Fracture Mechanics of Concrete Structures
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INFLUENCE FACTORS MATRIX METHOD FOR SIMULATION OF FRACTURE PROCESS OF CONCRETE

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Abstract
Based on analysis of failure mechanisms of concrete at its internal structural levels, a fracture process model of concrete is proposed. Then a numerical method named *Influence Factors Matrix method* is developed and is used to simulate the effect of specimen's size on the fracture behavior of concrete.
Key words: Concrete, fracture process, simulation, model, size effect

1 Introduction

The internal structure of concrete is very complicate. It has been proven that it is helpful for concrete to subdivide the internal structure into three levels, i.e. micro-level, meso-level and macro-level according to Wittmann (1987). At micro-level the structure of hardened cement paste is studied. At meso-level the important structural elements are pores, cracks, inclusions and interface. At macro-level concrete is considered as an elastic and homogeneous material in engineering applications.

The key feature of fracture in concrete is the nucleation of relatively large fracture process zone (FPZ) around the crack tip. The failure occurring within FPZ can also be characterized by the three internal structural levels. At micro-level the intrinsic fracture process of concrete includes the break of Si-O bond at the contact points of C-H-S sheets, cleavage fracture of CH crystalline and cracks along the hydrated cement particles. At meso-level the reinforcing phases such as aggregates or/and fibers are pulled out from matrix or/and broken. Therefore, it is appropriate to study the failure phenomena inside FPZ and to evaluate the effects of FPZ on the macroscopic fracture behaviors from viewpoint of internal structural levels.

2 Fracture process model of concrete

According to the analysis of failure mechanisms based on internal structure of concrete by Zhang and Wu (1997), a fracture process model is composed of the following hypotheses.

1. Linear elasticity: Undamaged concrete body outside FPZ is assumed to be linearly elastic;
2. Criterion for emergence of FPZ: FPZ emerges as soon as the stress intensity factor K_I at crack tip is equal to the fracture toughness of cement matrix(K_{IC})

$$K_I = K_{IC} \tag{1}$$

3. Criterion for saturation of FPZ: FPZ is saturated as soon as the crack tip opening displacement ($CTOD_{tip}$) is equal to the maximal pull-out displacement (w_m) of reinforcing phase;
4. The constitutive law of FPZ in concrete is simplified by the pullout force-displacement relationship

$$F=f(w) \tag{2}$$

On the basis of the hypotheses above the fracture process of concrete can be described qualitatively. For example, fracture process of a notched three-point bended beam is illustrated in figure 1.

(a) Firstly, when the load is relatively small, the stress intensity factor K_I at notch tip is less than the fracture toughness of cement matrix

($K_I<K_{IC}$). According to hypothesis 2, no FPZ emerges. The notched beam behaves as a linearly elastic body and the load-deflection (or CMOD) curve is linear.

(b) Secondly, when the stress intensity factor K_I at notch tip is gradually increased and becomes equal to the fracture toughness of cement matrix (K_{IC}), FPZ appears ahead of the notch tip. Hereafter load-deflection (or CMOD) becomes nonlinear.

(c) During the development of fracture process zone,

$$K_a + K_c = K_{IC} \tag{3}$$

where K_a and K_c stand for the stress intensity factors at the frontal end of FPZ caused by external force and closing force of reinforcing phase such as aggregates or/and fibers inside FPZ, respectively.

(d) As FPZ develops, the notch tip opening displacement ($CTOD_{tip}$) increases. When $CTOD_{tip}$ is equal to w_m, FPZ is saturated.

(e) Then saturated FPZ moves ahead as further extension of crack.

