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# LATTICE TYPE MODELS FOR FRACTURE: METHODOLOGY AND FUTURE PROSPECTS

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**Abstract**
In lattice-type models the material is discretized as a network of linear elements. Fracture can be simulated by either removing the elements, or through a step-wise reduction of the stiffness. For fracture this simple and straightforward technique was re-introduced in theoretical physics in the late 1980s. Main aspects of such models are the discretization, i.e. the selection of the type of linear elements and their connectivity, the incorporation of the (statistical) information from the material structure and the selected fracture criterion. An advantage of the lattice models is their inherent simplicity, and in many cases, by omitting numerical iteration loops, a seemingly precise approximation of pattern growth that matches well with experimental observations can be obtained. The models can also be adapted for analyzing moisture flow, breakdown of electrical networks, and also combinations of these.
Key words: Lattice model, network geometry, material structure, 3 level approach, fracture law,

## 1 Introduction

Fracture is a timely research topic. After the development of linear elastic and plastic crack models in the past decades, many researchers are focusing on

non-linear materials like concrete, rock, ceramics and other composites. Some of these materials exhibit elastic-brittle behaviour at a local level (i.e. grain level or meso-level), whereas at a global scale (macroscopically) a so-called softening behaviour is observed. The development of the various models goes often hand in hand with the development of new advanced testing techniques, such as the emergence of closed-loop test methods, even for so-called class-II materials that exhibit snap-back phenomena. By starting the analysis from the structure of the material under consideration, crack growth processes can be simulated in great detail. The disadvantage of such meso- (or sometimes even micro-) scale approaches is the enormous computational effort needed, and, moreover, the experimental determination of the local properties of the materials is not always straightforward. Nevertheless, the meso-level models seem a simple and promising tool for materials engineering, and interest has increased over the years.

The first to develop a meso-level model for concrete were Roelfstra et al. in 1985, rapidly followed by many others, e.g. Zubelewicz & Bazant (1987), Stankowski (1990), Bazant, Tabbara, et al. (1990), Berg & Svensson (1991), Vonk (1992), Schlangen & Van Mier (1992), Beranek & Hobbelman (1994), Wang (1994) and many others. All these models have in common that in some way the micro- or meso-structure of the material is incorporated in the analysis. Underlying all these models are either the finite element method, the finite difference method or molecular dynamics. Moreover, in any of these models a local fracture criterion must be set, and differences are met there as well. Some models are based on assumptions from the classical strength of materials theory, plasticity theory, linear elastic fracture mechanics, non-linear fracture mechanics, or any other energy concept.

Quite promising for micro- and meso-level analysis of material behaviour is the lattice approach, or network approach. These models are certainly not new, but with the development of powerful digital computers, the technique has regained interest, and due to its flexibility in incorporating the structure of the material under consideration makes it a very powerful tool indeed, see Herrmann et al. (1989), Schorn & Rode (1989), Jirásek & Bazant (1995), Schlangen (1993) and Vervuurt (1997). Very important is also that the model is based on simple linear elements from which the lattice is constructed. These elements have very simple local behaviour laws, which makes the model readily accessible for everyone. In particular the combination of lattice modelling with experiments has proven to be very powerful, not in the least place because realistic crack geometries are generated which can be compared to experimental crack geometries (see Van Mier (1997)), but also because we should prevent that a virtual fracture world develops. Note that geometry is

one of the few experimental parameters that is not affected by theoretical biases. In this overview the elements making up a lattice model are presented. Some present day achievements and directions for the future will be discussed. No explicit examples are shown, but then, *ideas prevail above knowledge.*

## 2 Constructing the lattice

Constructing a lattice or network starts from a grid of nodal points in a plane (2D) or in space (3D). The grid points can be used directly to define the mechanical (or any other physical) relationships between either neighbouring or more distantly located points. The grid can be set up from simple geometric rules, with no direct relationship between the geometry of the grid and the material that is to be analyzed. Alternatively, one can try to incorporate the structure of the material under consideration directly into the lattice. In the present paper we will limit ourselves to 2D lattices. In principle the discussion can be extended to three-dimensional cases, but this is subject of future study and development.

