Fracture Mechanics of Concrete Structures,
Proceedings FRAMCOS-2, edited by Folker H. Wittmann,
AEDIFICATIO Publishers, D-79104 Freiburg (1995)

SIMILARITY LAWS FOR CONCRETE STRUCTURES UNDER DYNAMIC LOADING

J. Mazars, J. F. Dubé and S. Ghavamian
Laboratoire de Mécanique et Technologie et GRECO Géomatériaux,
Ecole Normale Supérieure, Cachan, France

Abstract

This paper proposes an overview of different aspects of the similarity problems and their applications to concrete structures under dynamic loading.

Useful for experimental studies on reinforced concrete structures, the applications presented concern a shaking table test on a one-third scale structure. Based on non linear simulation, an analysis of the behaviour of the prototype (scale 1) is deduced.

1 Introduction

The use of testing facilities to both characterise and analyse the response of concrete structures under dynamic loading such as earthquake, is in practice performed on reduced scale structures.

In that case, several problems have to be solved in order to obtain similar behaviours when the specimen tested is compared to the full scale structure under study.

The basic assumptions commonly adopted (Wang et al. 1993) for similarity laws are :

1 - same stress field,
2 - same acceleration.

They require adjustments on both the mass distribution and the time scale of the input signal.

But if, for the linear behaviour those assumptions are sufficient, it is not the case when non linearity occurs.
In concrete structures, non linearities are generally linked to the microcraking of concrete and the yielding of R-bars.

When damage occurs, there is strain localisation ; then to insure the same relative size of the localisation zones, it is necessary to adapt the concrete mixture of the model.

Such a problem is the concern of the French research program CASSBA (Bisch & Coin 1994) involving a shaking table test on a 1/3 scale structural wall building.

This paper proposed through the CASSBA case, an overview of different aspects of the similarity problems and, using both, experimental results and numerical simulations, an analysis of the relevance of the choices carried out.

2 Similarity framework

To realise a model at reduced scale with a good faithfulness, it is imperative to examine all similarity conditions which come from equations governing all the phenomena :

1. Dynamic equations
2. Constitutive equations for the material behaviour
3. Boundary conditions.

The first group, of equations constitutes the mechanical similarity, the second group the rheological similarity and the third one, the site similarity.

2.1 Mechanical similarity

Considering a given structure submitted to an acceleration $\Gamma(t)$; assuming at the first stage the linear elastic behaviour, the equation of motion is :

$$M\ddot{X} + C\dot{X} + KX = -\Gamma(t)\, Mu \qquad (1)$$

X =X (t) : displacement vector, is the solution of the equation (1),
M : mass matrix,

K : stiffness matrix,
C : damping matrix, its formulation is a problem, generally solved considering C as a linear combination of M and K.
u : unit vector in the direction of the loading at the locations where acceleration is imposed.

2.1.1 Similarity relations

In the following, the subscript p is used for the prototype and the subscript m for the model, the scale of which is $1/\alpha$; $\alpha>1$ is the reduction factor of all the dimensions of the prototype.

- Mass matrix

The assumption : "same stress field", implies that gravity loads, which act on surfaces, satisfy :

$$M_m = M_p / \alpha^2 \tag{2}$$

However if the dimensions verify the scale $1/\alpha$, the volume does it by the ratio $1/\alpha^3$. Then, to obtain $M_m = M_p/\alpha^2$ requires, either to change the specific mass, or to add masses.

- Stiffness matrix

The assumption "same stress field" and the one "same material behaviour" induce : "same strain field". Assuming a discretized representation of the structure, the classical relation between strain and displacement X at Gauss points is given by the matrix relation $\varepsilon = BX$ With the expression for stress, $\sigma = D\,\varepsilon$, the local stiffness matrix is : $K(X) = \int_V B^t D B \, dV$

$$X_m = X_p/\alpha \text{ induces } B_m = B_p\alpha \text{ and } dV_m = dV_p/\alpha^3 \text{, which implies}$$
$$K_m(X_m) = K_p(X_p)/\alpha \tag{3}$$

- Natural modes.

They come from the solution of the equation

$\|K - M\,\Omega^2\| = 0$ which gives n natural frequencies $1/\Omega_i$. From (2) and (3) it comes :

$$\Omega_{im} = \sqrt{\alpha}\,\Omega_{ip} \tag{4}$$

- Damping matrix.