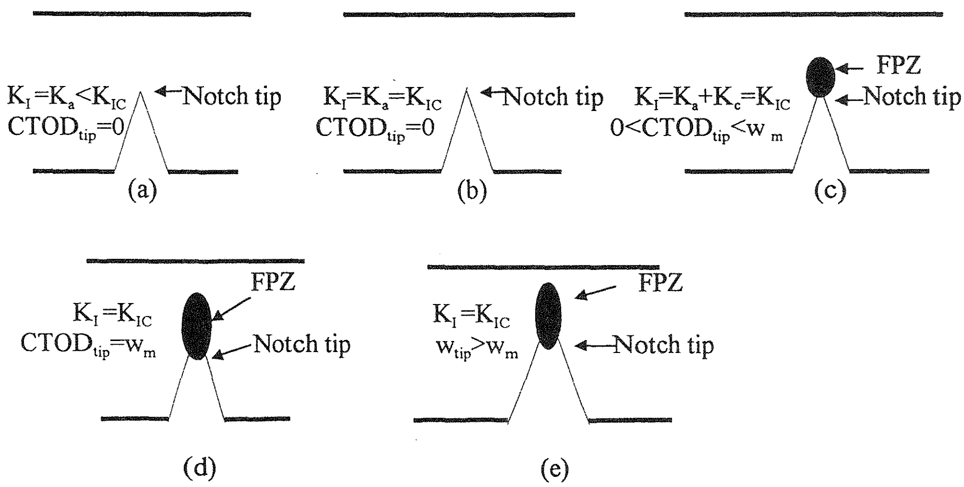

Fig. 1. Illustration of fracture process model of concrete

3 Numerical method for simulation of fracture process of concrete -*Influence Factors Matrix Method*

The fracture of concrete is a very complicate process. Inside the FPZ of concrete, the closing force caused by aggregates or/and fibers is coupled with the opening displacement at the zone acting the closing force. This coupling brings much difficulty to simulation and analysis of fracture process of concrete. In the present paper, a method named *Influence Factors Matrix Method* is developed to solve the coupling problem and to achieve the simulation and the analysis of fracture process of concrete.

3.1 Influence factor

The concept of influence factor is illustrated in figure 2, where a crack is opened by external forces. A pair of forces P acting at site AA' on the crack surfaces influences the displacement at other sites on the crack surfaces. Take site BB' for example, the influence factor of site AA' on BB' is defined as

$$f_{BA}=w_{BA}/P \tag{4}$$

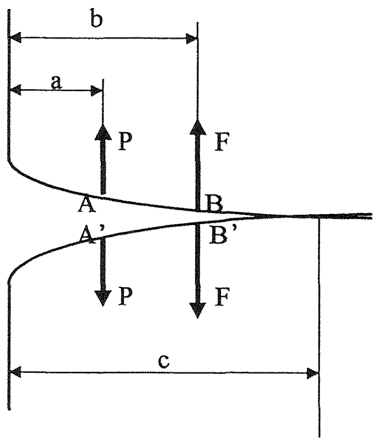

Fig. 2. Illustration of influence factor

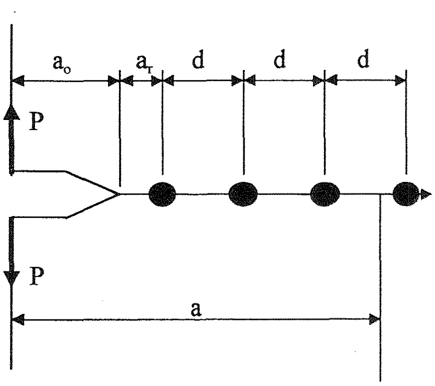

Fig. 3. Illustration of influence factors matrix method

The displacement at site BB' induced by forces at site AA' can be expressed as

$$w_{BA}=f_{BA}\cdot P \tag{5}$$

According to Tada et al (1973), the influence factor between different sites on the crack surface in specimens with any shape has the form of

$$f_{BA} = \frac{2}{E} \cdot \int_0^c (K_P / P) \cdot (\partial K_F / \partial F)_{F=0} dc \quad (6)$$

where P and F are two pairs of forces acting at site AA' and BB', K_P and K_F are stress intensity factors caused by the forces P and F, respectively. E stands for elastic modulus of the materials and c the length of crack .

3.2 Influence factors matrix method

The crack extension in figure 3 is taken into consideration, where the continuously distributed closing force in FPZ is simplified by discrete closing forces. If the spacing (d) between discrete closing forces is sufficiently small, the error caused by the simplification could be negligible in the similar manner to finite element method.

For the convenience of explanation, the crack extension in half infinite plate is analyzed. In figure 3, a_0 and a represent length of initial crack and extended crack, a_r and d represent distance between the tip of initial crack and the first closing force site and the spacing between closing force sites.

1. When $P < \frac{\pi}{2\phi} \cdot \sqrt{a_0} \cdot K_{IC}$, $K_I < K_{IC}$, the initial crack does not begin to extend.

2. When $P \geq \frac{\pi}{2\phi} \cdot \sqrt{a_0} \cdot K_{IC}$, $K_I \geq K_{IC}$, the crack begins to extend, while the tip of the crack does not reach the first closing force site. In this case,

$$P = \frac{\pi}{2\phi} \cdot \sqrt{a} \cdot K_{IC} \quad (7)$$

$$a = a_0 + \Delta a$$

where Δa represents the extension of crack.

3. When the crack tip extends into the closing force sites, $a_0 + a_r + (i+1) \cdot d > a > a_0 + a_r + i \cdot d$, some closing force sites are activated. Here the force sites are numbered, the site of external load is given the number of 0, the closing force site nearest to the crack tip is numbered as 1, and the next closing force sites are given the numbers of 2, 3 .... . f_{s0} represents the influence factor of 0# site on itself, f_{I01} represents the influence factor of 0# site on 1# site, and so on.

