Fracture Mechanics of Concrete Structures,
Proceedings FRAMCOS-2, edited by Folker H. Wittmann,
AEDIFICATIO Publishers, D-79104 Freiburg (1995)

DYNAMIC ANALYSIS OF REINFORCED CONCRETE SLABS : COMPARISON OF SIMPLIFIED METHODS

F. Toutlemonde and P. Rossi
Division Bétons et Ciments pour Ouvrages d'Art, Laboratoire Central des Ponts et Chaussées, Paris, France

Abstract

High strength and regular reinforced concrete slabs have been shock-tested using a shock tube, and compared to quasi-statically loaded slabs using the same device. Different failure modes have been observed, according to concrete strength and loading rate. In a first part of this paper, the failure load and scheme are evaluated using different simplified methods: as a reference, the French design code, then the technique of the equivalent single-degree-of-freedom (SDOF) oscillator, and a method based on the yield lines theory which turns out to be the most realistic, provided the various possible failure mechanisms are accounted for. Secondly, predicting the dynamic serviceability of the slabs is dealt with, either using an analytical viscoelastic modal superposition, or an elasto-plastic SDOF oscillator equivalence. However, these methods are limited to a homogeneous deformation stage of the structure. In conclusion the necessity and requirements of refined analysis methods are emphasized.

1 Shock-tube tested reinforced concrete slabs

1.1 Research significance

In order to validate computational methods for concrete structures in dynamics, a model structure test has been defined, Toutlemonde et al. (1993). It consists in loading circular concrete slabs with a pressure step using a shock tube. Complete details of samples and boundary conditions are given in Toutlemonde (1994). Ordinary (O) and high performances (HP) reinforced concrete (RC) slabs have been tested. One major result is the possible onset of a shear failure mode, in competition with the global bending failure mode, Toutlemonde (1993), depending on concrete strength, i.e. on the steel to concrete bond, and on the loading rate, Toutlemonde & Rossi (1994).

Similar changes in the collapse mechanism of dynamically loaded structures have often been reported, e.g. for rectangular concrete slabs, Miyamoto et al. (1992), and for circular fiber-reinforced concrete slabs, Gambarova & Schumm (1994). However, predicting the correct failure mode and ensuring a ductile mechanism with a consistent reinforcement design still appears as a challenge, Woodson (1993).

In this paper, some simple analytical design or computation methods are compared, using the RC circular slab tests as a benchmark. Ultimate and serviceability limit states are successively considered, that is, predicting the dynamic bearing capacity of the structure, and computing its response up to the onset of irreversible undesired deformations. Shortcomings and further requirements are discussed.

1.2 Test results

The samples dimensions and the loading are recalled (Fig. 1.). Reference data are obtained for slabs tested quasi-statically (maximum strain rate of about 6.10^{-6} s^{-1}). Thresholds in the global load-deflexion behaviour of the slabs are defined as follows, they correspond to major decreases in the global slab rigidity. A first crack appears, which takes place at a pressure of 102 kPa for an ORC slab and 147 kPa for HPRC. Rebars yielding begins at 211 kPa for ORC and 289 kPa for HPRC. Then the "ultimate" slab behaviour takes place, corresponding to the onset of a block mechanism with major cracks wide open, at 475 kPa for ORC and 490 kPa for HPRC. Finally the slab fails, at a pressure of 641 kPa for the ORC slab (direct shear and rebars pull out, Fig. 2.) and 608 kPa for HPRC (mixed shear and bending mode, some twisted rebars are still anchored).

