

## **A STUDY ON APPROXIMATION METHOD OF TENSION SOFTENING CURVE OF STEEL FIBER REINFORCED CONCRETE**

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### **Abstract**

To determine the tension softening curve of steel fiber reinforced concrete (SFRC), direct tensile tests were performed. From the results of these tests, bilinear model was assumed for the tension softening curve. Using this, a non-linear finite element method analysis was performed and experimental results of the bending test were closely reproduced. Then, a method was proposed for determining the tension softening curve using the results of the bending test. This method focuses on the correlation between the experimental load-displacement curve and the tension softening curve, and derives the tension softening curve from the equilibrium equations for the crack cross section.

### **1 Introduction**

Compared to plain concrete, steel fiber reinforced concrete (SFRC) has much higher fracture toughness. The high toughness of SFRC comes from the fact that the steel fibers transmit the tensile stresses across the cracked surface and prevent the crack from opening. Therefore, to take full advantage of this property of SFRC, it is necessary to determine the

transmitted stress after the initiation of cracks. The relation between the transmitted stress after crack initiation and the width of the crack is known as the tension softening curve, and is an important parameter in fracture mechanics for concrete. Methods that have been presented for determining this curve include the direct tensile test method (Wang et al., 1990), methods using back analysis to find the optimum parameter from experimental results such as bending tests (Nomura et al., 1990), and methods based on the J integral (Li et al., 1987; Uchida et al., 1991). The research described in this paper involves the use of the direct tensile test for determination of the tension softening curve. To evaluate the method, four point bending test simulation was performed using the non-linear finite element analysis, and a comparison made with experimental results. Furthermore, from the comparison of the results of the tensile and bending tests a new method for determining the tension softening curve was produced.

## 2 Experimental procedure for the test

### 2.1 Specimen preparation

The fibers used for the specimens were deformed fibers, and were 30mm long and 0.6mm nominal diameter. Three kinds of fiber volume ratio ( $V_f=0.5\%$ ,  $1.0\%$  and  $1.5\%$ ) were prepared. The SFRC was made using ordinary Portland cement and its water-cement ratio was 50.3%. Taking into account the length of the fibers, the maximum aggregate size was set at 20mm. The specimens were cast in steel molds with inner dimensions of 10 x 10 x 40 cm. After curing, specimens were removed from the curing tank the day before testing. Prior to testing, 5cm was cut from each end of the specimens resulting in a 30cm specimen length. A 10mm deep notch was cut into two faces of the specimen in the middle cross section to enable measurement of crack mouth opening displacement and control the stable test.

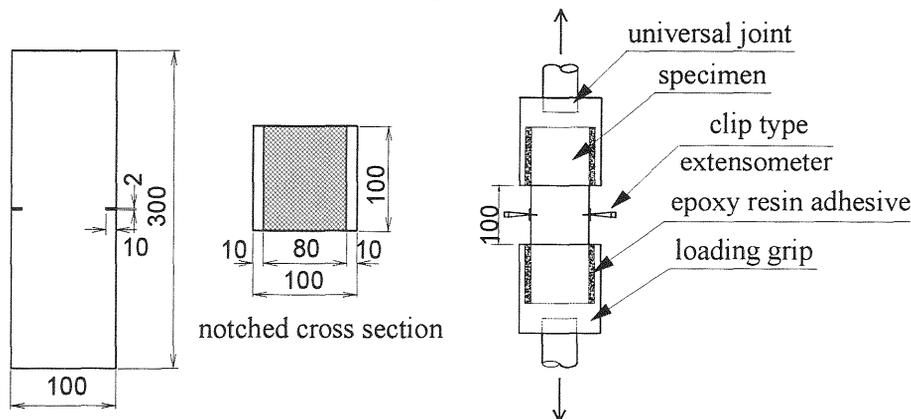


Fig.1. Dimensions, mm, of specimens and details of the loading fixture

## 2.2 Test procedure

The direct tensile test was performed with reference to Wang et al. (1990) as shown in Figure 1. Loading grips were attached to the both ends of each specimen using epoxy resin adhesive, and the specimen mounted in the test machine with universal joints. Load was applied controlling the displacement speed of the crosshead of a universal testing machine. A clip type extensometer was placed across the notch to measure crack opening displacement. After completion of the test, the area of the fracture cross section was measured using a vernier calliper and the tensile stress calculated. Three test specimens were used for each test.