When the crack tip extends between the i and i+1 site, the opening displacements at the force sites are

$$\begin{aligned}
w_P &= f_{s0} \cdot P + f_{I01} \cdot F_1 + f_{I02} \cdot F_2 + \cdots + f_{I0i} \cdot F_i \\
w_1 &= f_{I01} \cdot P + f_{s1} \cdot F_1 + f_{I12} \cdot F_2 + \cdots + f_{I1i} \cdot F_i \\
w_2 &= f_{I02} \cdot P + f_{I12} \cdot F_1 + f_{s2} \cdot F_2 + \cdots + f_{I2i} \cdot F_i \\
&\vdots \qquad\qquad\qquad\qquad \vdots \\
w_i &= f_{I0i} \cdot P + f_{I1i} \cdot F_1 + f_{I2i} \cdot F_2 + \cdots + f_{si} \cdot F_i
\end{aligned} \tag{8}$$

where $w_P, w_1, w_2 \ldots\ldots w_i$ are the opening displacements at force sites, P stands for external force, $F_1, F_2 \ldots\ldots F_i$ are closing forces.

Then, the stress intensity factor (K_I) is obtained as,

$$\begin{aligned}
K_I &= K_P + K_{F_1} + K_{F_2} + \cdots + K_{F_i} \\
&= A_P \cdot P + B_1 \cdot F_1 + B_2 \cdot F_2 + \cdots + B_i \cdot F_i
\end{aligned} \tag{9}$$

where, A_P, B_1, $B_2 \ldots\ldots B_i$ are the stress intensity factors due to unit external force, 1# closing force, 2# closing force......i# closing force.

The relationship between the closing force and the crack opening displacement can be represented by a function,

$$F = f(w) \tag{2}$$

or its inverse function

$$w = g(F) \tag{10}$$

So coupled equations with i+1 unknowns can be composed, as follows;

$$\begin{cases}
f_{I01} \cdot P + f_{s1} \cdot F_1 + f_{I12} \cdot F_2 + \cdots + f_{I1i} \cdot F_i = g(F_1) \\
f_{I02} \cdot P + f_{I12} \cdot F_1 + f_{s2} \cdot F_2 + \cdots + f_{I2i} \cdot F_i = g(F_2) \\
f_{I03} \cdot P + f_{I13} \cdot F_1 + f_{I23} \cdot F_2 + \cdots + f_{I3i} \cdot F_i = g(F_3) \\
\quad \vdots \qquad\qquad\qquad\qquad \vdots \\
f_{I0i} \cdot P + f_{I1i} \cdot F_1 + f_{I2i} \cdot F_2 + \cdots + f_{si} \cdot F_i = g(F_i) \\
A_P \cdot P + B_1 \cdot F_1 + B_2 \cdot F_2 + \cdots + B_i \cdot F_i = K_{IC}
\end{cases} \tag{11}$$

By solving these equations the i+1 unknowns P, F_1, F_2......F_i can be determined. Then we can calculate the opening displacement at external force site and the closing force sites. If the opening displacement at 1# closing force site (w_1) is larger than the maximal opening displacement (w_m), 1# closing force site is deactivated, and abandoned. Then the number of the remaining closing force sites is re-arranged and coupled equations with i unknowns are composed again. By solving these equations, P, F_1 ...... F_{i-1} and w_P, w_1......w_{i-1} are determined. Comparing w_1 and w_m again, if $w_1 > w_m$, repeat the above procedure until $w_1 \le w_m$ is satisfied. By gradually increasing the length of the crack, the external load, displacement, the crack profile and the evolution of FPZ can be determined, and the fracture process of concrete is simulated.

4 Applications

By comparing the theoretical results on the fracture process of concrete and experimental results, the effects of material and geometrical parameters on the fracture behavior can be quantitatively studied. It may provide some theoretical clues for new concrete materials research and improvement of design methods of concrete structures. Here the effect of specimens' sizes on the fracture behavior of concrete is studied using above-mentioned fracture process model.