As said before usually the damping matrix is formed using a linear combination of the mass matrix and the stiffness matrix :

$C = \alpha M + \beta K$; values of α and β are determined in order to find on the first 2 natural modes (frequencies $1/\Omega_1$ and $1/\Omega_2$) the reserved value for damping, ξ.

These considerations lead towards :

$$C = \frac{2\xi}{\Omega_1 + \Omega_2}[\Omega_1 \Omega_2 M + K]$$ Then (2), (3) and (4) implies :

$$C_m = \frac{1}{\alpha\sqrt{\alpha}} C_p \quad (5)$$

- Time scale :

From equation (1) we can write respectively for the prototype and the model :

$$M_p \ddot{X}_p (t_p) + C_m \dot{X}_p(t_p) + K_p(X_p) X_p(t_p) = -\Gamma_p(t_p) M_p u_p \quad (6)$$
$$M_m \ddot{X}_m (t_m) + C_m \dot{X}_m(t_m) + K_m(X_m) X_m(t_m) = -\Gamma_m(t_m) M_m u_m \quad (7)$$

One can notice that to satisfy the scale implies $u_m = u_p/\alpha$, then considering the similarity relations obtained before, the only way to make (7) consistent with (6) is to impose : $t_m = t_p/\sqrt{\alpha}$ (8)

- Conclusion : The mechanical similarity is respected if :

1 - masses are added in order to obtain $M_m = M_p/\alpha^2$ as the weight of the structure verifies the ratio $1/\alpha^3$ (assuming that densities of materials are the same) then $M_m = \frac{M_p}{\alpha^3} + m$, with $m = \frac{M_p}{\alpha^3}(\alpha - 1)$;

2 - the time scale is changed in order to obtain for the model $t_m = t_p/\sqrt{\alpha}$

The second point is easy to be satisfied, it consists in a "compression" of the accelerogram used as the load.

The respect of the first one is not so easy. For that the best would be to distribute the mass m on the whole structure which is equivalent to change the material density. The other way is to add these masses at the different floors but this unperfected distribution induces errors.

This point can be a problem in satisfying the similarity.

2.1.2 Toward a generalisation using forces similarities.

Considering equations (6) and (7) and the time "compression" (8), one can observe that different forces, inertia forces $M\ddot{X}$, damping forces $C\dot{X}$ and internal forces KX, verify the similarity ;

$$F_m = F_p/\alpha^2 \quad (9)$$

This must be considered as a rule for the general non linear case which implies at each step of the movement to ensure (9) and, as seen here after, what is called for the material behaviour, "Rheological similarity".

2.2 Rheological similarity

The main aspect usually considered is the mechanical behaviour of the material which must be the same for the model and the prototype. This means that at any time the irreversible processes, such as damage which affect the stiffness matrix, or plasticity which creates permanent strains, must have the same distribution. This implies to have between prototype and model :

- the same stress field
- the same strain field, including permanent and localisation aspects.

Then, for concrete like-material, full rheological similarity means to respect same behaviour, and to insure the similarity for strain localisation.

2.2.1 Size effect and non locality of damage.

It is now well-known that for a same material there are 2 phenomena which play a size effect role : one is linked to the volume and the other to the size of damage localisation (Mihashi, Otamura & Bazant 1994). It has been shown that both are mainly related to the heterogeneities size inside the material.

Bazant - Pijaudier-Cabot (1989) and later Berthaud et al (1994) have shown that the size of the localisation zones is more or less proportional to the size of biggest grains of the concrete mixture. Assuming that the volume effect is governed by the same kind of rule, 2 points must be satisfied by the model's concrete mixture :

- same behaviour as that of the prototype and,
- aggregates similarity (the size of the grains must satisfy the scale factor $1/\alpha$).

2.3 Site consistency

Boundary conditions play a fundamental role in the functioning of a structure under loading. Particularly, the seismic load is created by acceleration imposed on a system through its support; so the base of the structure is obviously very important, but what must be done to respect similarity ?

When this link is perfect for the prototype (structure embedded into the support) , the same conditions are required for the model.

When the link is imperfect, rotation and/or displacement, are then possible at the base of the structure.

The case of possible sliding (not very common) requires for similarity the satisfaction of the same rule for friction (same friction coefficient if Coulomb laws are assumed).

