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NON LOCAL COMPUTATIONAL MODELING OF THE SOFTENING BEHAVIOR OF EARLY AGE CONCRETE

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Abstract

The softening behavior of early age concrete is studied with the aid of a gradient plasticity model where mode-I crack generation of a simple tension specimen is simulated by a smeared cracked zone. In this regard a new and simple softening function is proposed which complies with the other available softening rules but versatile enough to be applicable for early ages as well as for matured concrete. The use of separate gradient influence functions are demonstrated which increases the scope of the gradient plasticity theory. In this context a semi-empirical formula for the estimation of fracture energy of concrete is proposed. It has been shown that with the aid of the proposed model we can thoroughly investigate the strain softening behavior of early age concrete.
Key words: Fracture, strain localization, non-local, gradient plasticity.

1 Introduction

Computational modeling of the constitutive relation of concrete incorporating fracture and strain localization is one of the most addressed problem in the recent past. As a result, a number of powerful computational techniques emerged of which gradient plasticity model by de Borst and Mühlhaus (1992) and Pamin (1994) is a very promising one.

However, their works were concentrated mainly on establishing the basic mathematical background of the computational modeling and numerical algorithm using simpler material models. They demonstrated the application of gradient plasticity theory using very ideal cases having very limited practical application. The gradient plasticity technique is a computational technique suitable for finite element application of strain localization problems. In this model non-locality is obtained by incorporating a second gradient of some damage parameter in the damage criterion. In this respect a number of factors are necessary to consider before such a model is applied in a real case which includes global material properties as well as parameters specific to the particular type of non-local formulation. The basic material parameters that are of interest are the tensile strength, the rate of degradation of strength after the peak, characteristic length, unit fracture energy etc. Depending upon the type of formulation there are additional parameters of interest.

In this paper focus is given on the use of gradient plasticity model in investigating the fracture and strain localization problems of early age concrete through the development of an appropriate softening rule. In this regard, an approximate formula for the estimation of fracture energy is given which incorporates a parameter called gradient influence factor (g) specific to the gradient plasticity model. It is demonstrated that such evaluation of fracture energy enables us to use any reasonable form of g instead of taking it dependent on the softening rule as shown by Pamin (1994).

2 Softening Behavior Modeling

In gradient plasticity modeling, the failure function has the following form,

$$f(\sigma,\kappa,\nabla^2\kappa) = \varphi(\sigma) - \bar{\sigma}_g(\kappa,\nabla^2\kappa) \tag{1}$$

where $\phi(\sigma)$ is a function of stress components (usually stress invariants) and σ_g is the yield strength which depends on both κ and $\nabla^2\kappa$. Here κ is a measure of damage or plastic flow and $\nabla^2\kappa$ indicates Laplacian of κ with respect to the global coordinate system. For one-dimensional case $\phi(\sigma)$ is simply the axial stress σ. In two-dimensional analysis $\phi(\sigma)$ may take various forms such as Rankine's principal stress criterion or von Mises criterion etc. In this paper, fracture of early age concrete in simple tension is studied and a vertex enhanced (Pamin, 1994) Rankine's principal stress criterion of failure is adopted. The form gradient dependent yield strength, $\bar{\sigma}_g(\kappa)$, adopted in this paper is,

$$\bar{\sigma}_g = \bar{\sigma}_t(\kappa) - g(\kappa)\nabla^2\kappa \qquad (2)$$

where $\bar{\sigma}_t(\kappa)$ is a given standard softening rule which is the local component and $g(\kappa)$ is a given gradient influence function which is the non-local component. Therefore, this general form of gradient plasticity is simply the addition of a local term and a non-local term. As it is seen from Eqn. (2) the degradation of yield strength depends on $g(\kappa)$ as well as on $\bar{\sigma}_t(\kappa)$. Hence it is logical that the load displacement response or the fracture energy consumed will be governed, in part, by the gradient influence factor $g(\kappa)$ and thus, the selection of the form of $g(\kappa)$ is an important consideration in gradient plasticity formulation. For a one dimensional problem with linear softening (softening modulus h is constant) and constant g, de Borst and Mühlhaus (1992) obtained a relation,

$$g = -l^2\bar{\sigma}_t'(\kappa) = -l^2 h \qquad (3)$$

where l is the internal length scale such that the crack band width, w, is given by

$$w = 2\pi l \qquad (4)$$

When the softening rule $\bar{\sigma}_t(\kappa)$ is nonlinear and g is taken according to the first part of Eqn. (3) then Eqn. (4) becomes approximate. An exact analytical solution for such a case is a mathematically formidable task. In all the previous studies with gradient plasticity approach, $g(\kappa)$ has always been taken according to Eqn. (3) so that the width of the fracture process zone can be approximated by Eqn. (4).

