DISCRETE VERSUS SMEARED CRACK ANALYSIS

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Abstract
Recent developments in numerical analysis of cracking are briefly reviewed and summarized.

1. Recent Developments in the Analysis of Cracking

In the recent past a number of exciting developments have changed the field of failure analysis by reconciling in part the fundamental differences between plasticity and fracture. The formation of discontinuities, which has been addressed within the framework of localization analysis at constitutive level, has made significant progress through analytical solutions. They are available for a wide spectrum of plasticity and damage formulations based on loading functions. Recent advances in spectral analysis of rank-one updates have essentially resolved the detection of singularities due to localization [22,23]. Moreover, the geometric interpretation of these singularities in terms of Mohr coordinates has added valuable insight into the formation of continuous and discontinuous failure modes and their orientation [33].

The basic differences of failure analysis really start at that point, when weak and strong discontinuities initiate at the material level.

The simple classification into discrete and smeared concepts does no longer do justice to the multitude of failure analysis approaches. The discrete crack and its extension with linear and nonlinear fracture mechanics has been replaced in concrete mechanics by the fictitious crack concept [9], and its physical manifestation in terms of decohesive and frictional interface contact [27,4,35–40]. The shortcomings of the fixed orthotropic crack concept [20,24] in smeared failure analysis have been improved considerably by rotating crack and multicrack concepts [15,7,2,41] . Another layer of complexity was recently added to the anisotropic smeared crack formulation by the microplane formulation [42,43] which projects material degradation on a finite number of 'microplanes' onto the continuum level via kinematic constraint and energy arguments. Parallel to the smeared crack proposals, there have been a number of significant contributions in continuum damage mechanics [14,10,3], with renewed attempts to capture the degradation of stiffness and strength in tension by an equivalent continuum formulation, similar to isotropic and anisotropic softening plasticity formulations [8,6]. Recently the smeared formulations have enjoyed an additional twist in the form of 'non-local' extensions [1]. Various regularization attempts have been advanced to suppress the formation of spatial dicontinuities and to replace discontinuous by continuous failure modes. Strain gradient theories [5,30], rate-sensitive material formulations [17,32], and higher order continuum concepts such as the micropolar Cosserat theories [34] have been advanced vigorously in recent years in order to remove weak discontinuities from the solution domain. In this course of action the continuum properties are essentially preserved in the static and kinematic relations at the expense of failure modes which might be subjected to significant distortion because of the underlying regularization strategy.

The spatial discretization in the form of finite elements introduces another layer of static or kinematic constraints which play a fundamental role in the description of the failure process. This is the main point of departure where the failure analysis methodology may introduce considerable bias into the prediction of the actual failure mode [19,29]:

(a) The discrete crack methods trace failure on a single or a finite number of surface interfaces. Its propagation requires in the general case continuous remeshing in order to advance the crack configuration (remeshing is not necessary if the crack path is know in advance, or if cracks are pre-introduced between all continuum elements in the mesh [44,45,35], although this is at the price of a much higher number of nodes and a pre-established number of crack paths). In the general case, the remeshing procedure causes problems with remapping the state of internal variables ahead of the crack, and with healing or

fusing previously cracked regions which may pose severe uniqueness problems in the case of crack branching.
(b) The smeared crack methods do not require remeshing, but necessitate element enhancements in order to capture the kinematic constraints of continuous failure modes, such as incompressible deformations in J_2-plasticity. Element enhancement or element alignment become mandatory when discontinuities are to be captured. In fact, the recent proposals go one step further and introduce strong discontinuities through Heavyside distribution functions [26,11] which represent the formation of discontinuities in the displacement field across the element domain. This approach is quite distinct from the more physical formulation of embedded cracks [12,13], which starts from the variational statement of internal discontinuities in order to acommodate an internal crack in the element domain. In fact, it is the latter, which appears very promising, but which needs to mature further to be applied in engineering practice.

In summary, a number of important findings have changed the world of discrete and smeared finite element crack analysis since its incipience by Scordelis et al.(1967) [18] and Rashid (1968) [20]. The most significant developments are the current attempts to lift weak and strong discontinuities from the material level onto the level of elements and structures by combining the best of the two worlds in smeared and discrete failure analysis.

2. References

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