The case of rotation is more common on real sites (due to soil behaviour). On shaking table that kind of movement is possible when the structure is not fixed and when the loading is sufficient to create uplifts. This situation can be modellized as a local moment-rotation behaviour :

$$M = k\,\Theta, \qquad (12)$$

The "dimension" of M is a mass-dimension product. Then, there is a $1/\alpha^3$ ratio between model and prototype. To respect similarity, Θ must remain the same, so for $k = M/\Theta$ the $1/\alpha^3$ ratio is required.

3 Application to the CASSBA case

The LMT Cachan, within the GRECO Géomatériaux (a national research network), was involved in the French research seismic program CASSBA ("Conception et Analyse Sismique des Structures en Béton Armé", Mazars 1994a, Bisch 1994). Based on a shaking table experiment performed at CEA Saclay on an 8 storeys 1/3 scale model, the aim of the research was, 1) a better understanding of the behaviour of structural wall constructions, 2) an improvement of the non linear modelling of structures of such kind.

3.1 The CASSBA Experiment (scale 1/3) and its modelling

The experiment leads to large quantities of information (3 levels of loading, 120 channels recorded) and the main observations during and after the tests were :

- the major effect of the table-model contact (without any fixing) which allowed uplifts during the movement,

- the appearance of cracks mainly located at lower parts of the walls.

The analysis of results required a lot of work, in particular to estimate the local behaviours and their consequences on the response of the structure. To treat in deep these aspects a combination of experimental and numerical results has been necessary.

To simulate the non linear behaviour we have used a damage model (La Borderie et al. 1993), which incorporates two scale damage variables, one for damage due to tension D_1, the other for damage due to compression D_2, and which includes a recovery stiffness procedure and the description of anelastic strain :

$$\varepsilon = \varepsilon^e + \varepsilon^p = \text{elastic} + \text{permanent, strain} \quad (11)$$

$$\varepsilon^e = \frac{\sigma^+}{E_0\,(1 - D_1)} + \frac{\sigma^-}{E_0\,(1 - D_2)} + \frac{\nu}{E_0}(\sigma - \mathrm{Tr}\,\sigma\,\mathbb{1})$$

$$\varepsilon^p = \frac{\beta_1\,D_1}{E_0\,(1 - D_1)}\frac{\partial f}{\partial \sigma} + \frac{\beta_2\,D_2}{E_0\,(1 - D_2)}\,\mathbb{1}$$

E_0 is the initial Young's modulus, ν is the Poisson ratio.
σ^+ and σ^- are respectively the "traction-tensor" and the "compression-tensor". D_1 and D_2 are respectively the damage variables of traction and compression. β_1 and β_2 are constants and $f(\sigma)$ allows to manage the closure of cracks (La Borderie al. 1993).

The discretization used is based on a multilayered description with 27 beams elements(or 54 in a 2nd stage) of 18 layers (reinforcements are located on layers on each side of the model and the lowest element is "semi-rigid" Fléjou 1993) , see figure 1. In the standard version each layer behaves uniaxially, but in a more recent version the addition of shear effects allowed to simulate the distortion of plane section (Crisfield 1984, Dubé 1994). For the CASSBA structure it was shown that these effects are minor.

The dynamic calculation (code EFiCoS - La Borderie 1993) uses a Newmark implicit algorithm (Bathe and Wilson 76) . The material parameters used were determined from characteristics measured before the test : f_c = 34 MPa, f_t = 3 MPa, E_c = 32000 MPa for concrete and $E_s = 2\ .\ 10^5$ MPa, f_e = 496 MPa for steel. Based on the response of the structure under free vibrations, a structural damping ξ = 5% is considered.

3.2 Identification procedure for the connection modelling

The structure has been tested with 3 different levels of the same accelerogram, which corresponds respectively to a maximum acceleration of 0.1g - 0.36g - 0.5g. An important effect on the global behaviour of the contact structure-support was observed. It is due, 1) to the fact that the connection was without any fixing, which allowed uplifts, 2) to the damage of the base of the structure due to previous loading related to transportation (the model was built out of the shaking table).

A preliminary calculation performed using a 3D F.E. model (with contact elements at the base to simulate the connection) in order to deduce the Moment - Rotation behaviour of the connection, leads to unrealistic simulations, particularly at the lowest level 0.1g (Mazars et al. 1994b).

This confirms that transportation had introduced a permanent curvature of the footing, observed and measured during the test, which changes a lot the table-model contact conditions.

