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# NUMERICAL CONCRETE APPLIED TO INVESTIGATE SIZE EFFECT AND STABILITY OF CRACK PROPAGATION

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## Abstract

In this paper the numerical concrete approach is used to simulate crack formation and crack propagation in uniaxial tensile concrete specimens. In this model concrete is represented by three principal phases, each having its own mechanical properties. The mechanical properties of each phase are assumed to be statistical variables following Gauss's distributions. The influence of the length of the specimen on the global (macroscopic) mechanical properties is studied. It has been found that the macroscopic tensile strength decreases sligthly as the length of the specimen increases. The mean value of the tensile strength of the weakest phase i.e. the interface controls the global tensile strength of the composite system. This analysis shows that the ductility of the composite material is strongly dependent on the strength differences of the phases forming the system. Remarkable damage occurs all over the composite system prior to the formation of a narrow band of real cracks. The energy dissipated by microcracking before the maximum load is reached (pre-peak damage energy) increases with the length of the specimen. After a certain critical length of the specimen, the fracture process can not be controlled any more. The system then is unstable.

## 1. Introduction

Fracture process in concrete-like materials is a rather complex phenomenon. This complexity arises principally from the heteregeneous nature of the material. Consequently, a realistic study of the mechanisms of cracking necessarily must take into account the mesostructure of the material. In this way the interaction between the aggregates and the matrix can be considered on the basis of their different mechanical properties. Results obtained at the mesolevel, will serve as a basis for a better understanding and a more realistic description of the global behaviour (macrolevel) of the material. Different models taking the heteregeneous character of concrete into account have been developed in order to describe fracture. There are continuous models (e.g.: Podvalnyi (1974), Buyukozturk (1993)), lattice-based models (e.g.: Schellekens (1992), Rode (1993), Schlangen and van Mier (1993), Tsubaki and Abdeen(1994)) and stochastic models (e.g. Mihashi (1983), Zaitev (1993)).

Some years ago we developed an alternative approach, the numerical concrete (Roelfstra et al. (1985). This approach allows us to take the composite structure including the interface into consideration in a realistic way.

The main objective of the present contribution is to investigate, at the mesolevel, size effect and stability of crack formation in concrete specimens subjected to a uniaxial tensile load. By means of the numerical concrete, the internal structure of concrete is represented by a continuous system consisting of three different phases. The properties of the different phases as well as their distribution in a given volume can be adjusted to describe a special type of concrete.

## 2 Generation of numerical concrete

### 2.1 Simulation of the composite structure

In this numerical approach, concrete is considered as a composite material, composed of three major phases. The first phase, the dipersed one, represents the aggregates exhibiting a certain shape and size according to a predefined distribution. Maximum aggregate size (Dm) and volume content are two other parameters characterizing the first phase, as in the case of normal concrete, these values are 32 mm and 75 %, respectively. Because of the computer limits (mass storages and computing-time), only aggregates with a diameter larger than 4 mm are considered. The remaining smaller aggregates are incorporated in the second phase.

The aggregates are randomly dispersed in a binding matrix representing the second phase consisting on cement mortar composed of hardened cement paste and sand-grains having sizes smaller than 4 mm. In cement-based materials, during hydration a transitional zone around aggregates is formed. This zone has been studied in great detail by several authors (Maso (1982), Scrivener and Gartner (1988) ,Scrivener et al. (1988)). It has been found that this transitional zone is much more porous than the bulk matrix This zone plays a predominant role in the fracture process. In the numercial approach, this transitional zone is introduced as third phase. It consists on a thin layer of 0.86 mm thickness inserted between inclusions and the surrounding matrix. Actually an analysis in 3D of the proposed task seems to be impossible on the mesolevel because of limitations of avaliable computer facilities.

Therefore a 2D representation of the composite structure of concrete has to be simulated and subdivided by finite elements. First of all, a regular network consisting of equilateral triangles of 1 mm side length is generated. Second, knowing the 2D aggregate-size distribution and the aggregate concentration (from 4 to 32 mm), the number of aggregates having a diameter lying in the range [D(i-1),D(i)] can be determined. For the sake of simplicity, the inclusions are assumed like to be hexagonal. Thereafter, the inclusions are distributed inside the network by positioning their centres at random on the nodes of the network. By this adopted computer-generation technique, the corresponding finite element mesh of the composite structure is generated automatically. Finite elements are equilateral triangles of the same size with a node at each corner. Cross sections of the generated composite structures normal to the tensile loading direction are allways 3 times the maximum aggregate-size (3xDm). The length of the structure is variable. Fig.1 shows two computer-generated structures. In the upper part, the whole structure is a fully 3-phase material. For the sake of clarty, the finite elements representing the transitional zones are not represented, in the figure. They appear as "white channels". These structures are composed of ten-thousands of finite elements. Because of the computer limitations, for structures longer than 240 mm, another type of composite structure representation was chosen: the first 3xDm mm of the length of the specimen is regarded as 3-phase material, the length of the specimen is then varied by adding a homogeneous part, composed of coarser finite elements having effective properties of the composite material. In the lower part of Fig.1, a typical example is shown.