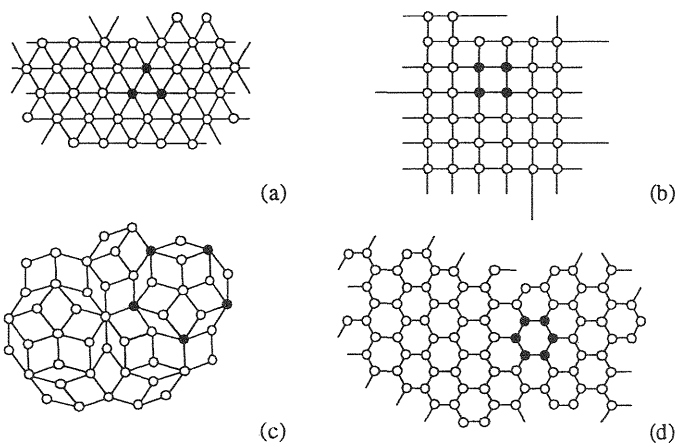


Fig.1. Basic geometrical grids with threefold (a), fourfold (b), fivefold (c), or sixfold (d) symmetry.

### 2.1 Geometrical constructs in a plane

In Figure 1, four examples of basic plane grid patterns are plotted. The smallest basic grid cell is a triangle (Figure 1a). Using equilateral triangles, a plane can be filled completely, and no gaps exists. The same can be done with a grid constructed from square cells (Figure 1b), or hexagonal cells (like a

honeycomb, as shown in Figure 1d). Fivefold symmetry is more difficult, and gaps between the basic grid cells will appear. In 1973 Penrose constructed a plane tiling with fivefold symmetry (see for example in Penrose (1989)), that was later found in nature as well (and which is now known as a quasi-crystal). This so-called Penrose tiling is depicted in Figure 1c. Note that in this latter example, a mixture of four-noded grid boxes is used.

At the atomic level, these grids can be used immediately, viz. atoms are often found at regularly spaced intervals. When modelling materials at higher dimensional levels, for example the micro- or macro-level, additional steps must be taken as will be explained in section 3.

Next to the above sketched regular grids, irregular grids can be constructed. One of the methods is based on a Voronoi construction, and was developed by Mourkazel & Herrmann (1992). Starting from a regular grid, for example a regular square grid, a point is selected at random in each of the grid cells. Next, the three points that are closest to one another are connected. Or stated differently, dependency relations are prescribed for the nodes that are most closely spaced. In Figure 2, the principle of constructing a network with random interval length is depicted. Because in the mechanical analysis, the interval length should not be too close to zero, a node can be selected in a sub-box which is defined inside a grid cell. By varying the size of the sub-box, one can change the randomness, see for example in Vervuurt (1997) and Chiaia et al. (1997).

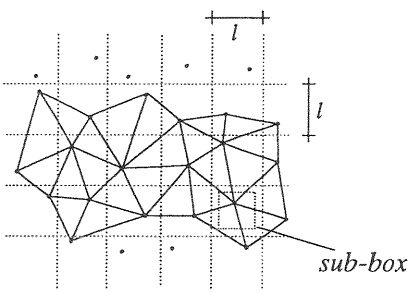


Fig. 2 Construction of a triangular lattice with random interval length.

In the above networks, always a node is connected to all its closest neighbours. A different option is not to limit the connectivity to the closest nodes, but to specify that $n$ connections are made to more distant nodes in the grid as well. Examples of such networks can be found in literature, e.g. Burt & Dougill (1977) and Berg & Svensson (1991).

## 2.2 Dependency relations

In the following, the discussion is limited to mechanical analysis. In describing the dependency relations in a lattice, one can consider the grid points as lumped material points, or, as an alternative, one might consider the linear elements as rigid (non-deformable) elements. According to the first point of view, which is shown schematically in Figure 4a, the deformations $u$ and $v$, as well as the in-plane rotation $\varphi$ between two grid points $i$ and $j$, must be related to forces and bending moments. In the most general case we must consider the relation between normal force $N$ and normal displacements $u_i$ and $u_j$, between the shear force $D$ and the shear displacements $v_i$ and $v_j$ as well as the nodal rotations $\varphi_i$ and $\varphi_j$, and the relation between bending moments and the shear displacements and nodal rotations, as explained earlier by, among others, Herrmann et al. (1989), and Schlangen & Garboczi (1997). They are all well known relations from linear elasticity, and the main parameters are the cross-sectional area $A$ of the element connecting two grid points, and the moment of inertia $I$. Next to that the grid spacing $l$ is of importance as it defines the length of the elements. Not necessarily all these relations are incorporated in the model. If only the normal force-normal displacement relation is used, a so-called central force lattice is constructed. Force and displacement are coupled through a spring constant $K = EA/l$, where $E$ is the Young's modulus of the material. In fact, the lattice resembles a truss, where rotations in the nodes can occur freely. Such models are known for a long time indeed. Hrennikoff (1941) used them to estimate stresses in problems in elasticity.