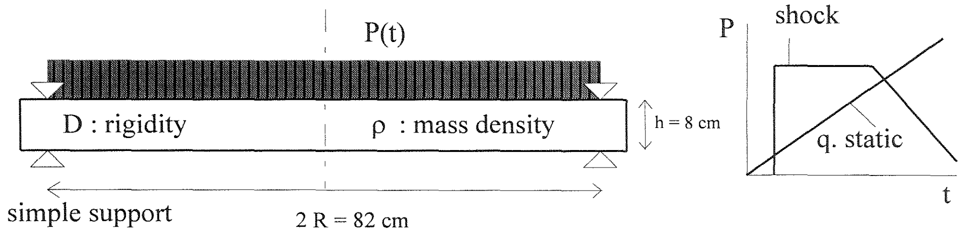

Fig. 1. Schema of the test

Corresponding threshold values for slabs tested through successive shocks are given below, as average values for 2 slabs. First visible cracks are observed after shocks with a mean pressure of 232 kPa for ORC slabs, 203 kPa for HPRC slabs. However for ORC the global slab rigidity has already come down for a smaller pressure (about 160 kPa), *cf* Fig. 3. The beginning of rebars yielding is detected with strain gauges at a mean pressure of 280 kPa for ORC slabs, and 388 kPa for HPRC ones. An ultimate stage is obtained when the global rigidity reaches a bottom level, at 362 kPa for ORC and 484 for HPRC. Afterwards, some main cracks among the previously created ones progressively open. A typical bending failure is observed at a pressure of 712 kPa for HPRC slabs (Fig. 4.). For ORC slabs a mixed shear-bending failure is observed at a mean pressure of 831 kPa, following a global bending ultimate state which was obtained at the previous shot with a mean pressure of 665 kPa. Complete details and analyses can be found in Toutlemonde (1994).