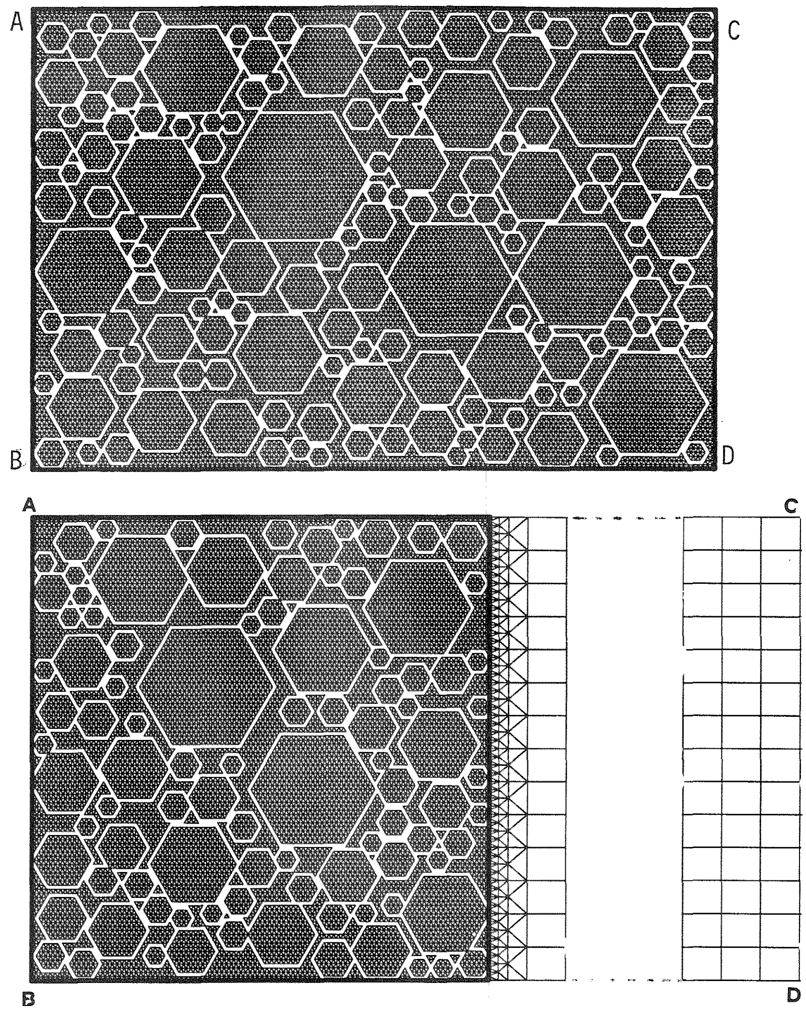



Fig. 1. Computer generated composite structures

## 2.2 Simulation of material properties

2.2.1 Aggregates

In the present work, normal concrete is considered. In this case, strength and modulus of elasticity of aggregates are much higher than those of the binding phase and of the transitional zone. Several experimental investigations have shown that cracking occurs predominantly along the interfaces and in the binding matrix. It can be said that the probability that an aggregate cracks is very low compared to the probabitlity of failure of the matrix or of the transitional phase. Owing of these observations, it has been assumed that the aggregates behave in a linear elastic way without the possibility to fail. During the numerical simulations of direct tension tests, it has been observed that tensile stresses inside an aggregate never exceeded 7 MPa, which can be considered to be a lower band of the tensile strength of aggregates used in normal concrete. A Young's modulus of 60000 MPa has been used in the analysis as determined experimentally (Wittmann et al. (1993)).

### 2.2.2 Matrix and transitional zone

Nowadays, it is accepted that cement-based materials cannot be considered as brittle materials, but rather as strain-softening materials. A specimen subjected to tensile stresses, will not fail suddenly if at a given finite volume the tensile strength of the material is reached. But deployment of microcracks will occur in this volume. A fracture process zone is then formed. Microcracking in this zone develops gradually as the imposed deformation increases. During this process, the microcracked zone is still a cohesive material capable to support tensile stresses, the material is in a softening state. Its tensile load-bearing capacity diminishes gradually as deformation increases. At a given threshold of the deformation, a real crack will appear in this volume; the tensile stress transfer capability then is zero. Among cohesive-crack based models existing in the literature, the so-called Fictitious Crack Model (FCM) developped by Hilleborg et al. (Hillerborg et al.(1976), Petersson(1981), Hillerborg (1983)) is the most suitable, because it describes in a realistic way the fracture process in these materials.

