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ROUND ROBIN ANALYSIS ON MODELLING OF OVER-REINFORCED CONCRETE BEAMS - CALCULATION OF THE LOAD DEFORMATION BEHAVIOUR OF CONCRETE BEAMS WITH THE BDZ-MODEL

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Abstract

RILEM Technical Committee has organized an ongoing Round Robin research program to study the response of over-reinforced concrete beams subjected to four-point loading conditions. The major objective of the program is to predict the load-deformation behaviour of over-reinforced concrete beams based on the mechanical parameters of the respective composite constituents. Therefore, a wide range of experimental and theoretical investigations were carried out at different research facilities.

At University of Leipzig, an analytical study was conducted to compute the load-deformation response of over-reinforced concrete beams using the so-called "BDZ-Model". The BDZ-Model employs fracture mechanics parameters and takes into account the failure localization in a certain damage zone to predict the fracture behaviour of concrete under compressive loading.

Keywords: Over-reinforced concrete beams, four-point bending, compression zone, size effect, localisation

1 Introduction

RILEM Technical Committee 148 SSC "Strain Softening of Concrete" has organized a Round Robin research program to study the response of over-reinforced concrete beams subjected to four-point loading conditions. The major objective of the program was to investigate if existing analytical models are suitable to predict the experimental response of over-reinforced concrete beams.

Therefore, experimental tests were carried out at the Structural Laboratory at Aalborg University, at the Department for Structural Engineering and Materials of the Technical University of Denmark and at the Stevin Laboratory of Delft University of Technology in 1996. Over-reinforced beams with different material properties were used to examine the softening behaviour of concrete in the compression zone (Ulfkjaer et al.(1997)). So far, experimental results from those beam tests have not been published. However, the mechanical properties of the used concrete were determined from uniaxial tension, uniaxial compression and three-point bending tests, the properties of the used steel were determined under uniaxial tension. Those material parameters were given to the participants of the Round Robin program that used analytical methods for the prediction of the load-deflection behaviour of the tested beams. This procedure was chosen to obtain independent results from the theoretical and experimental investigations.

This report summarizes the results from a theoretical study conducted at University of Leipzig as part of the Round Robin program. During the study the so-called "BDZ-Model" and the given material parameters were used to predict the load deformation behaviour of the tested beams.

2 Experimental program on over-reinforced concrete beams

During the experimental program beams with two different dimensions were subjected to four-point loading conditions. The smaller sized beams had a cross-sectional area of 200 x 100 mm^2, a total length of 3800 mm and a span of 3600 mm. All larger beams had a cross-sectional area of 400 x 200 mm^2, a total length of 7500 mm and a span of 7200 mm. The smaller beams were reinforced with eight stranded wires, the larger beams with nine 20 mm deformed rebars. The smaller sized over-reinforced concrete beams were made of normal strength concrete (NSC), high strength concrete (HSC) and steel fibre reinforced concrete (FRC), the larger beams were made of NSC.

All four-point bending tests were performed deformation controlled. The feedback control signal was provided by the stroke and a deformation sensor (LVDT), which was mounted to the specimen surface in the area of the projected compression zone to record the "Crack Opening Displacement (COD)".

3 Introduction of the BDZ-Model to describe the behaviour of concrete under flexural compression

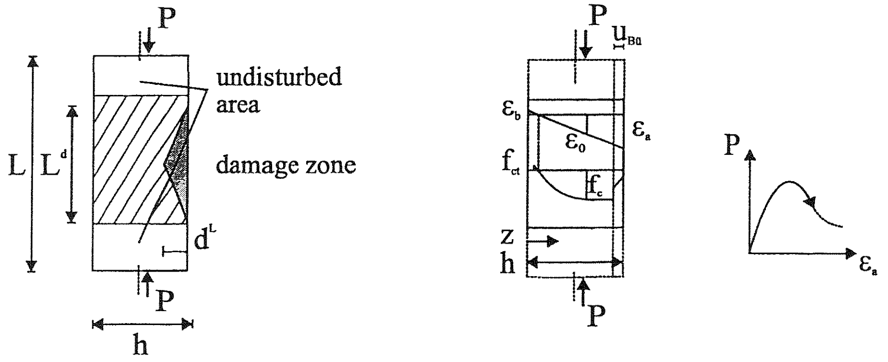
The Biegedruckzonen (BDZ) - Model was developed by Meyer (1997) to predict the flexural behaviour of specimens under compressive loading conditions. The model was proposed based on experimental results from a study conducted at University of Leipzig. The test program of the study was designed to investigate the deformation capacity of concrete in the compression zone. Therefore, concrete prisms with different strength and sizes were subjected to eccentric lateral loading conditions. The test set-up and loading conditions were designed such that the compression zones generated in the tested prisms were comparable to damage zones in four-point bending beams. The test specimens were made of three concretes with different strength and stirrups as transversal reinforcement to provide two different reinforcement ratios.