To model as well as possible the contact, the only solution was to identify directly from the real response of the structure. An automatic procedure of identification was used to obtain the best adapted parameters to model the connection behaviour. This procedure, developed at LMT Cachan (code SIDILO, Pilvin 1983), is based on a combination of 2 orthogonal "cost functions" able to represent the distance between an experimental curve and its simulation for a given set of parameters. The best set of parameters is that which insure the minimum values for the cost functions.

For CASSBA different strategies of identification have been used, depending on the constitutive relation used for the connection (linear, multilinear,...), depending on the data base considered (displacement at the top for 1,2 or 3 levels of loading). See Mazars, Dubé et al (1994a, 1994b) to have more details on that.

Because it is sufficient for our presentation, we have chosen here to work with the simplest moment-rotation model, a linear elastic one ($M=k\Theta$, with K=Cst, see Dubé 1994) identified on the response at the level 0.1g. It gives good simulations for other 2 levels (see figure 1 for 0.5 g).

3.3 Simulation of scale 1 using similarity laws

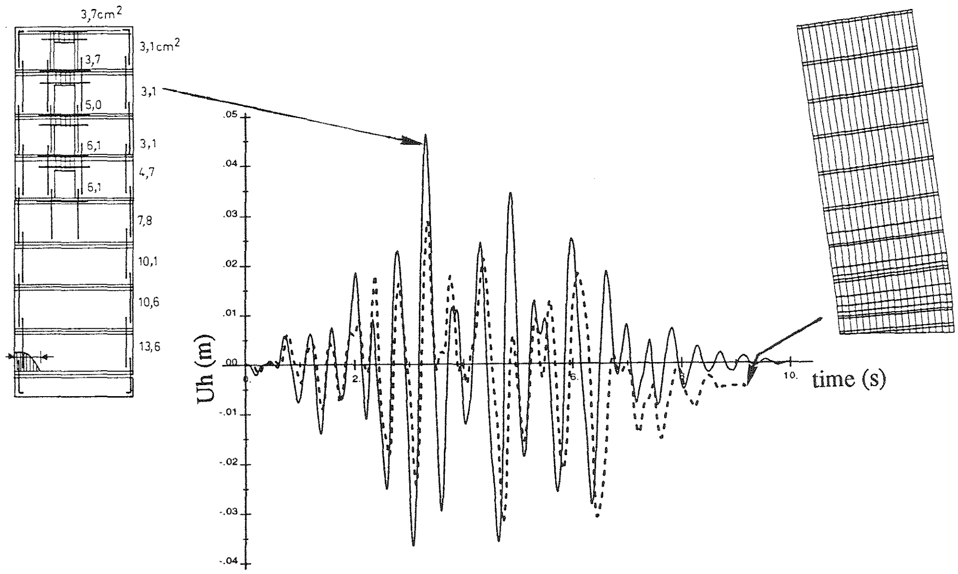
Before and after this experiment the main question was : " is it possible to extrapolate to real constructions observations and conclusions done on the CASSBA model ?". The first answer was : "yes, for a construction put in same conditions, because the similarity has been respected". Which means that there are :

- Mechanical consistency, by satisfying, the scale for all dimensions of the structure (concrete and R-bars), the compression of the time scale ($1/\sqrt{3}$) and the mass ratio ($1/3^2$);
- Rheological consistency, by satisfying the same material behaviour including localisation similarities (aggregates ratio : 1/3);
- Site consistency, the real site must verify the same rules for the interface behaviour than those activated during the experiment.

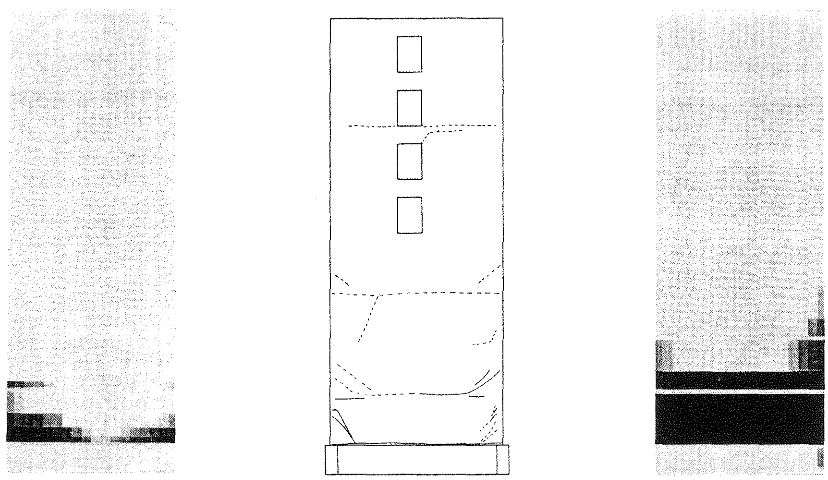
The experiment on scale 1 is not what is concerned here. Obviously if it was possible the use of scale 1/3 was not done. The only thing possible is a numerical experiment using the similarities laws.