The matrix and the transitional zone when subjected to tensile stresses are assumed to behave in a linear-elastic way untill the corresponding predefined tensile strength is reached. At this tensile stress threshold, the behaviour of the material obeys a predefined strain softening law. Two principal techniques can be used to perform a numerical analysis based on the FCM : a discret crack approach and a smeared crack approach. In the present work, the numerical treatment of the fracture process of a composite structure subject to tensile load has been performed with the non-linear FE-package MARC (MARC (1994)). A special subroutine,TENSOF (MARC (1994)) is available in the package to handle the cracking process, the smeared-crack concept is adopted. It must be pointed out, that no "crack rotation" is permitted, i.e. if at a given Gauss point the principal stress exceeds a critical value (tensile strength) a crack initiates normal to the principal axis with an angle, this angle is held constant during the remaining part of the analysis.

A detailled description of this subroutine can be found in (Konter (1988). The material parameters needed are: Young's modulus, tensile strength, Poisson's ratio, fracture energy $G_f$, shape of the strain-softening diagram and a shear retention factor. The mortar matrix and the transitinal zones can never be considered as perfectly homogeneous systems. For this reason, in both phases, tensile strength and Young's modulus are assumed to be statistical variables. In each phase, the tensile strength distribution is assumed to obey a Gaussian distribution function with given mean value and a corresponding standard deviation. Young's modulus also follows a Gauss distribution, adequately correlated with the tensile strength distribution. In the present work, fracture energy is assumed to be constant. Experimental results show that fracture energy must also be considered as a statistical variable because with considerable scatter. But statistically speeking a well-defined correlation with the statistical distribution of other mechanical properties is not avaliable so far. A more realistic approach will be to introduce a statistical distribution for the fracture energy, without any correlation with distributions of the other properties.

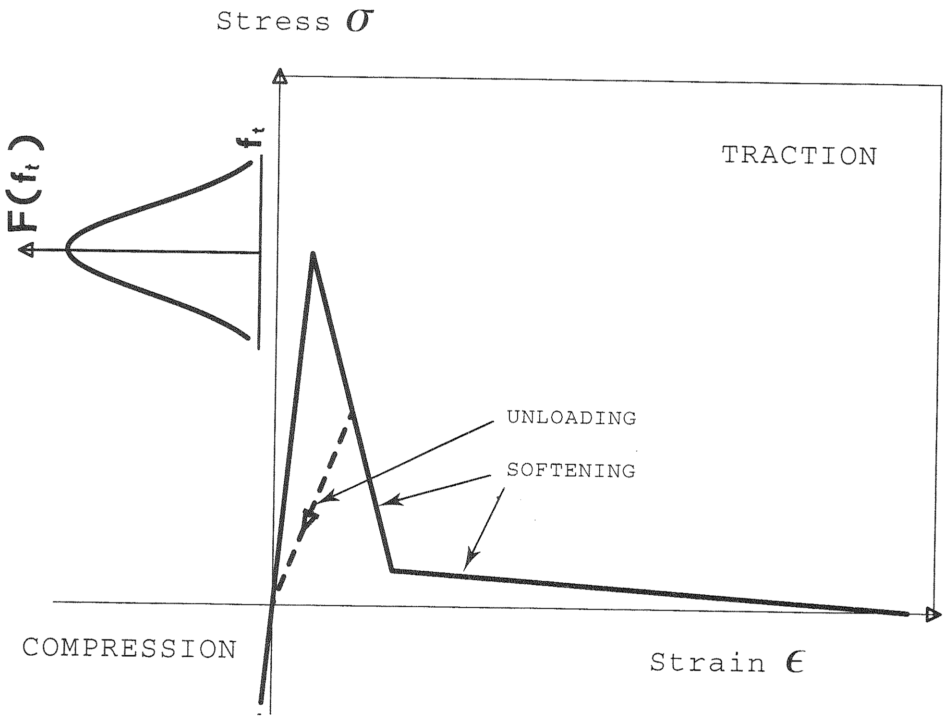



Fig. 2. Stress-strain relation and unloading branch

As it is shown in Fig.2, a bilinear approximation to the strain softening curve is adopted. The strain softening diagram is adjusted to the size of the finite elements to preserve the predefined fracture energy. Table 1 gives the mean values of the mechanical properties and their corresponding standard deviations (in parenthesis) used in the numerical simulation.