During the experiments, deformations localized in a certain damage zone. Outside this damage zone, the concrete stayed throughout the entire test procedure mechanically intact. Therefore, it was concluded that two different sections contribute to the overall deformation of the members, the actual damage zone and the intact surrounding area.

The fracture behaviour in the damage zone without transversal reinforcement can be described with the Compression Damage Zone (CDZ) Model by Markeset (1993). This model is based on fracture mechanics considerations and predicts the response of concrete in a damage zone with two failure modes, the tensile and shear failure mode. Accordingly, the descending branch of the stress-strain curve for concrete under compression is determined by tensile failure caused by the development of tensile splitting cracks in the direction of the applied load and shear failure with the occurence of an inclined shear-band.

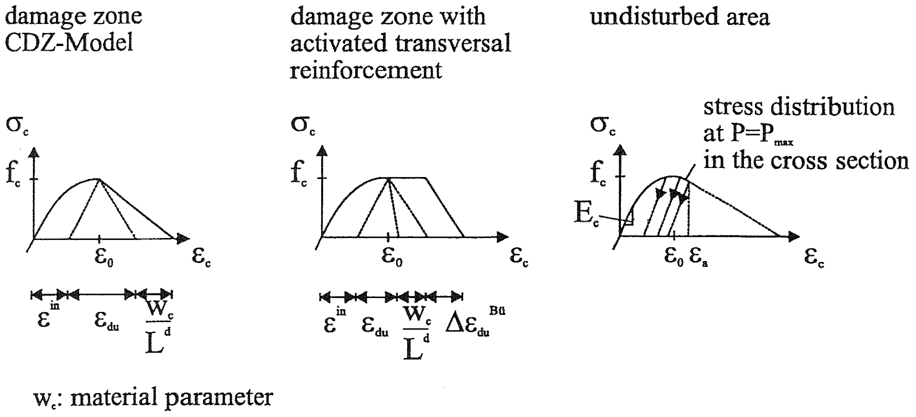
However, the experimental program illustrated also that transversal reinforcement increases the ductility of concrete in the post-peak region during a deformation controlled test. Therefore, energy dissipation by the transversal reinforcement during lateral specimen deformations should be considered for the prediction of the descending branch of the stress-strain curve for concrete under compression. The transversal reinforcement is only effective to prevent longitudinal tensile cracking in a certain core.

Regarding the overall deformation of a structure the behaviour of the material outside the damage zone should not be disregarded. Beyond the peak load fracture localizes in a damage zone whereas stresses and strains of the material in the surrounding area gradually decrease.



a) Specimen under eccentrical compressive loading

b) Distribution of strain and stress in the damage zone with transversal reinforcement



c) Modules of the descending branch

Fig. 1. Modules of the BDZ-Model

According to these findings, the so-called Biegedruckzonen (BDZ)-Model was proposed to predict the response of members under flexural compression with four modules (see Fig. 1). The first module defines the concrete response up to the peak stress based on the stress-strain curve as given in EC 2. The concrete fracture behaviour in damage zones without transversal reinforcement is modelled using the CDZ-Model. For the prediction of the fracture behaviour of concrete in members with transversal reinforcement the response of both, the confined and unconfined concrete sections is considered.

The BDZ-Model as described in this section was used to model the response of the tested four-point bending beams. However, since lateral reinforcement was not used for the beams of this experimental program, the module for lateral confinement effects, i.e. increasing ductility, was disregarded in this analysis.