For comparison purposes four point bending tests were performed with a notched beam which dimensions were 15 x 15 x 53 cm (span was 45cm). A notch 10mm deep was cut in the bottom surface of the specimen in the middle cross section and the crack opening displacement was measured using a clip type extensometer placed across the notch.

## 3 Test results

The typical results of the direct tensile test are shown in Figure 2. From the figure it can be seen that in all cases, after an initial steep drop following the maximum value of transmitted stress, the slope of decreasing stress becomes less inclined. However, the rate of decrease is different depending upon the fiber volume ratio. To be specific, the smaller the fiber volume ratio, the larger the drop in transmitted stress after the peak value. Based on the results of these tests, a bilinear model was applied that resembles the relation between the transmitted stress after crack initiation and the crack mouth opening displacement, in other words, the tension softening curve (Figure 3). The bilinear model can be represented by the three factors of initial softening point  $f_t$ , the break-point  $\gamma f_t$ , and the gradient  $\lambda$  following

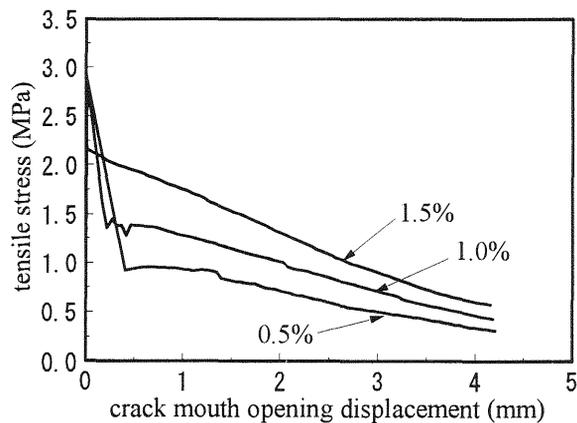


Fig.2. Experimental results of direct tensile test

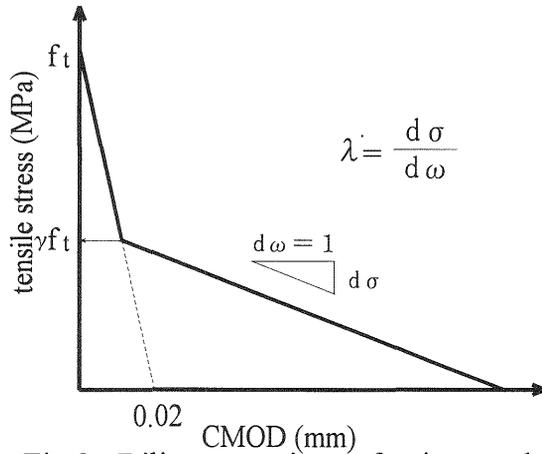


Fig.3. Bilinear tension softening model

the break-point. However, the initial gradient of the tension softening curve (the gradient between the initial point and the break-point) is considered to be the same as that for plain concrete. As the opening displacement is very small at the stage prior to commencement of softening, it was ignored when making the tension softening curve model.

When uniaxial tensile stress acts on the SFRC specimen, the transmitted stress increases linearly before crack initiation. As the stress reaches its maximum value, transition to the tension softening condition occurs. This initial softening point is taken as the tensile strength. According to Rokugo et al., (1993), the break-point of the tension softening curve is the limit of the resistance of the concrete matrix to the tensile stress, after which the steel fiber reinforcing comes into effect. Therefore, when evaluating the performance of SFRC in terms of the fracture toughness after crack initiation, rather than the value of tensile strength  $f_t$ , it is the break-point strength  $\gamma f_t$  that is important. The strength ratio  $\gamma$ , defined as the ratio between the tensile strength and the break-point strength, is shown in Figure 4 in relation to the compressive strength. The strength ratio  $\gamma$  is