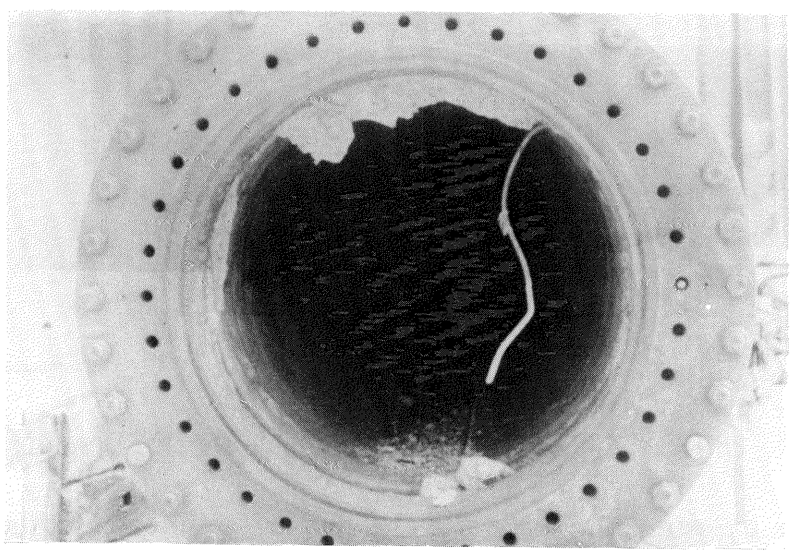

Fig. 2. ORC slab tested quasi-statically. Direct shear and rebars pull out

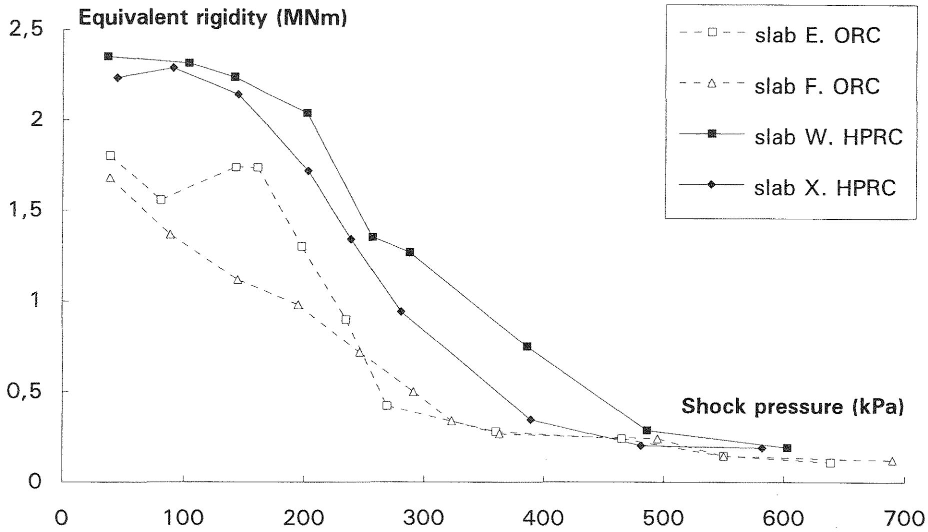

Fig. 3. Equivalent rigidity of RC slabs after successive shocks

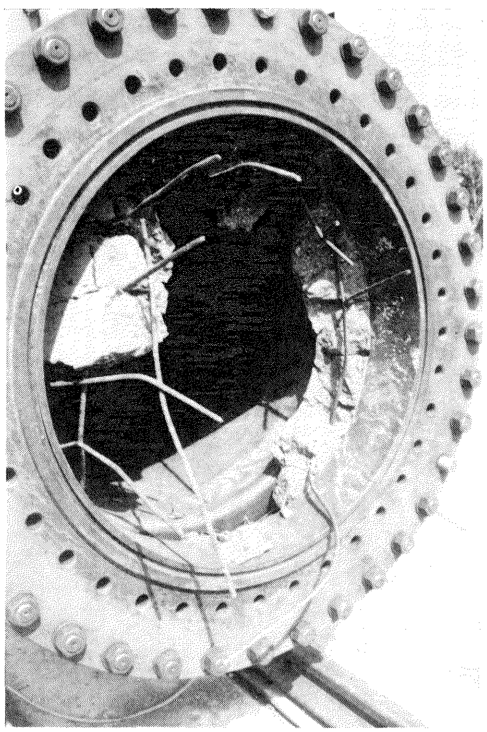

Fig. 4. HPRC slab tested through successive shocks. Bending failure

Correctly estimating these slabs behaviour, and predicting the serviceability and ultimate loads using simple methods is dealt with hereafter.

2 Evaluation of the bearing capacity

2.1 BAEL design code

A first approach of the slabs design is given using the French BAEL 91 design code. The ultimate limit state is defined by the maximum rebars strain, equal to 10 ‰. Given the reinforcement area, the steel yield strength (600 MPa), and the concrete strength (50 or 115 MPa), the ultimate (capable) bending moment can be computed. It is equalled to the externally applied moment, which is computed assuming that the structure remains elastic. So we get, using current notations (eq. 1):

$$M_u = p_u(3+\nu)R^2/16 = d.(1-0.4\alpha_u)A_s F_{su} \quad \text{with } \alpha_u = A_s F_{su}/0.8dF_{bu} \tag{1}$$

So the ultimate pressure p_u is estimated to 396 kPa for the ORC slab and to 405 kPa for the HPRC one. Even for quasi-statically tested slabs, these values seem rather conservative (the empiric safety ratio is about 1.5, and even 1.75 for dynamically tested slabs). They do not account for 2D effects, nor for shear failure. This first approach is thus compared to a more current and adapted dynamic design method.

2.2 Single-degree-of-freedom elastoplastic equivalent oscillator

This method is described e.g. in CEB (1988). Equivalent mass, load and rigidity are defined. Nevertheless for RC slabs the ultimate limit state is refered to an ultimate deformed state of the central section of the slab. The capable moment is then equalled to the external elastically computed one, which turns to give values very similar to previous ones. In fact it predicts p_u = 394 kPa for ORC and 404 kPa for HPRC. But this limit corresponds neither to the onset of rebars yielding, nor to the beginning of a block mechanism characterizing the failure mechanism of the slab.

2.3 Yield lines method

It is thus attempted to use the yield lines method, widely developed by Salençon (1983), which can account for the possible failure modes (fig. 5). After cracks have developed in the center of the slab, an ultimate rigidity is obtained, D*, which is computed using CEB (1988) formulas (about 0.21 MNm). It allows evaluating the deformation energy available just before failure (eq. 2):

$$W = \pi p^2 R^6(7+\nu)/384D^*(1+\nu) \quad \text{or } p = \sqrt{384(1+\nu)WD^*/\pi R^6(7+\nu)} \tag{2}$$