4 Calculation of the load-deformation behaviour of concrete beams with the BDZ-Model

The input-parameters for the calculation with the BDZ-Model are presented below. Concrete strengths of the beams are assumed to be the mean value given for high and low friction. Strain ε_0 at concrete strength was determined according to a formula proposed by Popovic (1973). Both ε_{du} and w_c were chosen with reference to Markeset (1993) and Meyer (1997). The fracture energy G_F was taken as determined by Ulfkjaer (1997) as a mean value from the uniaxial tests and tests on beams under three-point bending except for the fracture energy of steel fibre reinforced concrete. For the calculation with the BDZ-Model the stress-strain relationship is taken as a three-linear curve for the stranded wires or the ribbed steel bars respectively.

Table 1. Input-Parameters for the BDZ-Model

	SN	SH	SF	LN
Input Parameters	small beam NSC	small beam HSC	small beam FRC	large beam NSC
E-Modul [MN/m^2]	26659	49304	47103	26659
h [m]	0,154	0,154	0,154	0,32
b [m]	0,1	0,1	0,1	0,2
L [m]	0,6	0,6	0,6	1,2
k^1 [-]	5	5	5	5
f_{cm}* [MN/m^2]	22,1	139,8	121,9	22,1
ε_0 [-]	0,00203	0,00322	0,00311	0,00203
ε_{du} [-]	0,00482	0,00099	0,03628	0,00482
w_r [m]	0,0006	0,0004	0,0004	0,0006
A_{sl} [m^2]	0,001452	0,001452	0,001452	0,002827
f_{s1} [MN/m^2]	1800	1800	1800	650
ε_{s1} [-]	0,0070	0,0070	0,0070	0,00284
f_{s2} [MN/m^2]	2200	2200	2200	650
ε_{s2} [-]	0,0100	0,0100	0,0100	0,02500
f_{s3} [MN/m^2]	2450	2450	2450	730
ε_{s3} [-]	0,0350	0,0350	0,0350	0,10000
k [-]	2,6	2,6	2,6	2,6
α [-]	0,9	0,9	0,9	0,9
γ [1/mm]	0,25	0,25	0,25	0,25
r [mm]	1,23	1,23	1,23	1,23
G_f [N/m]	91,05	118,50	3780,00	91,05

- E-Modul determined according to EC 2
- Stress-strain curve of steel is given with a three-linear course
- k, α, γ, r: Parameters of the CDZ-Model

The deformation in the area between the two loads was calculated with the BDZ-Model. In the descending branch of the moment-deformation curve the total deformation is composed of the deformation in the damage zone and the material outside (Fig. 3).

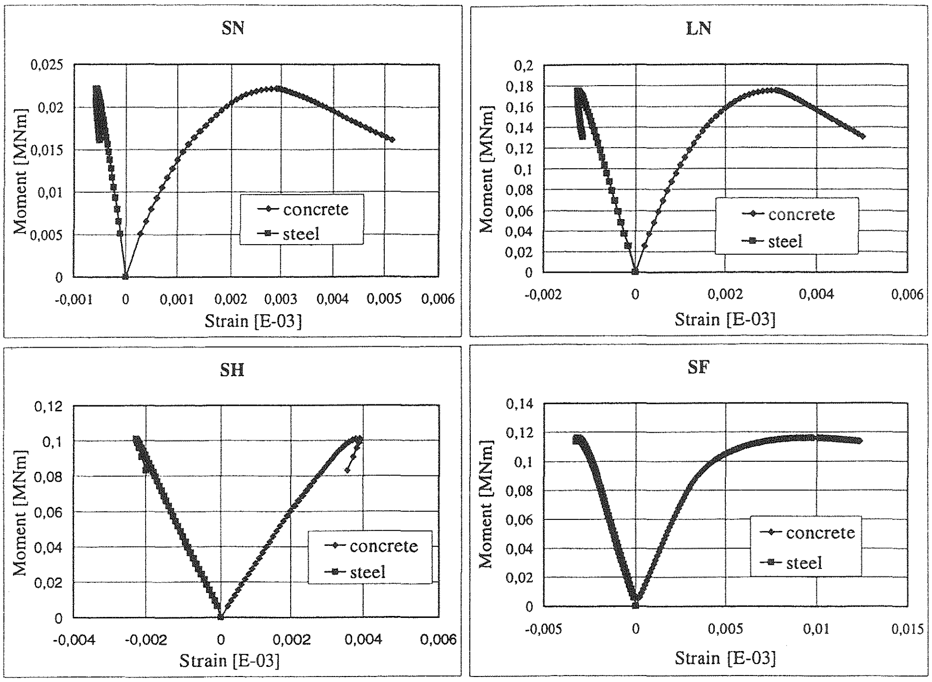


Fig. 2. Calculated moment-strain curves for concrete and steel between the loading platens