Table 1 : Mechanical properties of the phases

| Phases | Aggregate | Matrix | Transitional zone |
|---|---|---|---|
| $G_f$ (N/m) | - | 60 (-) | 30 (-) |
| $f_t$ (MPa) | 7 (-) | 4 (1) | 2 (1) |
| E (MPa) | 65000 (-) | 25000 (500) | 4...(1) |
| $\nu$ | 0.2 (-) | 0.2 (-) | 0.2 (-) |
| Shear ret. fact. | - | 0.0001 (-) | 0.0001 (1) |

## 3. Results and discussions

### 3.1 Evolution of the cracking process

Computer generated composite structures as shown in Fig.1 are loaded in uniaxial tension. Nodes lying along the side AB of the structure are fixed. Tensile loading is performed by increasing gradually the horizontal displacement (direction of loading) of the nodes situated on the CD side of the structure. In principle, with this simulated displacement-controlled test, it is possible to follow the post-peak response of the loaded system untill final rupture.

Fig.3 shows crack patterns obtained for a composite structure of 48 mm length. Patterns (a), (b) and (c) correspond to deformation steps of 0.0040, 0.010 and 0.014 mm, respectivelly. It must be outlined that in these figures "all cracks" are drawn, this means closed and opened cracks. As the load increases, some existing cracks close and other initiate in other regions. In patterns (a) and (b), all cracks are fictitious and not real. As it can be seen, at low load levels (Fig.3a), cracks occur predominantly in the interface "normal" to the direction of load. At higher levels, further cracks appear in the transitional zones some inclined with respect to the direction of load and other appear in the matrix trying to 'bridge' the damaged interfaces of closest aggregates. This crack-bridging effect is more pronounced in the crack pattern (c).

Fig.4 shows the evolution of real cracks at three sollicitation levels. No real crack appears under a deformation level of 0.01 mm. Even if at low load level a swarm of fictitious cracks invades the whole structure, (execpt inclusions) only few fictitious cracks will further develop to real cracks. As it can be observed in Fig.4, real cracks have a tendancy to localize in a narrow band, running around inclusions, perpendicular to the direction of load. The evolution of these bands of real cracks will finally form the real macrocrack provoking colapse of the structure.

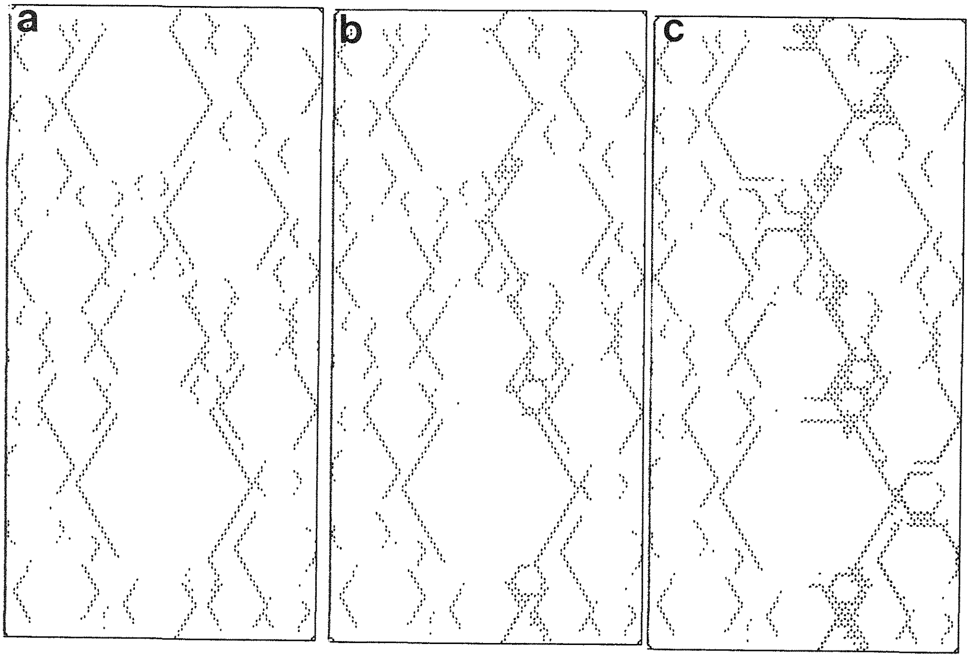



Fig. 3 . Crack patterns at three different displacement states.
(opened and closed fictitious cracks).