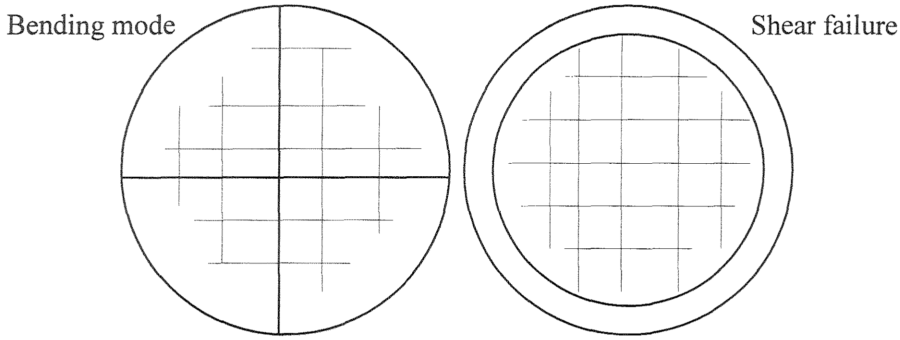

Fig. 5. Yield lines method (circular RC slabs)

This elastic energy is to be equalled to the total plastic energy required to open major cracks, either in a bending mode or in a shear mode. For the bending mode, the ultimate moment is computed according to BAEL 91, and is multiplied by the corresponding concentrated rotation which is derived from a crack opening of 2 mm varying linearly in the height of the slab. For the shear failure, the energy is the sum of a term related to concrete and of a term standing for rebars debonding. A crack opening of 10 µm seems reasonable to compute the shear dissipated energy, multiplied by the tensile strength of concrete on a 45° inclined crack plane. For rebars debonding, the limit bond stress is integrated over the bars surface possibly concerned by pull-out, i.e. along l=2 cm (no proper anchoring). Thus the shear failure energy reads (eq. 3):

$$W = \left(2\pi R.h\sqrt{2}.f_t.\delta_u/2\right) + \left(24.l.2\pi\varnothing.\tau_u\right),\ \tau_u = 0.6\psi_s^2 f_t\ ,\psi_s = 1\ \text{(degraded bond)} \quad (3)$$

In dynamics, the same formulas can be applied. The capable bending moment may be roughly estimated as 20% higher, consistently with the increase in steel tensile strength, while the shear failure energy can be evaluated as 50 % higher, due to the increase in concrete tensile strength. Comparing the energies and experimental vs computed ultimate load values (Table 1) shows that a shear failure is more probable for an ORC slab statically tested, that a bending mechanism is predominant for a HPRC slab dynamically loaded, and that for a shock at 831 kPa both mechanisms are possibly present for an ORC slab. These results are consistent with test results. Even if the definition of ultimate crack openings remains arbitrary in this analysis, the slab failure mode and bearing capacity are correctly evaluated, provided all possible mechanisms are considered.

Table 1. Failure energies and ultimate loads. Yield lines method

Loading rate	Quasi- Static				Shock Tests			
Concrete	ORC		HPRC		ORC		HPRC	
Failure mode	Flex.	Shear	Flex.	Shear	Flex.	Shear	Flex.	Shear
W (J)	458	454	467	688	550	680	560	1030
p_u (kPa)	638	635	658	799	700	778	721	978
exp. p_u (kPa)		641	608		> 665	< 831	712	

3 Computation of the structural behaviour under service loads

3.1 Modal analysis

However, this is not sufficient for structures where dynamic service loads have to be accounted for. In this case, computing the structural response has to be achieved. So, a first description of the slabs response through successive shocks using the classical technique of modal analysis is carried out. The equation of an elastic circular slab reads (eq. 4):

$$\frac{\partial^4 w}{\partial r^4}+\frac{2}{r}\frac{\partial^3 w}{\partial r^3}-\frac{1}{r^2}\frac{\partial^2 w}{\partial r^2}+\frac{1}{r^3}\frac{\partial w}{\partial r}+\frac{\rho h}{D}\frac{\partial^2 w}{\partial t^2}=-\frac{p(t)}{D}-\frac{\rho h}{D}\frac{\partial^2 v}{\partial t^2} \tag{4}$$

with w the local deflexion, r the radius and v the support displacement. The solution is computed, *cf* Schaffar (1970), as the superimposition of the first three eigenfunctions, which are linear combinations of Bessel's functions, multiplied (for a Heaviside pressure signal) by a sine-type time function. With a simple support at $r = R$, it reads (eq. 5):