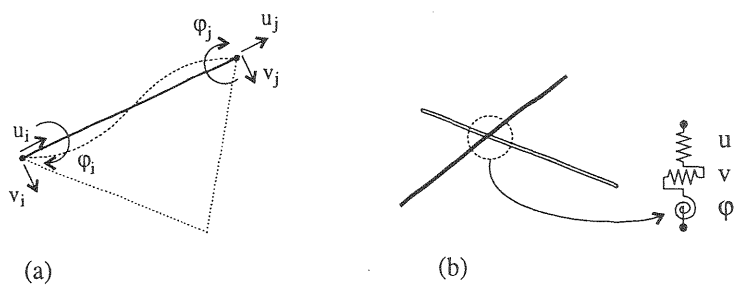


Fig. 3 Lattice analysis based on relations between nodes (a) and an alternative where the relation between the linear elements is considered (b).

The disadvantage of such models is that the Poisson's ratio is constant and equal to 1/3. When shear forces and/or bending moments are incorporated in the formulation, a Poisson's ratio smaller than 1/3 is obtained, depending on the cross-section area, the shape and the length of the linear elements (beams in

this case). For further information, the reader is referred to Schlangen & Garboczi (1997) and Vervuurt (1997).

In the above view (Figure 3a), the relation between the grid points is established through linear elasticity. One might also assume that the connecting elements are rigid, and that all processes take place in the grid points, as was done by Heyden (1996). Heyden looked to the structure of cellulose fibre materials. However, one might consider calcium silicate hydrates at a micro-scale, or SIFCON (Slurry Infiltrated Fibre Concrete) at a larger scale, as well. Fibre interactions occur at the locations where the fibres meet. These are not necessarily the outer ends of the fibres. Instead, one or more spring-type connections might exists between two fibres at any location along its length as depicted in Figure 3b. The spring connection may consist of a two-dimensional spring in combination with a rotational spring, at least when the model is limited to two dimensions.

## 3 Incorporating information of the material structure

As mentioned, the geometrical constructs of section 2.1 resemble real material structures only at the atomic level. The atoms are located in the grid points (nodes), and the direct connections are established through stress-separation relationships, see for example Kelly & MacMillan (1986). It is common practice to simplify the force-separation relation to a sine function, for example,

$$\sigma = \sigma_{max}.\sin[\frac{\pi}{l}(x - l_0)] = \sqrt{\frac{\gamma E}{l_0}}.\sin[\frac{\pi}{l}(x - l_0)] \qquad (1)$$

where $\gamma$ is the surface energy, and $x$ is the coordinate defining the separation between the two nodes. $l_0$ is the atomic spacing where the system is in equilibrium. When the spacing $l$ (which is equal to the lattice spacing) becomes too small, repulsion occurs, whereas under extension, after an increase of the bond stress, softening behaviour is observed. This softening behaviour must not be confused with softening of concrete at the macro-level. The stress-separation law is depicted in Figure 4. Eq. (1) forms a fracture law at the atomic level, but for concrete, rock and many other materials we are interested in modelling the fracture behaviour at a larger scale. The geometrical constructs must then be transformed in order to take into account the structure of the material at the level of observation.

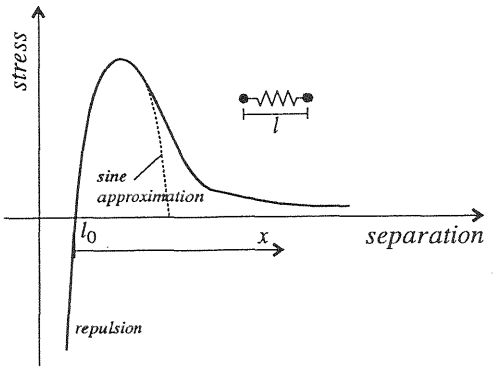


Fig. 4 Stress-separation law for atomic bond.

When the relationship between the rigid linear elements in a lattice is described at their contact points, the elements are the fibres and through springs the bond between the fibres and some type of matrix material is described. At a very small level, for example the fibrous structure of CSH in hardened cement paste, one might assume that Van der Waals forces act at the cross-points between the fibres. As the distance between the fibres is regulated by the amount of water in the material, the RH might be the controlling factor in the attraction force between the fibres.