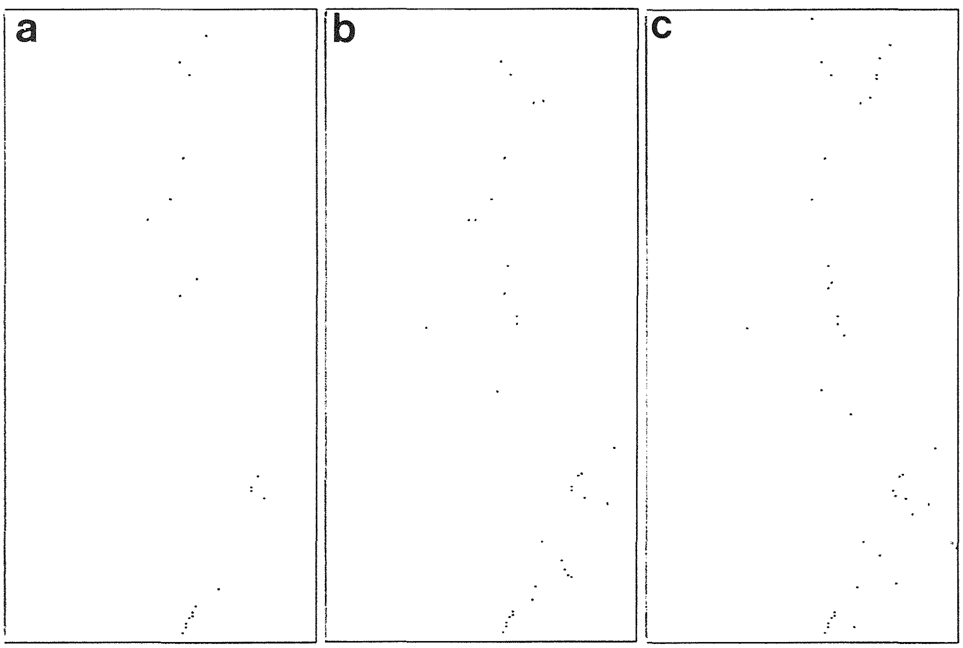



Fig. 4. Patterns of real cracks at three different displacement levels.

### 3.2 Load-displacement curves

Simulation of the behaviour under uniaxial tension if the test is run displacement-controled is carried out for composite structures with increasing length varying from 32mm (1xDm) to 240mm (7.5xDm). Fig.5 shows the load-displacement responses. As it can be seen in this figure, specimens shorter than 240mm behave in a stable way. But the slopes of the descending branches of the resulting load-displacement diagrams are getting steeper as the length of the specimen increases, in other words the composite structure looses apparently in ductility as its length increases.The slope of the post-peak regime obtained for the structure of 240 mm is nearly vertical. This means the system is close to instable failure. To find the critical length at which the structure looses its stability, the length of the structure must be further increased. Because of computer-limitation evoked in section (2.1), the composite-homogeneous structure (Fig.1b) with a length of 300mm is used. The corresponding load-diplacement diagram is shown (dashed curve) in Fig.5. The numerical tensile test can not be controled any more if the imposed displacement exceeds 0,0209 mm. This means that mechanical system has become instable.

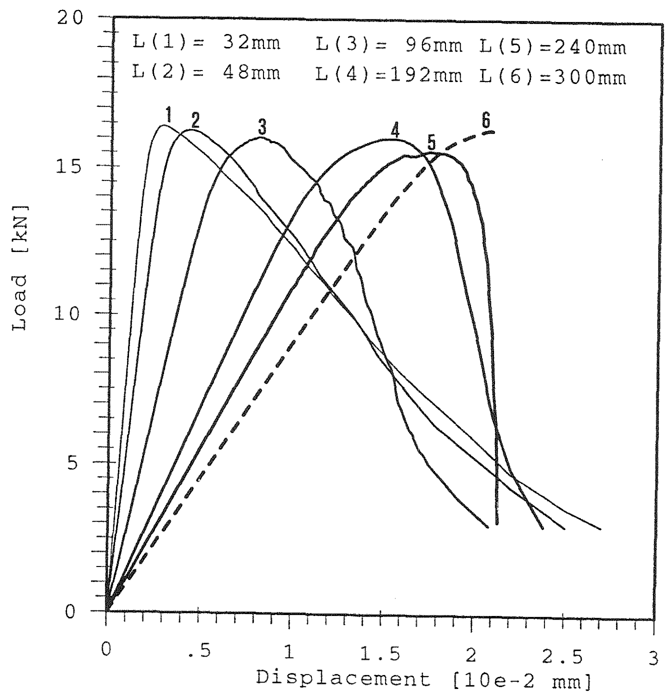



Fig. 5. Load-displacement curves of composite structures with length varying from 32 to 300 mm.