$$w(r,t)=\frac{pR^4}{D}\sum_i A_i.\left[1-\cos\left(c_i t\sqrt{D/\rho h}\right)\right].\left[J_0\left(r\sqrt{c_i}\right)+\alpha_i I_0\left(r\sqrt{c_i}\right)\right]$$

$$\begin{aligned}\text{with}\quad &\alpha_1=-4.36\ 10^{-2}\quad \alpha_2=6.78\ 10^{-4}\quad \alpha_3=8.37\ 10^{-6}\\ &A_1=-7.17\ 10^{-2}\quad A_2=8.55\ 10^{-4}\quad A_3=-3.52\ 10^{-5}\\ &\mathrm{R}\sqrt{\mathrm{c}_1}=2.18\qquad \mathrm{R}\sqrt{\mathrm{c}_2}=5.44\quad \mathrm{R}\sqrt{\mathrm{c}_3}=8.63\end{aligned} \tag{5}$$

In order to account for the observed damping of vibrations, a more general solution is searched using a complex pulsation, which consists in multiplying the cosine function by an exponential term $\exp(-\eta_i t/T_i)$, with η_i the logarithmic decrement and T_i the period of the ith eigenmode. Since only one global damping factor can be identified experimentally, the same η_i/T_i ratio is adopted and fitted using the measured damping on first mode oscillations. The slab rigidity D is fitted for each shot from the quasi-static load-decreasing stage (fig. 3) or from a previous static test.

Using these empiric D and η/T factors, it is possible to compute the slab response and to determine the deviation with characteristic values of the experimental signals: the maximum deformed shape of the slab, or the initial velocities, Toutlemonde (1994). An average standard deviation of 14.7% is obtained for the prediction of the maximum central deflexion, with an error always smaller than 30%. The estimation of the initial velocity is correct (average standard deviation of 29.3%) but less precise, especially for high pressure shocks. It may then be concluded that for slabs where strain localization has not yet created a block mechanism (i.e. before the "ultimate stage"), a viscoelastic tangent computation provides a correct - but non predictive - estimation of the structural response.

Finally, it seems that the global damping factor underestimates frictional dissipative phenomena, especially when cracks have appeared, Toutlemonde & Rossi (1995). Thus it is suggested to account explicitly for permanent strains, as one of the major consequences of concrete cracks and of rebars yielding, in a refined three-dimensional analysis.

3.2 Elastoplastic oscillator

Since shortcomings of the viscoelastic tangent approach are its lack of predictivity and the absence of permanent deflexions, a simple alternative elastoplastic method has then been employed, namely the equivalent SDOF elastoplastic oscillator, CEB (1988). A bilinear load-deflexion diagram is defined (k and k^* stand for the equivalent elastic rigidity and tangent stiffness in the hardening range, respectively). It allows computing the deflexion obtained during a shock. In particular, when the yield limit (load P_r, deflexion w_r) has been exceeded, the maximum deflexion for a Heaviside load signal (plateau value: F) reads (eq. 6):

$$w = w_r + \frac{F-P_r}{k^*} + \sqrt{\left(\frac{F-P_r}{k^*}\right)^2 + \frac{F^2}{k.k^*}\left[1-\left(\frac{w_r k}{F}-1\right)^2\right]} \qquad (6)$$

This method has been employed successfully for estimating the global structural response (fig. 6), at least up to the onset of rebars yielding. At this stage, localized high strains make the approximation of a single mode rather inadequate. The problem is, that this limit of validity is not predicted by the method itself (the computed failure load is 400 kPa).