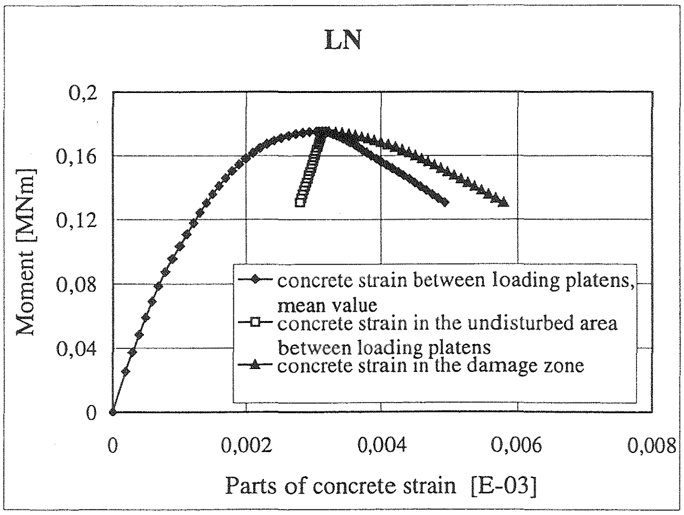
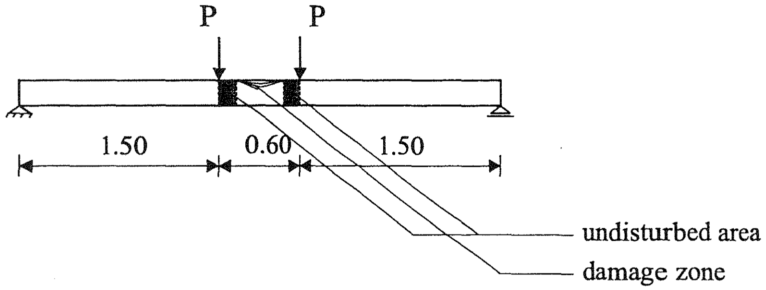


Fig. 3. Moment-strain curves of concrete between the loading platens for the large beam made of NSC

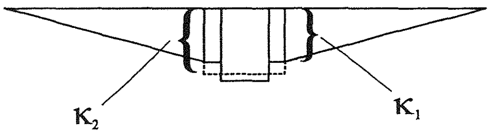
The load-deflection curve and the deflection δ_{10} in the middle of the system was determined based on the force method as shown in equation (1) and Fig. 4 . The values κ_i were used for the curvature of the beam at the edge of the zone with a constant moment (κ_1) and for the mean curvature inside the damage zone (κ_2). The values of the moment curve $\overline{M}$ were calculated based on the unit load "1".

$$\delta_{10} = \int \kappa \cdot \overline{M}\, dx \tag{1}$$

Small beam



“0”-System: Curvature determined by calculation with the real load P



κ_1: Curvature of the beam at the edge of the damage zone
κ_2: Mean curvature inside the damage zone

“1”-System: Moment curve $\overline{M}$ determined by calculation with the unit load “1”

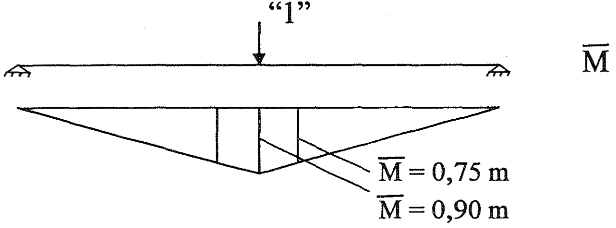


Fig. 4. Determination of the load-deflection response with the force method

Deflection of small beam is given by equation (2).

$$\delta = 2 \cdot \left(\frac{1}{3} \kappa_1 \cdot 0{,}75 \right) \cdot 1{,}50 + 2 \cdot \left(\frac{1}{2} \kappa_2 \cdot (0{,}75 + 0{,}9) \right) \cdot 0{,}30 \tag{2}$$