### 3.3 Effective mechanical properties

Fig.6 shows stress-strain diagrams corresponding to the load-displacement curves of Fig.5. As a first observation, we can state that the effective stiffness (slope of the linear part of the diagrams) of the reported curves is, at least in the prospected length range, a constant; with a mean value of 32100 MPa. The effective tensile strength of the composite structures can be defined to be equal to the maximum tensile stress of the $\sigma-\varepsilon$ diagrams. Fig.7 shows the resulting effective tensile strength as function of the composite structure length (form 32 to 240 mm). In accordance with Weibull theory and experimental results (Trunk (1995)), tensile strength decreases gradually as the length of the structure increases (depth and width of the structures remain constant).A curve fitting of the obtained numerical points leads to the following relationship :

$$(f_t/f_{to}) = (l_o/l)^{**}(1./49) \qquad (1)$$

where $l_o$ =32mm and $f_{to}$=2.064 MPa are used as reference values.
It can be underlined, that the statistical mean value (2 MPa) of the tensile strength of the weakest phase , the transitional phase, dictates the effective tensile strength of the composite system. The validity of this

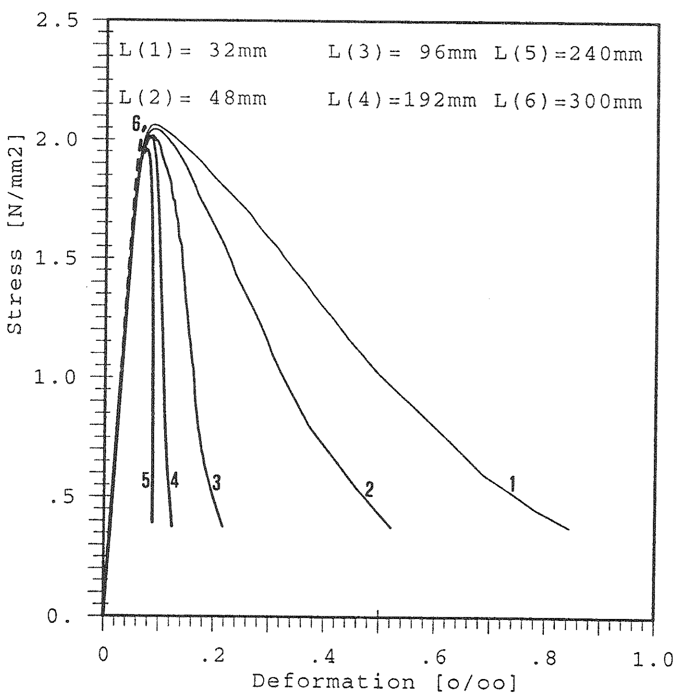
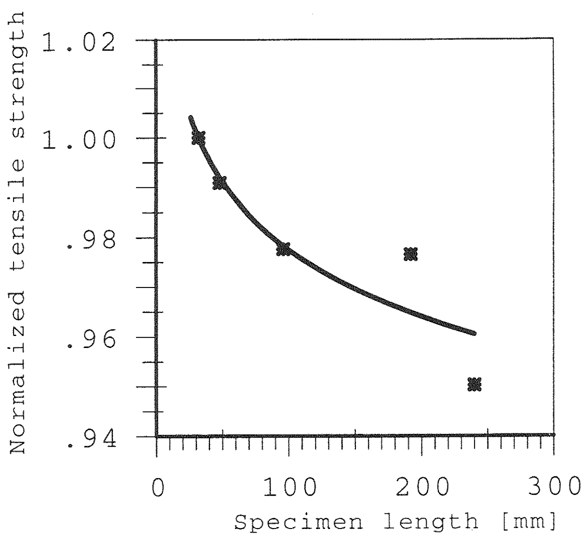



Fig. 6. Stress-strain diagrams corresponding to curves of Fig. 5.



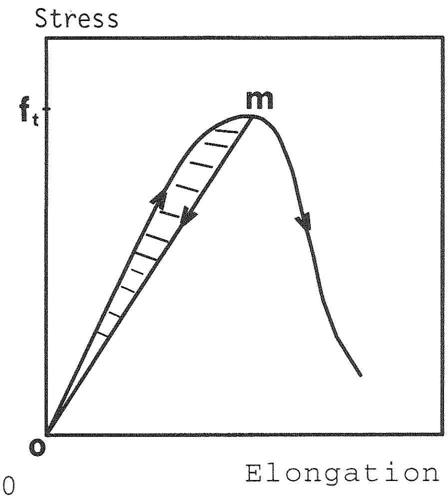




Fig. 7. Effective tensile strength of composite structures as function of the length.