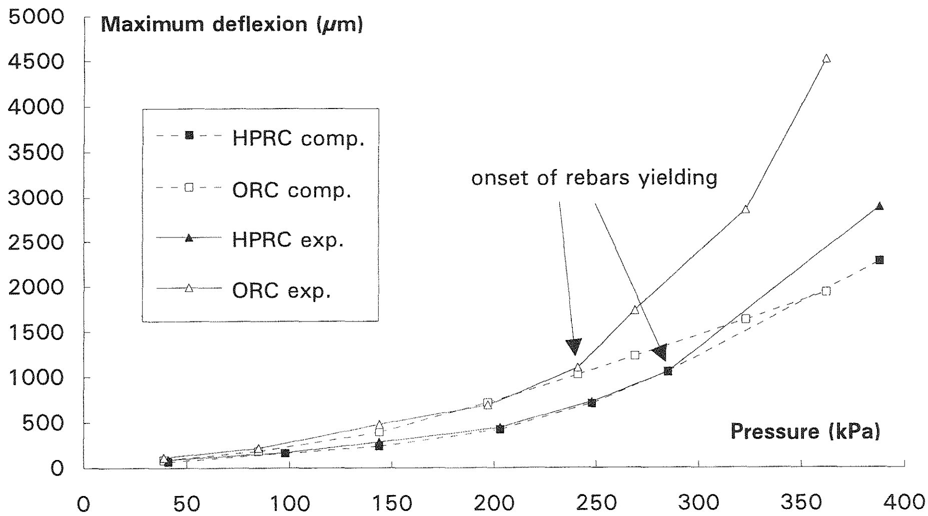

Fig. 6. Deflexion vs loading pressure. SDOF oscillator / experiment

4 Conclusions. Refined analyses

Results obtained on shock-tube tested RC slabs have been used as a benchmark to compare some simplified analytical design or computation methods. The yield lines theory has proved very effective in predicting static and dynamic failure loads and modes. Yet some arbitrary values have been assumed and should be furtherly justified. Checking the serviceability of a structure under a "smaller" shock requires other computational methods. Tangent modal analysis has proved effective but generally not predictive. The SDOF equivalence has a limited domain of validity. Further refined physically-based analyses have to be carried out.

5 References

BAEL 91, French design code for public works contracts - Reinforced concrete, Journal officiel de la République Française, 92-03.

CEB (1988) Bulletin d'information n° 187 **Concrete structures under impact and impulsive loading. Synthesis report.** Lausanne, 186 p.

Gambarova, P.G. and Schumm, C. (1994) Impulsive punching of fiber-reinforced concrete slabs, in **ASCE Structures Congress XII** (eds N.C. Baker & B.J. Goodno), ASCE, New York, vol. 1, 252-257.

Miyamoto, A., King, M.W., and Fujii M. (1992) Nonlinear dynamic analysis of impact failure modes in concrete structures, in **Fracture mechanics of concrete structures** (ed Z.P. Bazant), Elsevier Applied Science, London & New York, 651-656.

Salençon, J. (1983) **Calcul à la rupture et analyse limite.** Course of anelastic structural design, Presses de l'ENPC, Paris, 366 p.

Schaffar, M. (1970) **Calcul de la déformation dynamique de plaques circulaires encastrées ou appuyées, uniformément chargées par un échelon de pression.** ISL report 25/70, Saint-Louis, France, 60 p.

Toutlemonde, F., Boulay, C. and Gourraud, C. (1993) Shock-tube tests of concrete slabs. **Materials and Structures, 26**, 38-42.

Toutlemonde, F. (1993) Dynamic failure modes of concrete slabs: experimental evidence and questions, in **Structural concrete slabs under impulsive loads** (ed Krauthammer), Fortifikatorisk Notat Nr 211/93, Trondheim, 69-76.

Toutlemonde, F. and Rossi, P. (1994) Shock-tested R.C. slabs: significant parameters, in **ASCE Structures Congress XII** (eds N.C. Baker & B.J. Goodno), ASCE, New York, vol. 1, 227-232.

Toutlemonde, F. (1994) **Résistance au choc des structures en béton: du comportement du matériau au calcul des ouvrages.** Doctoral thesis, ENPC, Paris, 396 p.

Toutlemonde, F. and Rossi, P. (1995) Analysis of shock-tube tests on concrete slabs, in **Concrete under severe conditions. Environment and loading** (ed K. Sakai), CONSEC'95, Sapporo, Japan, aug. 2-4.

Woodson, S.C. (1993) Response of slabs. In-plane forces and shear effects, in **Structural concrete slabs under impulsive loads** (ed Krauthammer), Fortifikatorisk Notat Nr 211/93, Trondheim, 51-68.