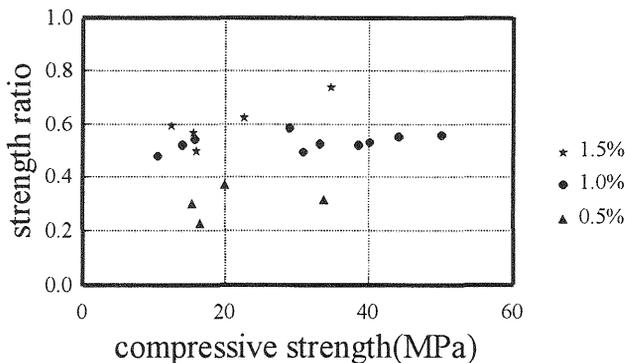


Fig.4. Strength ratio  $\gamma$  to compressive strength

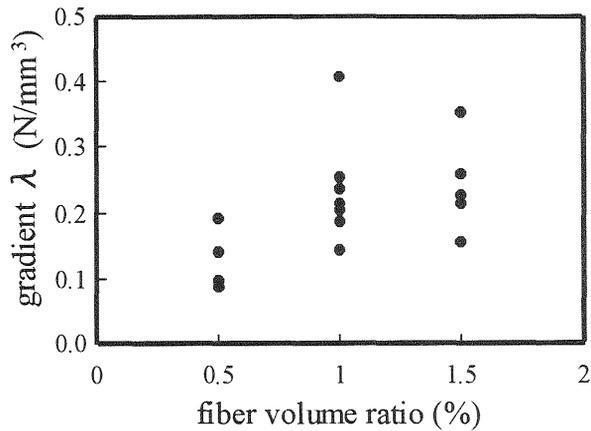


Fig.5. Gradient  $\lambda$  to fiber volume ratio

unaffected by the compressive strength, but depends on the fiber volume ratio.

The gradient  $\lambda$  following the break-point on the tension softening curve is also an important factor when considering the performance of SFRC. To be more specific, for the same value of break-point, the smaller the value of gradient  $\lambda$ , the larger the area under the strain softening curve (= fracture energy), therefore the larger the failure toughness. The value of  $\lambda$  is defined as the reduction in transmitted stress per 1mm crack opening displacement (see Figure 3), and the relation of  $\lambda$  to the fiber volume ratio is shown in Figure 5. The value of  $\lambda$  in the case of 1.0% fiber volume ratio is large compared with the 0.5% case. However, no appreciable difference was observed for the case of 1.5%.

Considering the experimental results in this section, when the fiber volume ratio is small, the drop in strength immediately following the commencement of softening is large and the gradient after the break-point is small. On the other hand, when the fiber volume ratio is large, the opposite tendencies are evident. For any given type of fiber, the end of the tension softening curve, or in other words the limit of crack opening width, can be considered the same, so the larger the fiber volume ratio, the larger the failure toughness.

## 4 Non-linear FEM analysis

### 4.1 Outline of the analysis

As a means of evaluating the SFRC tension softening curve obtained from the tests, an approach based on fracture mechanics was adopted. More precisely, a bending test simulation was performed using non-linear FEM incorporating the tension softening curve, to confirm the applicability. For the analysis model, a smeared crack model was used, and an orthotropic

body element was adopted for the crack initiated element. For the purposes of the analysis, the tension softening curve was treated as strain softening, and by using the principle of equivalent length described in reference (Cervenka et al., 1993), the size effect of the element was eliminated.

#### 4.2 Tension softening curve and stress-strain curve

The tension softening curve used for the analysis is the bilinear model shown in Figure 3. From the test results for 1.0% fiber volume ratio, the tensile strength was taken as  $f_t = 3.1(\text{MPa})$ , the strength ratio  $\gamma = 0.50$ , and the gradient after the break-point  $\lambda = 0.20 (\text{N/mm}^3)$ . In order to express the localization of the cracking, in cases where the displacement of the crack mouth opening decreases the opening width after crack initiation, an unloading path pointing to the origin was followed. For the compression side stress-strain relation, a quadratic stress-strain curve was used, as for plain concrete.

#### 4.3 Results of the analysis

The results of the analysis using the bilinear tension softening curve are shown in Figure 6. The full lines in the figure indicate the FEM analysis results, and the dotted lines, the four point bending test experimental results for a 1.0% fiber volume ratio. The load-displacement (crack mouth opening displacement) curve resulting from the bending test was well reproduced. That is to say, it was confirmed that a relatively precise tension softening curve was obtained from the direct tensile tests. However, the direct tensile test method does not always lead to stable results, and a certain amount of improvisation with the test equipment is also required. Therefore, it cannot be described as an easy test method, and a simpler means of determining the tension softening curve is necessary. For this reason, the method presented below was produced for the approximation of the tension softening curve from the load-displacement curve.