To describe the softening behavior of concrete a number of softening rules are available in the literature among which the one proposed by Cornelissen et al (1986) is widely used (Pamin 1994, CEB 1996). In this type of softening rule the tensile strength $\bar{\sigma}_t(\kappa)$ becomes zero at ultimate damage κ_u while a certain non-zero value of $\bar{\sigma}_t'(\kappa)$ remains at κ_u (here the prime indicates derivative with respect to κ). When numerical calculations are carried out the value of $\bar{\sigma}_t'(\kappa)$ is arbitrarily set to zero when a point in the structure reaches ultimate damage. This sudden change in $\bar{\sigma}_t'(\kappa)$ resulted in some numerical disturbances and prevented numerical calculation to be carried out after the mostly strained point reaches the ultimate damage. In a complex structure, a particular point may yield and reach to ultimate damage while the overall structural response still remains far from reaching collapse condition. In such a case the numerical analysis should be able to continue to get the desired

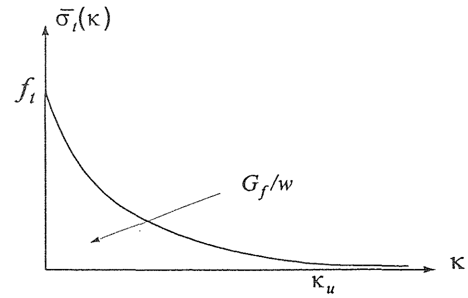


Fig. 1 Softening rule

overall structural response and locally stressed points should not pose any numerical divergence resulting from singularities in the softening rule. To overcome such difficulties the following softening rule is proposed (Fig. 1),

$$\bar{\sigma}_t(\kappa) = \frac{f_t}{c_1 + 1}[c_1 e^{-c_2\left(\frac{\kappa}{\kappa_u}\right)} + \tfrac{1}{2}\{e^{-4\left(\frac{\kappa}{\kappa_u}\right)^3} + e^{-4\left(\frac{\kappa}{\kappa_u}\right)}\}] \tag{5}$$

The characteristics of this function is that near ultimate damage κ_u the value of $\bar{\sigma}_t(\kappa)$ becomes asymptotic to κ axis and gradually reduces to zero instead of suddenly becoming zero at κ_u (see Fig. 1). For c_1=3.5 and c_2=9.0 the above Eqn. (5) becomes virtually identical with the function proposed by Cornelissen et al (1986) except that near κ_u the function does not intersect the damage axis but asymptotically becomes zero.

In earlier works the value of gradient influence factor $g(\kappa)$ was taken proportional to $\bar{\sigma}_t'(\kappa)$ (Pamin 1994, Amanat and Tanabe 1996) although the basic gradient plasticity formulation does not impose such a constraint. For the suitable values of constants c_1 and c_2 a large non-zero value of $\bar{\sigma}_t'(\kappa)$ remained at or near κ_u which resulted in a large residual value of g at or near ultimate damage. This high value of g prevented stress to drop to nearly zero since g is a part of the yield strength. The softening function proposed by Cornelissen et al (1986) suffers similar problem. For this reason a separate suitable form of g became necessary. In the present study an exponentially decreasing g as given by the following equation is used.

$$g(\kappa) = (g_0 - g_c)e^{-c_2\frac{\kappa}{\kappa_u}} + g_c \quad \text{where} \qquad g_0 = -l^2\bar{\sigma}_t'(0), \;\; g_c = g_0/50 \tag{6}$$

There are also other reasons for taking a separate function for g. Study shows that a softening rule together with a g according to Eqn. (3) gives very good numerical convergence. But in a few cases the resulting softening stress-strain relationship shows unacceptable shape. If $g(\kappa)$ is chosen according to Eqn. (3) then it can be shown that (Pamin 1994) the softening modulus h can be expressed as,

$$h = \eta\, \bar{\sigma}_t'(\kappa) + l^2 \bar{\sigma}_t''(\kappa) \nabla^2 \kappa \tag{7}$$

where l is the internal length scale and η is a parameter that relates plastic multiplier λ with the damage parameter κ such that $\kappa = \eta\lambda$. Study shows that for suitable values of c_1 and c_2 for early age concrete $\bar{\sigma}_t(\kappa)$ has a higher curvature near κ_u i.e. h becomes high due to higher value of $\bar{\sigma}_t''(\kappa)$ according to Eqn. (7). This high value of h results in a steep drop of stress at the later stage of damage producing a hump in the load displacement response which is not acceptable. Such things actually manifests the necessity of a suitable practical form of g. The form of g given by Eqn. (6) is chosen in such a way that it remedies all these problems. In Eqn. (6) it has been assumed that its initial ($\kappa \approx 0$) value will be equal to the $-l^2$ times the classical hardening $\bar{\sigma}_t'(\kappa)$ and its ultimate value will be 1/50th of its initial value instead of being zero. In between it will follow an exponential decay. Thus the proposed softening rule of Eqn. (5) together with g as in Eqn. (6) gives the most desirable result. It should be kept in mind that the overall shape of g should have an exponentially decaying form (or similar) to comply with the experimental observation. Since the non-local nature of the gradient plasticity formulation will be lost if g=0 so we keep a small positive value (i.e. $g_0/50$) of g near κ_u.