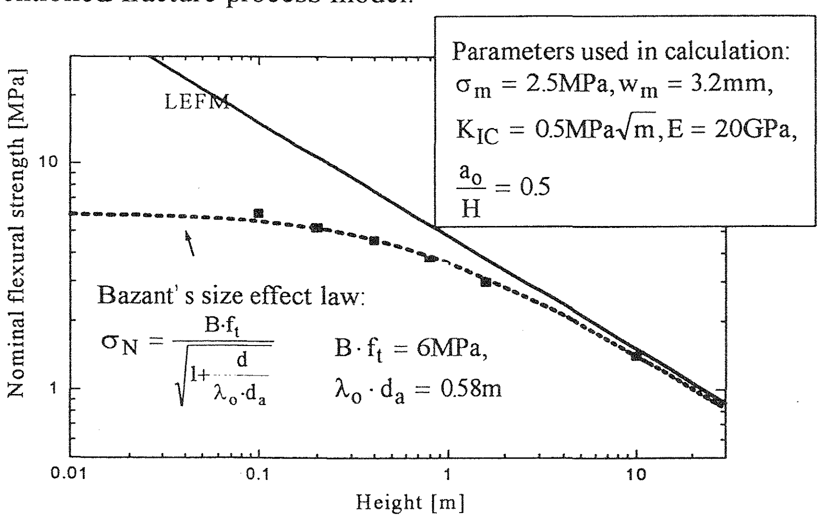

Fig. 4. Size effect of nominal flexural strength of geometrically similar three-point bended specimens

The simulated results on nominal flexural strengths of geometrically similar three-point bended specimens are represented by the dots in Figure 4. Parameters used in calculation are shown in the figure. The solid and dashed lines represent the size effect law of LEFM and Bazant (1984), respectively. The simulated results on the size effect of flexural strength are in good agreement with Bazant' s size effect law.

As the research on the size effect is going further, much more attention is being paid to the non-geometrically similar size effect of concrete' s strength. Because defects in the concrete structures or member are not always proportional to the size of the structures, the non-geometrically similar size effect of concrete's strength is more practical than the geometrically similar size effect.

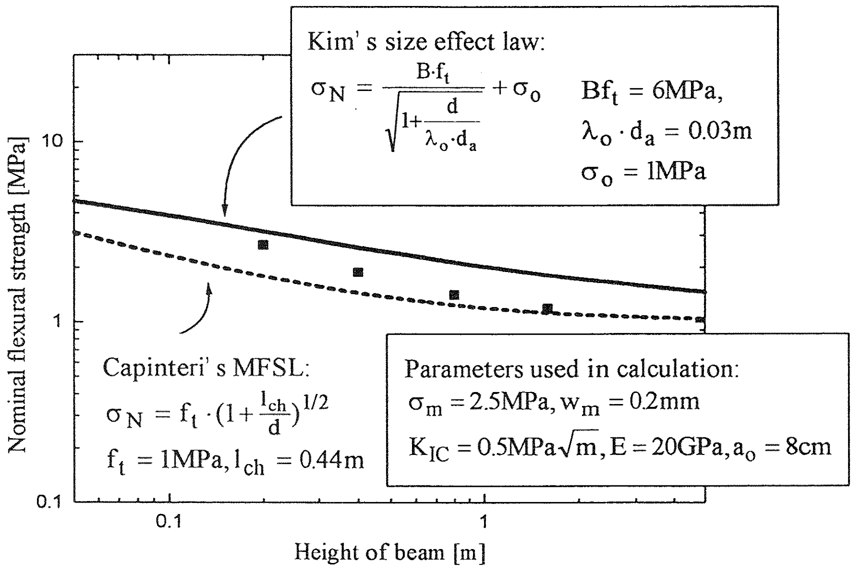

Fig 5 Size effect of nominal flexural strength of non-geometrically similar three-point bended specimens

The solid rectangular dots in figure 5 represent the simulated results on the size effect of three-point bended beams with notches. Parameters used in calculation are shown in the figure too. The solid and dashed lines represent the size effect laws of Kim (1994) by the modification of

Bazant's size effect law and Multifractal Scaling Law (MFSL) of Carpinteri (1996), respectively. By comparing figure 4 and 5, it is found that the non-geometrically similar size effect is quite different from the geometrically similar size effect. In the case of geometrical similarity, nominal flexural strength of concrete decreases with increase of specimen's size. As the size of specimen becomes very large, the size effect is the same as that of LEFM. But in the case of non-geometrical similarity, nominal flexural strength of concrete decreases with increase of specimen's size, and approaches some constant value as the size of specimen becomes very large. By observing figure 5, the simulated results are in better agreement with the size effect proposed by Kim and Carpinteri.

5 Conclusion

In the paper, a fracture process model of concrete and a numerical method named *Influence Factors Matrix Method* are proposed and used to study the geometrically and non-geometrically similar size effect of nominal flexural strength of concrete. The size effect by the simulated results is comparable to the size effect laws of Bazant, Kim and Carpinteri. it suggests that the fracture process model and *Influence Factors Matrix Method* are useful for the analysis of fracture behavior of concrete.

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