Fig. 8. Typical strain-stress diagram of a softening material.

conclusion is confirmed by other numerical experiments in which the mean value of the tensile strength of the transitional phase has been varied. In Fig.8 a typical complete, stable stress-deformation curve of a specimen loaded in tension is shown. Line (m-o), shows the unloading branch if the specimen is unloaded from the point at which it reaches its maximum load (m). The area of the dashed surface shown in this figure is the energy dissipated in the whole system by microcracking, before a real crack begins to initiate. This irreversible energy loss, (called in this paper pre-peak damage energy) is consumed by the formation of fictitious microcracks created in the whole system in the pre-peak regime. This amount of energy should be distinguished from the so-called fracture energy ($G_f$), which is the energy needed for a formation of a real crack. Fig.9 shows crack patterns of of 3 composite structures with lengthes of 32, 48 and 96 mm loaded at their corresponding peak-levels. It can be observed, that the number of fictitious microcracks increases as the volume (length) of the structure increases. From the results of the numerical experiments, the pre-peak damage energy is computed for each specimen length and shown in Fig.10. It is clear from this figure that this energy increasies with the volume (i.e. length) of the structure.
This function can be described by the following relation :

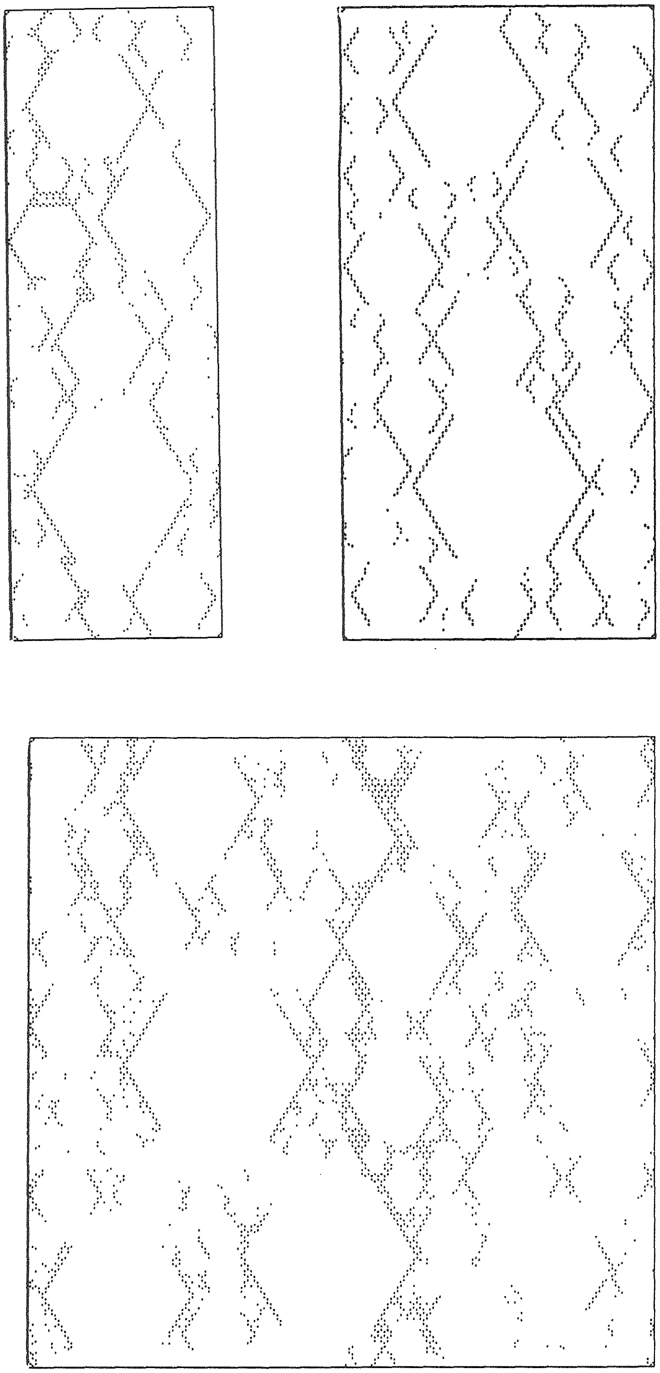


Fig. 9. Patterns of fictitious and real cracks (in opening or closing state) of 3 composite structures with different lengths. The load level corresponds to the peak of the load-dsplacement curves of Fig. 5.

$$y = 0.12\ x \tag{2}$$

in which, y [Nm] is the pre-peak damage energy and l [mm] stands for the length of the composite structure.