Gradient plasticity formulation involves another important parameter η which relates the plastic multiplier λ with damage parameter κ. If we adopt classical strain hardening hypothesis for calculation of damage accumulation then it is possible to calculate the corresponding value of η. However, taking other reasonable values of η will enable us to adopt any different strain hardening hypothesis and thus increase the flexibility in choosing the strain hardening hypothesis. Since η is a controlling factor in calculating damage accumulation, it is imperative that it has some effect on fracture energy. Although no definite quantitative relationship was presented, earlier study (Amanat and Tanabe 1996) qualitatively reveals that smaller value of η results in a higher fracture energy. In the present study an attempt has been made to give a quantitative description of the effect of η on fracture energy.

3 Calculation of Fracture Energy

If the width of the fracture process zone, w, is known then the fracture energy, G_f, is given by,

$$G_f = w \int_0^{\kappa_u} \bar{\sigma}_t(\kappa) \mathrm{d}\kappa \tag{8}$$

where κ_u is the ultimate damage at which $\bar{\sigma}_t(\kappa)$ drops to nearly zero. The above equation is valid on the assumption that during the process of progressive fracture the width of fracture process zone, w, is constant. In gradient plasticity this is approximately valid when $g(\kappa)$ is chosen according to Eqn. (3) such that w is approximated by Eqn. (4). However, when $g(\kappa)$ is chosen arbitrarily, Eqn. (4) becomes too approximate and G_f given by Eqn. (8) becomes erroneous. It is thus necessary to develop an appropriate expression for fracture energy since Eqn. (8) remains no longer valid. In this study it is assumed that the actual value of internal length scale varies according to the values of $g(\kappa)$ and $\bar{\sigma}_t'(\kappa)$ at any stage of damage and can be written as,

$$l(\kappa) \approx \sqrt{-\frac{g(\kappa)}{\bar{\sigma}_t'(\kappa)}} \tag{9}$$

Now Eqn. (8) is modified by taking w inside the integral and substituting $2\pi l(\kappa)$ for w, and by incorporating η in the expression of fracture energy calculation,

$$G_f \approx \frac{2\pi}{\eta} \int_0^{\kappa_u} l(\kappa)\, \bar{\sigma}_t(\kappa) \mathrm{d}\kappa \tag{10}$$

In such a case the term l in Eqn. (6) merely becomes a material parameter rather than the actual internal length scale.

4 Analyses of Experiments

To demonstrate the effectiveness of the proposed softening rule and the gradient influence factor the tension test of matured concrete by Willam et al (1985) is simulated here and the results are shown in Fig. 2 and Fig. 3. For this simulation one dimensional elements were used with c_1=6,

Table 1 : Values of material parameters

Age	9 hrs.	16 hrs.	24 hrs.	48 hrs.	72 hrs.	102 hrs.
E, MPa	400	1000	6000	8000	11000	14000
ν	0.15	0.15	0.15	0.15	0.15	0.15
c_1	2.0	2.0	2.0	2.0	2.0	2.0
c_2	8.0	6.0	7.0	7.0	7.0	7.0
κ_u	0.03	0.03	0.022	0.02	0.02	0.02
f_t, MPa	0.14	0.25	0.8	1.0	1.5	2.2
l, mm	1.6	2.0	1.0	1.1	1.0	0.9

c_2=5.0, E=22100 MPa, specimen length L=107 cm, η=1.0. Fig. 2 clearly demonstrates that the proposed softening rules and the gradient influence factor are capable of simulating the behavior of hardened concrete. Fig. 3 shows the growth of localization band obtained for different combination of κ_u-l. For the present case (Willam et al 1985) no direct or indirect data regarding the width of the localization band was available. However, if we obtain any such data on localization band from experiments then we can use those data to compare with Fig. 3 and determine the appropriate values of parameters.

Analyses were also made for concrete test specimens at ages 9, 16, 24, 48, 72 and 102 hours (Lokuliyana 1992). The values of different material parameters that were taken for the numerical simulations are shown in Table 1. The Young's modulus was calculated from the initial slope of the stress strain curve obtained experimentally and an averaging was done over the values calculated from each of the six strain-gages across the fracture process zone. Due to the lack of experimental data, Poisson's ratio was taken as 0.15. Other values such as κ_u and l etc. are taken on the basis of experiment in such a way that the numerical results best fits the experimental stress-strain data for three different gage lengths.