Now let us return to the structure of the material when the model with bending beams, bars or springs is used. The effect of the material structure can be incorporated in a number of ways. First of all, one can prescribe *random material properties* (i.e. Young's modulus $E$ or breaking threshold $f_t$ (or local strength) of the lattice elements, or simply in the presence of bonds). For example, a Gaussian or Weibull distribution can be used for the breaking threshold, see Van Mier et al. (1997). The second approach could be called *element distortion method.* Starting from a regular grid (Figure 1), elements are distorted according to some geometrical rule. This method was for example applied by Vonk (1993) and Argyris et al. (1994). In both cases, hexagons were distorted to resemble irregularly shaped grains in the material. The third option is to *identify preferential stress flow directions* in a real material, for example dominant grain contacts. The lattice elements are then placed along these lines. This was done, for instance, by Beranek & Hobbelman (1994), for modelling clay fired bricks and concrete as an assemblage of spheres. The fourth manner is to *map a computer generated structure of the material on top of the lattice*, and to identify different material properties for lattice elements falling in the distinct phases. Computer generated assemblages of spheres were for example made by Stroeven & Stroeven (1996) and Schlangen & Van Mier

(1992). Finally, one might *map a digital image of a real material structure on the lattice*. This was done by Schlangen (1995). After the mapping, again different properties are assigned to elements falling in different phases of the material. Some researchers claim that the two last mentioned methods do not include the real load-carrying mechanisms in particle composites like concrete, e.g. see the criticism by Beranek & Hobbelman (1994). They claim that, at least in two-dimensional computations, the idea of rigid particles 'swimming' in a soup of matrix material is not realistic as direct particle contacts are omitted from such models. For compression, this point certainly makes sense, for tension however, this seems of lesser importance as will be addressed in the next section. The essential information of the lattice beams is reduced to the Young's modulus and a breaking threshold for all the different phases in the material. Porosity can simply be modelled by randomly omitting lattice elements as was done, among others, by Arslan (1995). The fineness of the lattice, i.e. the grid length, then determines the minimum pore size.

## 4 Fracture laws

After the lattice has been constructed, and the different material phases have been incorporated in the model, a law for fracturing the elements must be defined. At the atomic level, a stress-separation law must be implemented, as was already touched upon in section 3. At the micro- and meso-level different models must be specified. One might decide to compute the behaviour laws at these levels from atomic level simulations, but in view of the complex microstructures, the many unknowns and the capacity of current day digital computers this seems an impossible task. Therefore one must necessarily rely on phenomenological laws. A number of possibilities exist; most of them are related to existing macroscopic fracture laws such as the Rankine, Von Mises, Mohr-Coulomb criterion or other. The simplest case is to use a central force lattice with free rotations in the beams, and to specify a brittle breaking threshold under tension. In that case the criterion resembles the well known Rankine criterion. Thus fracture is governed by normal stress only

$$\sigma = \frac{N}{A} \leq f_t \qquad (2)$$

where A is the cross-sectional area of the lattice element, and $f_t$ is the tensile strength of the material. This tensile strength is for the material phase in which the particular lattice element is located. The easiest way to include this simple brittle breaking law in the computation is to remove the element from the mesh

as soon as the breaking threshold is exceeded. Crack growth is then simulated as the result of subsequent removals. The advantage is that the stress redistributions due to crack growth occur in a very controlled manner. Note, however, that the results obtained in this manner depend on the size of the mesh, a problem that remains to be solved.

The elastic brittle fracture law is depicted in Figure 5a. The important material parameters are the Young's modulus and the breaking threshold. As an alternative, one might consider to include plasticity (Figure 5b) or even softening at the level of the lattice elements (Figure 5c). Then of course an iterative approach is needed, although a step-wise reduction of the stiffness with constant breaking threshold in the case of plasticity, or a decreasing breaking threshold in the case of softening, may preserve the attractiveness of the step-wise approach. These latter simplifications are depicted as dashed lines in Figure 5b and 5c.

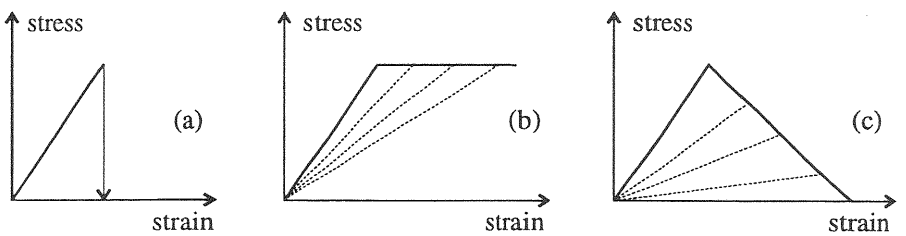


Fig. 5 Different type of material behaviour that can be assumed at the level of the lattice elements: (a) elastic-purely brittle; (b) elasto-plastic; and (c) elastic-softening.