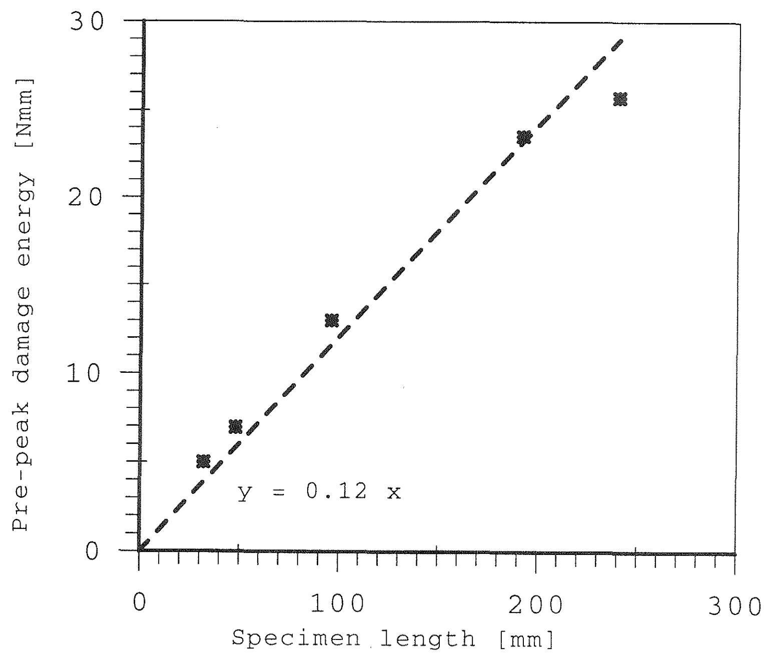



Fig. 10. Pre-peak damage energy as function of the specimen length.

**3.4 Influence of tensile strength on ductility**

Other numerical experiments have been performed by using an other mean value i.e. 3.5 MPa for the normal distribution of the tensile strength of the transitional zone. The other mechanical properties are kept constant. The stress-deformation curve obtained for the composite structure of 96mm length is shown in Fig.11. For comparison, the corresponding curve for a structure with a mean tensile strength of the interface of 2 MPa is drawn on the same figure. It can be observed, that in both cases, the effective tensile strength of the composite material roughly equals the mean value of the tensile strength of the weakest zone .

Fig.11 shows a remarkable difference in the behaviour of the two composite structures. The second composite structure has a much higher effective tensile strength than the first one, but its ductility is strongly reduced. An important result which comes out of this study is that the ductility of multi-phase materials is governed, among other factors (e.g.: roughness of inclusions (Sadouki and Wittmann (1988)), by the strength differences of the phases forming the material. The higher the difference the higher is the ductility. This can be explained as follows : cracks occuring in the weakest phase will be arrested if they are running into a stiffer phase, more external energy is required to further propagate them.

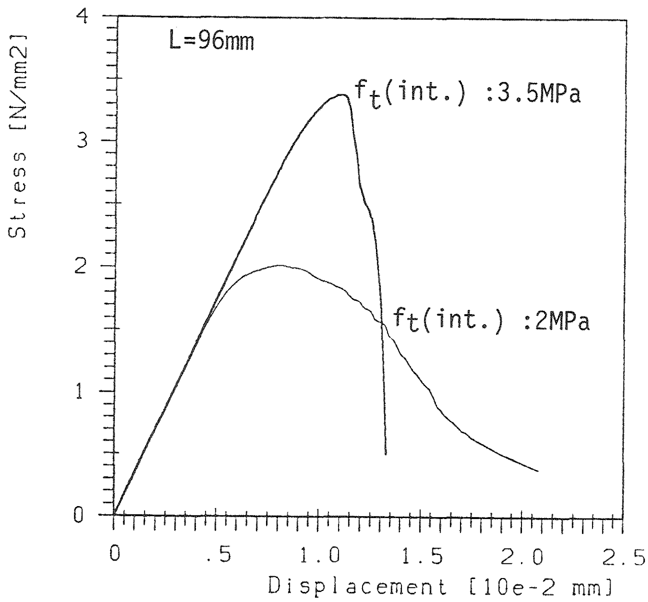



Fig. 11. Stress-displacement curves of two composite structures with a length of 96mm (3Dm). The tensile strength of the interfacial zone is 2 MPa (left) and 3.5 (rigth). The other mechanical properties are unchanged.

## 4 Conclusions

The composite nature of concrete-like materials can be simulated numerically in a realistic way, if the matrix, the aggregates and the interface are taken into consideration properly. This numerical model is called numerical concrete.

Ductility of a composite material depends on the ratio of the two major phases but it also depends severely on the strength of the interface.

If the length of a numerical concrete element is increased the resulting tensile strength decreases in agreement with experimental findings. This strength follows the prediction of Weibulls theory.

The pre-peak damage energy increases as the volume of the specimen increases.

Beyond a critical length of the specimen the system fails in a unstable way. This critical length depends on the material parameters of non-linear fracture mechanics. In unstable cracking conditions a loaded system can not benefit of fracture energy.

In any size effect law the transition from stable to unstable crack formation has to be taken into consideration.

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