The first credible point is to be confident on the modelling used: the good agreement shown before and reported in other papers (Mazars 1994), between simulations and experiments, constitute a good starting point.

From this we have to approach, using these 3 kind of consistencies, as well as possible the real situation (scale 1). What does it mean ?



a) Top displacements



''''' Test at 0.50g
—— Test at 0.36g
▬▬ Test at 0.10g

b) Damage maps

Figure 1, CASSBA program results on 1/3 scale building at 0.5g loading.
a) Top displacement comparison between experimental results (dash lines), and, numerical results based on the presented model (solid line).
b) Comparison between Damage maps obtained by numerical means, and the real fractures observed on the test specimen; on the left: "macro damage" (D>0.99) after 3. sec, on the right: global damage (D>0.) at the end of the experiment.

To be consistent with mechanical similarity, apart from the "decompression" of the time scale, it is necessary to respect the original distribution of masses; as mentioned in § 2.2 this point creates a difference with the model because the additional masses needed in the model were located exclusively on the floors.

About rheological similarity, we used the same constitutive relations and parameters, and to insure similarity for localisation, exactly (apart for the scale) the same discretization.

We have seen before that the site similarity will be respected if the same rules were used at different scales, which means in the framework of our modelling that k $(=M/\Theta)$ must respect the ratio $1/3^3$.

3.4 Analysis of the results obtained

The design of the construction from which the model has been deduced was done for an earthquake at 0.36g, thus this loading is chosen to compare the behaviour of the model and that of the prototype.

Since it is easy to change the model, the comparison concerns different situations, linear or non-linear behaviour, fixed or non-fixed connection with the support. Comments on this are given next.

- Quasi- identity in a linear regime

Concerning the results obtained from the prototype and the model, in some different situations among which embedded conditions of foundations and different values for structural damping, time-history top displacement curves show for every case a quasi superposition. Proving that similarity is totally verified in a linear regime.

- Good concordances for the experimental conditions

These conditions are, non-linear behaviour, of course, and structure non-fixed on the support (simulated as said before, with a linear moment-rotation law).

Figures 2 gives the results for the displacement at the top and the damage field at the end of the earthquake. Very few differences appear after 6 seconds which lead to small permanent deformations at the end but with no significant differences for the damage of the two structures.

- Significant differences for fixed support conditions

This case was not considered for the experiment. But simulation allowed to forecast what could happen if the structure was anchored with the support.

In that case one can see on figure 3 that damage on the prototype is further more distributed than in the model. The difference in the evolution of damage induce differences in the displacement at the top visible after 4 seconds. They concern, the maxi values on different peaks, the frequency, and finally leads for the model to a significant permanent displacement.