A more complex failure law can be used when beam elements are applied. In that case, one might consider a Tresca/Von Mises or Mohr-Coulomb type of fracture law. The first case was considered by Herrmann et al. (1989), and subsequently used by Schlangen & Van Mier (1992), Schlangen (1993) and Vervuurt (1997). An effective beam stress is computed with contributions from the normal load and bending moments following

$$\sigma_{eff} = \frac{N}{A} \pm \alpha . \frac{(|M_i|, |M_j|)_{\max}}{W} \leq f_t \qquad (3)$$

where $W$ is the section modulus, and $M_i$ and $M_j$ the bending moments in the nodes $i$ and $j$ respectively. For situations where the tensile component dominates, this criterion seems to work quite well. However, for compressive

failure problems are met. In that case a Mohr-Coulomb type failure law seems to make more sense, as was demonstrated first by Beranek & Hobbelman (1994), and was later confirmed by Van Mier & Van Vliet (1996). In that case the failure law is

$$F_{crit} = \frac{1}{2} F_t \cdot [\frac{-1 \pm \sqrt{1 + 4(D/N_{act})^2}}{(D/N_{act})^2}] \quad (4)$$

where $F_t$ is the maximum tensile load which can be carried by the lattice element. Using the failure law of eq. (4), the biaxial failure contour of concrete can be computed. A closed envelope in the compression-compression regime is found, whereas an open ended envelope is computed in the compression-compression regime when the failure law of eq. (3) is used. From macroscopic finite element analysis it is well known that the Mohr-Coulomb criterion is quite suitable for concrete and many types of rocks.

## 5 Trends and future developments

The advantage of the lattice type models, as well as continuum based models where the structure of the material is included in the model (i.e. numerical concrete), is the capability to compute (or simulate) macroscopic material laws. In particular for fracture problems, an important size and boundary condition dependency exists (see for example in Van Mier (1997)), which can be simulated by simply taking into account the experi-mental conditions of the experiments that are compared to the outcome from the simulations. These experimental conditions include loading platens, hinges, actuators, guiding devices and other elements normally considered as part of the testing environment.

The question is now whether the above mentioned lattice models are a sufficient tool to capture all aspects of material behaviour. Less is true of course. Problems where a hydrostatic compressive stress occurs, or when frictional effects may be expected, are more difficult to capture, as was the experience over the past few years. Moreover, many of the models have focused on particle composites, whereas important developments in material technology include fibrous materials and laminates as well. In principle the model can be used for such materials as well, provided of course that the essential additions, such as a frictional slip element, are included in the lattice model. Important seems to preserve the step-wise approach where in each step of the analysis a single element is removed, or a certain parameter in a lattice

element is changed (for example the aforementioned step-wise reduction of the Young's modulus of a lattice element).

Another important development is to include three-dimensional effects. To date most models were limited to two-dimensions. It is obvious that the fracture process in concrete and many types of rock is a complex three-dimensional growth process. Many details of the fracture process are missed when one limits the simulations to plane-stress. In the end, for problems such as anchor pull-out, a fully three-dimensional analysis is needed.

The other open questions concern dynamic, fatigue (including crack closure effects) and creep behaviour, as well as coupled problems like a combination of drying shrinkage and mechanical behaviour, or temperature effects (for instance from hydration in young hardening concrete) in combination with a mechanical analysis. Including such effects may eventually lead to a more complete description (or as some researchers would claim, prediction) of laboratory scale experiments. In this respect it should be mentioned that a rational procedure for estimating the properties of the lattice elements is an essential step in the development of this class of numerical fracture models.

So where would we find applications for the lattice type fracture models ? Several possibilities have been sketched in the past, namely, as tool for designing laboratory scale fracture experiments, as a tool to design new composite materials, as a pre-processor to determine the effective material models for macroscopic non-linear finite element analysis of full-scale reinforced concrete structures, or as a direct application for the analysis of structural details such as the anchor pull-out problem. From experiments new information is needed to tune the models, namely information on the complex crack growth process and crack patterns is needed. The fractal character of such patterns may be a tool in assessing the material parameters in lattice type models. This approach seems to prevail because a direct evaluation of geometrical patterns is needed only. As was argued before, (euclidian) geometry seems the only direct measurable quantity without theoretical bias.

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