micro crack zone, the density and tortuosity of micro cracks.

With fractal theory, the structure of micro crack zone is simplified to a dendritic structure, and its fractal dimension obtained.

In this paper, the most essential formulae of fractal theory were studied from another angle. According to the definition of G_f by TC50-FMC, a formula was deduced to express the relation between G_f and H, γ_s and D_c (see Eq. (3)). Theoretical analysis showed that G_f is an increasing function of γ_s, H and D_c. If the value of H is large enough, the size effect of G_f can be ignored.

The formula which describes the relation between G_f and the geometry parameters was also proved by experiment. The results show that:

1. The surface energy γ_s increases with concrete strength;
2. With the improvement of concrete strength, the structure parameter D_c in the micro crack zone decreases, and so does the magnitude of the size effect of G_f;
3. When the size of the specimen varies, the structure of the micro crack zone changes, and so does the value of D_c;
4. The thickness of the specimen also causes size effect of G_f.

4 References

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