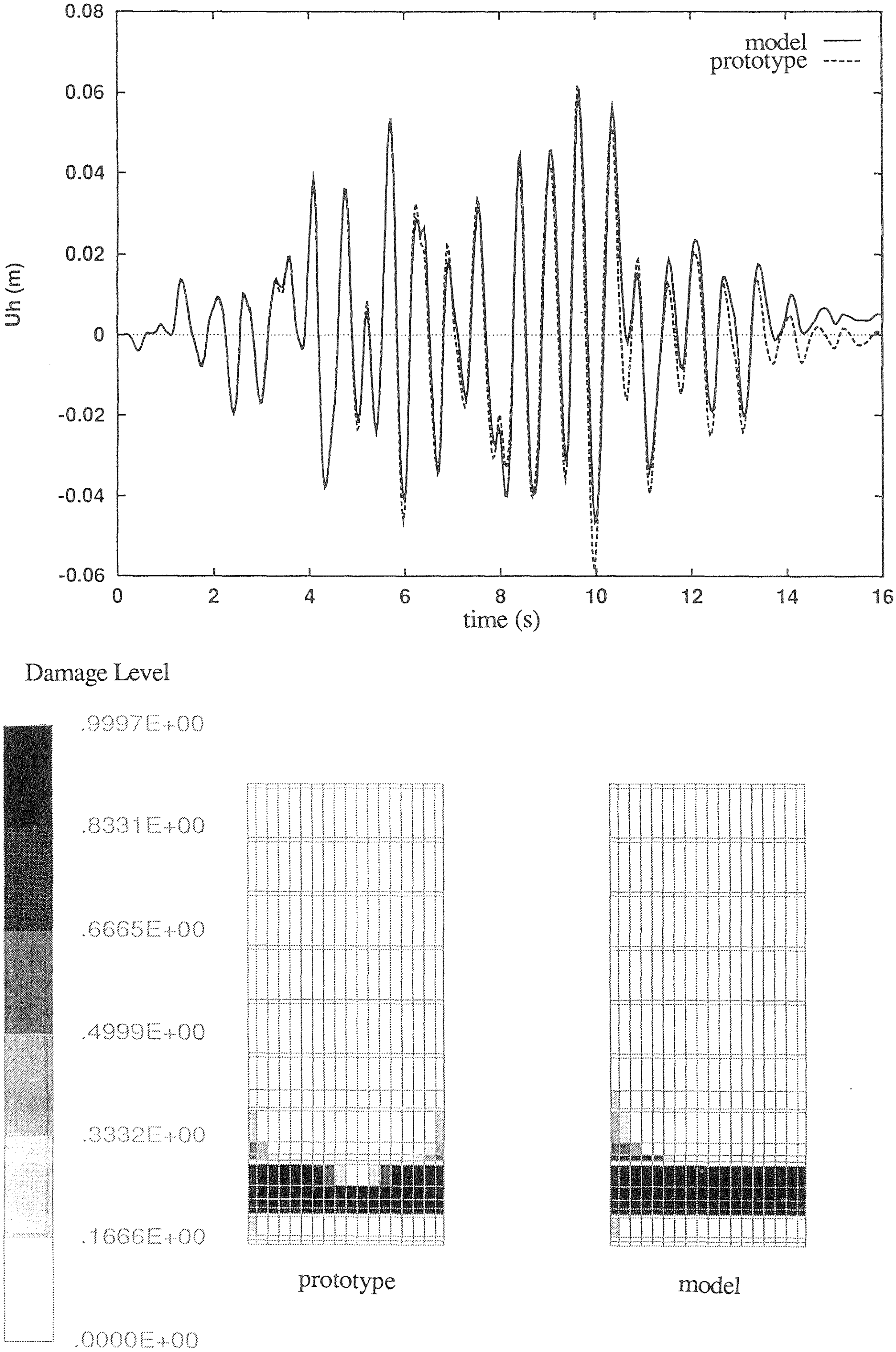


Figure 2. Top displacement and final damage-map comparisons between the two numerical analysis made on the model and the prototype (scale 1) at the following conditions: experimental base contact condition (moment/rotation behaviour), 0.36g load level.

An explanation for these differences
Considering a cantilever beam, it is easy to show that the moment on the support due to a concentrate loading at the end is greater than those created by the same load distributed along the beam. The concentration on the floor of additional masses, necessary for similarity, creates during the loading greater moment at the base of the wall at each floor. This effect which stays low, is sufficient to activate localisation in spite of distributed damage in that condition of fixed support (which was not the case when rotations were possible). However in both cases of support this effect can be responsible for the creation of permanent deformations due to the entrance into plastification of the R-bars.

This confirms what is now commonly admitted : softening materials are very sensitive to loading and boundary conditions and approximations, even light, can create different responses. The solution is to use adequate simulations before concluding.

4 Conclusions

Experiments in the field of structural dynamics, such as seismic engineering, generally require to work on reduced scale structures.

In that framework this paper proposed an overview of different aspects of the similarity problems and their applications to concrete structures.

Three main points must be considered :

- Mechanical similarity, which insures the consistency of the equation of motion which leads to act on the mass distribution and the time scale.

- Rheological similarity, which consists to insure the creation of the same stress and strain fields implying the same similar behaviour for strain localisation.

- Site consistency, which concerns the crucial problem of boundary conditions; interface rules (friction, rotation, ...) between structure and support must be the same in order to verify similarity.