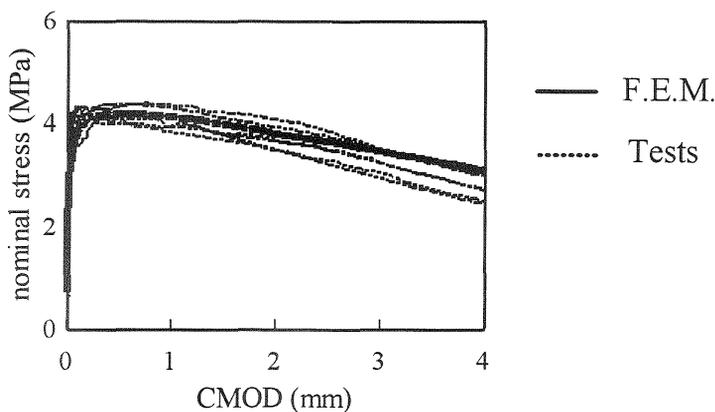


Fig.6. Bending test results & simulation on 1.0%  $V_f$

## 5 Proposed method for approximating the tension softening curve

Of the two lines making up the tension softening curve model, the first line, from the softening commencement point to the break-point, is mainly governed by the concrete matrix. The second line, running from the break-point, is the region where the reinforcing effect of the fibers is acting, and can be said to be the portion that contributes to the fracture toughness of the SFRC. Therefore, in determining the tension softening curve, it is the second line that is of prime importance. As was mentioned previously, the second line may be described by the break-point strength  $\gamma_f$  and the gradient  $\lambda$ .

During this current research, the values of both  $\gamma_f$  and  $\lambda$  were obtained from their relation with the tension softening curve and load-displacement curve.

### 5.1 Comparison of tension softening curve and load-displacement curve

A comparison was made of the tension softening curve obtained by experiment and the load-displacement curve. According to Nanakorn (1993), from test results obtained for 1.0% fiber volume ratio, the gradient of the line for the region of the load-displacement curve from the maximum load point to the point where the crack mouth opening displacement reaches 1mm, and the gradient after the break-point are linearly related. However, in cases where the maximum load is reached in the vicinity of 1mm crack opening displacement or after, or cases where there is a large drop between the first and second peaks, it is questionable as to whether the slope obtained truly represents the gradient of the load-displacement curve. Therefore, as shown in Figure 7, the slope  $\beta$  from the 80% to 60% second peak load point was taken to represent the gradient of load displacement curve. In this case, nominal stress (normalized moment) was made the vertical axis of the load-displacement curve. The results of the

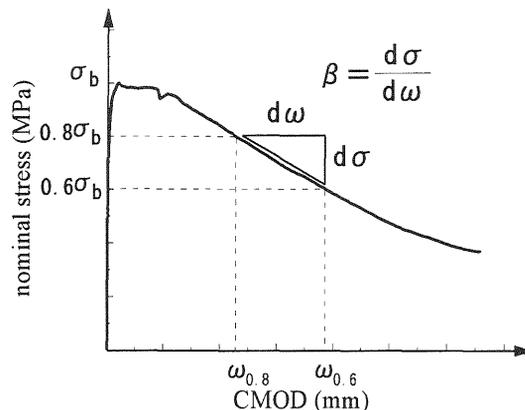


Fig.7. Definition of  $\beta$

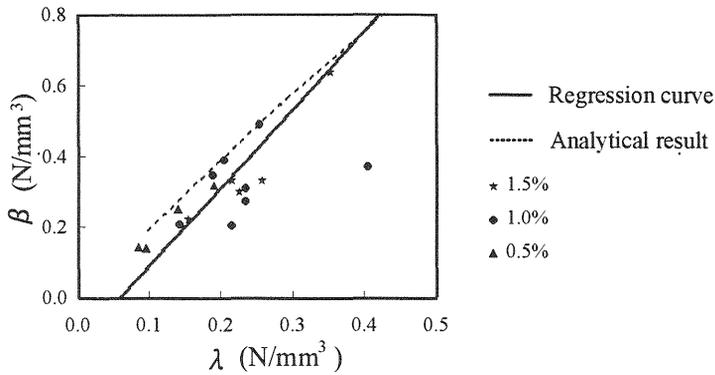


Fig.8. Relation between  $\beta$  and  $\lambda$

comparison of  $\beta$  with the gradient of the tension softening curve after the break-point  $\lambda$ , can be seen in Figure 8.

Calculating the correlation between  $\beta$  and  $\lambda$  using the method of least squares, resulted in the regression curve given by equation (1). The dotted line in the figure represents the results of the FEM analysis described in part 4 above.

$$\beta = 2.20\lambda - 0.13 \quad (1)$$

The correlation coefficient for this case was 0.915, indicating a strong correlation between  $\beta$  and  $\lambda$ . Using this relation, it is possible to obtain the value of  $\lambda$  from  $\beta$ .