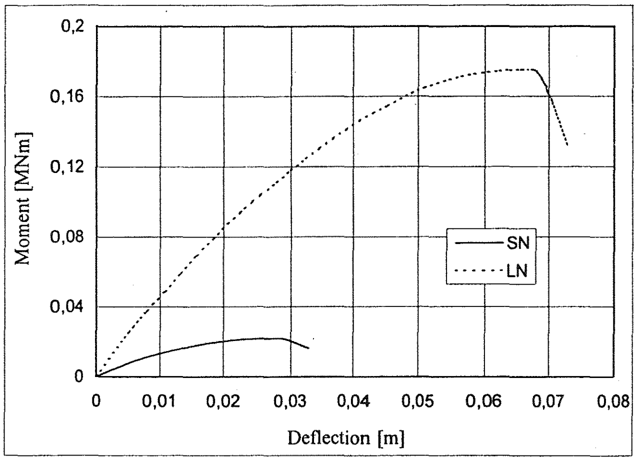


Fig. 5. Calculated moment-deflection curves of the test beams made of NSC

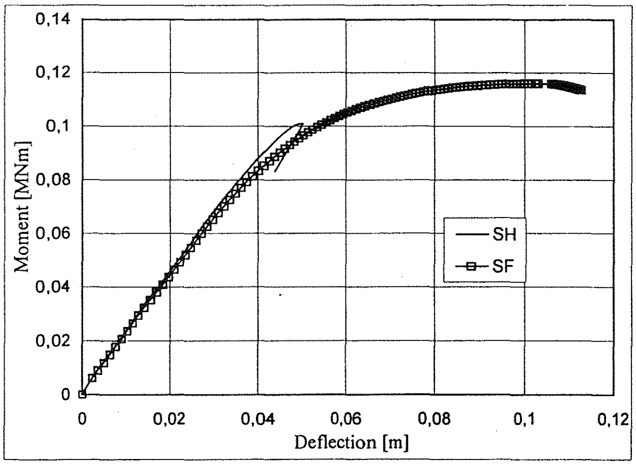


Fig. 6. Calculated moment-deflection curves of the test beams made of HSC and FRC

5 Conclusion

Based on the analysis conducted with the BDZ-Modell only beams made of normal strength (NSC) and fibre reinforced concrete (FRC) should exhibit a stable post-peak behaviour if tests are controlled by the deformation in the concrete compression zone between the two loading platens. For fibre reinforced concrete beams the analytical results depend mainly on the fracture energy G_F. In this test series G_F was only obtained from three-point bend tests and not from direct tensile tests. Therefore the ductility of the beams might be overestimated in this calculation.

Beams made of high strength concrete without fibres (HSC) fracture in a brittle manner at the peak load. The analysis indicated that a descending branch cannot be recorded for these specimens despite using a deformation controlled test set-up.

Regarding the different dimensions of beams made of NSC the deformation capacity of the concrete compression zone decreases with increasing cross-sectional areas. The total deformation capacity of the large beams is probably smaller compared to the capacity of the smaller beam. So, deformation capacity of the concrete compression zone seems to be size-dependent.

6 References

European Committee for Standardization (1992) **Eurocode 2 Part 1: Design of concrete structures - General rules and rules for buildings**.

Markeset, G. (1993) **Failure of concrete under compressive strain gradients**. NTH Universitetet i Trondheim.

Meyer, J. (1997) **Ein Beitrag zur Untersuchung der Verformungsfähigkeit von Bauteilen aus Beton unter Biegedruckbeanspruchung.** Thesis, Universität Leipzig.

Popovic, S. (1973) A numerical approach to the complete stress-strain curve of concrete. **Cement and Concrete Research**, Vol. 3, pp. 583-599, Pergamon Press, Inc..

Ulfkjaer, J. P., van Mier, J. G. M. and Stang, H. (1997) Invitation to a competition on: Modeling the behavior of over-reinforced concrete beams. RILEM Technical Committee 148 SSC "Strain Softening of Concrete" and ACI/ASCE committee 447 on "Finite Element Analysis of Reinforced Concrete Structures"