The analysis of the choices and the results obtained on the French experiment CASSBA (R.C. structural wall at scale 1/3, tested on a shaking table) show that the similarity on the experimental conditions was accurately respected. However simulations performed using a damge model show, that other site conditions (structure fixed on the support), could lead to create different damage and fracture paths on the model (scale 1/3) and on the prototype (scale 1). This seems to be mainly due to the location of masses added on the model in order to respect the mechanical similarity.

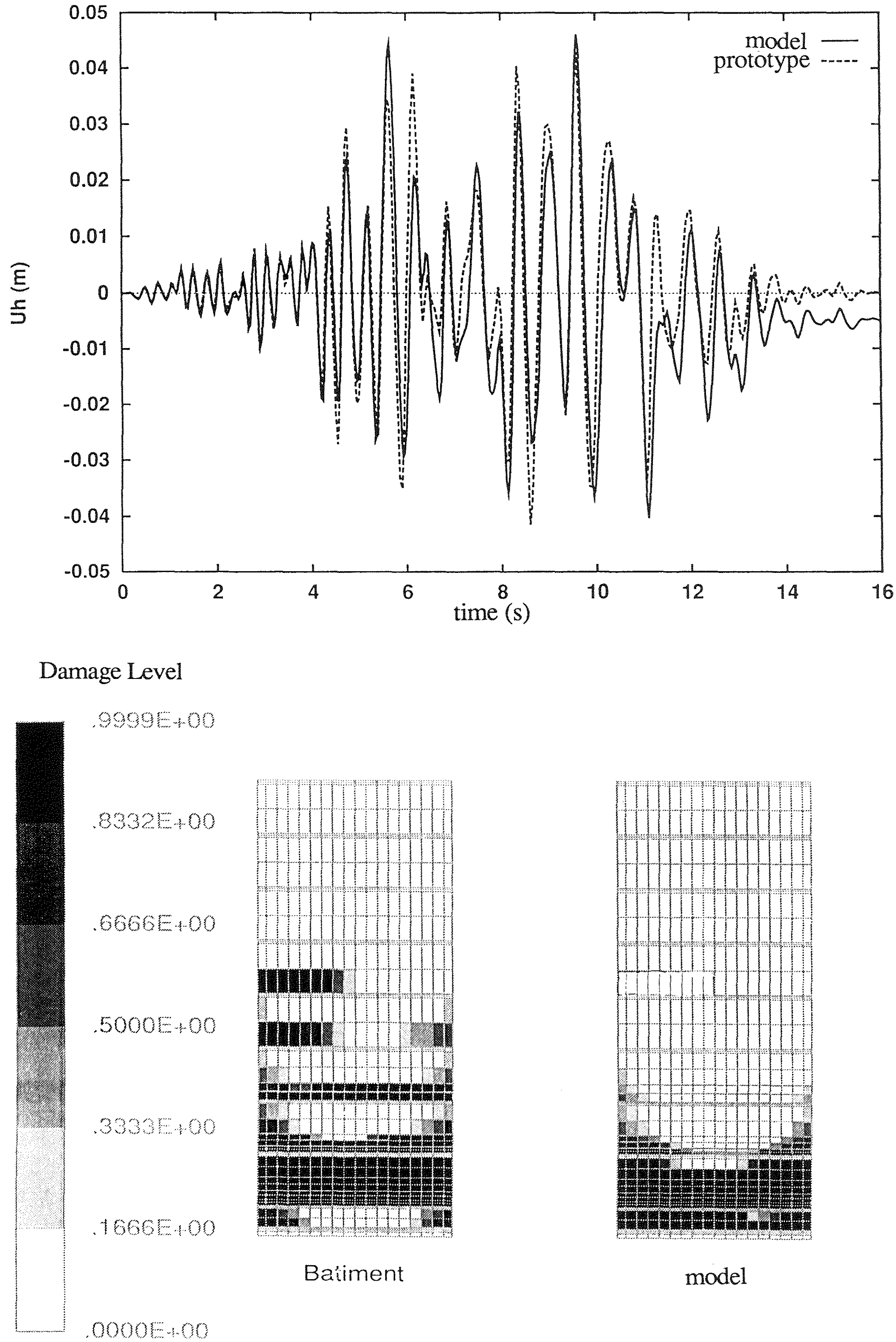


Figure 3. Top displacement and final damage-map comparisons between the two numerical analysis made on the model and the prototype (scale 1) at the following conditions: embedded base condition, 0.36g load level.