## 5.2 Equilibrium equations for the crack cross section

For the four point bending test, the strain distribution for the cross section where the crack initiates was assumed to be linear (Nanakorn, 1993) above the end of the crack (compression side). A quadratic curve type stress-strain curve was used for the compression side stress-strain relation, the same as in part 4. For the tension side, a straight line stress-strain relation was used as given in reference (Nanakorn, 1993). The stress distribution for the crack portion was based on the second straight line of the tension softening curve, the first straight line being ignored. For the form of the crack, a straight line was assumed giving a displacement of 0 at the end of the crack (crack depth  $\alpha h$ ) and the crack mouth opening displacement at maximum. The stress-strain distribution for the above crack section are shown in Figure 9. The equilibrium equations for the forces acting on the cross section are equations (2) and (3) below.

$$M_{ud} = \int_{-h/2}^{h/2} \sigma(y) \cdot y \cdot b \cdot dy \quad (2)$$

$$N_{ud} = \int_{-h/2}^{h/2} \sigma(y) \cdot b \cdot dy \quad (3)$$

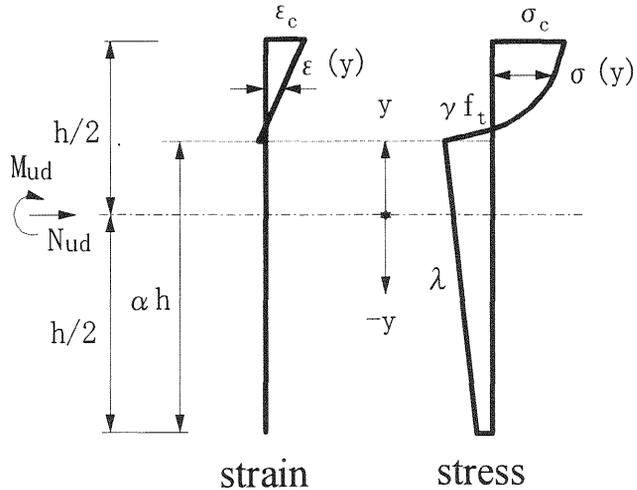


Fig.9. Stress and strain distributions

Where,  $M_{ud}$  and  $N_{ud}$  : the moment and axial tension acting on the center of the crack cross section

$h$  : height of the section,  $b$  : width of the section

$\alpha h$  : depth of the crack

The relation between the load and the displacement of the crack mouth opening is shown by a load-displacement curve. For a given opening displacement  $\omega$ , where load is  $P$ , the bending moment  $M_{ud}$  can be calculated from  $P$ . The axial stress  $N_{ud}$  is zero. That is to say, the unknown value in equations (2) and (3) above is the stress distribution  $\sigma(y)$ , which is represented by the compressive stress at the edge  $\sigma_c$ , the tensile stress at the end of the crack  $\gamma f_t$ , the crack depth  $\alpha h$  and the gradient  $\lambda$  of the tension softening curve. The value of  $\lambda$  can be obtained from the representative slope  $\beta$  of the load-displacement curve using equation (1). As the value of  $\gamma f_t$  is the analytical objective, it is necessary to decide on a value of either  $\sigma_c$  or  $\alpha h$  in order to solve the equilibrium equations and determine the stress distribution. Concerning the measurement of the crack depth  $\alpha h$ , a suitable method is unavailable, however, based on the assumptions of this section, the compressive stress at the edge  $\sigma_c$  can be evaluated from the compressive side edge strain  $\epsilon_c$ . The value of  $\epsilon_c$  can be measured directly using a strain gauge, which is easily handled. Consequently, by measuring the compressive edge strain along with the crack mouth opening displacement during the four point bending test, it is possible to determine the second line portion of the tension softening curve.

## 6 Summary

The following is a summary of this research.

1. The tension softening curve for steel fiber reinforced concrete (SFRC) was obtained from direct tensile tests.
2. Using non-linear FEM analysis incorporating the tension softening curve four point bending test was simulated, and close agreement with experimental results was achieved. In other words, a relatively accurate tension softening curve was achieved from direct tensile tests.
3. A method was presented for the approximation of the tension softening curve from the bending test results of the four point bending test. This method is used to determine the second straight line portion of the tension softening curve using the gradient of the load-displacement curve of the bending test, and the equilibrium equations for the crack cross section.

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