Experimental values of the fracture energy for these tests were calculated by Lokuliyana (1992) for ages upto 48 hours. In his analysis

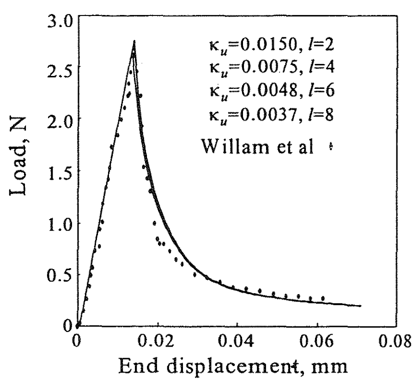


Fig. 2 Comparison of analysis and test results of Willam et al (1985)

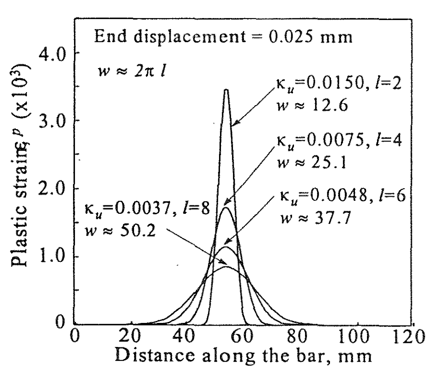
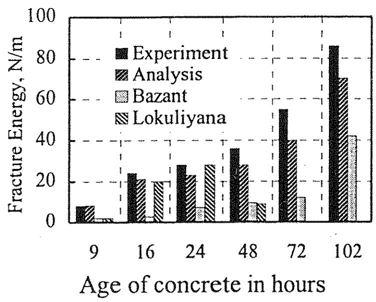


Fig. 3 Plastic strain distribution for different combination of κ_u and l



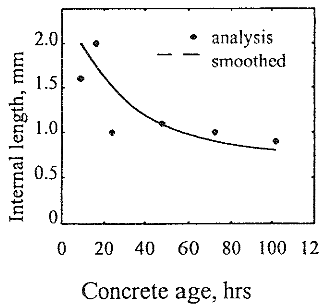


Fig. 4. Growth of G_f with age. Fig. 5. Internal length vs. age.

the experimental fracture energy was estimated on the basis of Eqn. (8). The experimentally obtained stress-strain diagram was used and the gage-length was taken as the width of the fracture process zone. In a similar way the fracture energy at 72 hour and 102 hour was calculated. The fracture energy value of the present analysis was calculated from the proposed Eqn. (9) using the values of material parameters given in Table 1. Fracture energy was also calculated from the empirical equation proposed by Bazant (1985),

$$G_f = 0.304(f_t + 8.95)\frac{f_t^2 d_a}{E} \tag{11}$$

Where G_f in kgf-cm/cm^2, f_t and E are in kgf/cm^2 and d_a is the maximum aggregate size in cm. Lokuliyana[9)] also proposed a similar empirical equation to calculate fracture energy of concrete at an early age,

$$G_f = (1.0 - 1.07 f_t)\frac{f_t^2 d_a}{E} \tag{12}$$

But the equation had a defect in a sense that if the value of tensile strength is higher than 0.934 MPa then it gives negative fracture energy. For both the above equations the respective authors did not mention any limit of their applicability and hence it is assumed that they are usable in the present study.

5 Fracture Energy and Internal Length Scale

As stated earlier, one of the primary objective of the present study was to investigate the fracture energy characteristics of early age concrete. Accordingly, fracture energy G_f was calculated for different ages of specimens following the procedure suggested before (Eqn. 10) and are shown in Fig. 4. It is observed that the energy predicted by the numerical analysis is closer to the experimental values whereas both the Eqn. (11)

and Eqn. (12) give much lower value of fracture energy. These equations are of empirical nature and both of the equations contain term like d_a and E. However, study (Saouma et al 1991) reveals that the maximum aggregate size has little or no influence on the fracture energy. G_f values found by analysis are observed to be somewhat smaller than the experimental values. This may be due to the fact no direct specific data were available regarding the fracture process zone and in Eqn. (8) w was taken as the gage length. Taking w equal to the gage length might be an overestimation.

A study of the values of different material parameters of Table 1 suggests that the important material parameters regarding the fracture energy at an early age are tensile strength f_t, internal length scale l and the ultimate damage κ_u. Although the value of Young's modulus changes significantly as the concrete gets matured from very early age its effect on post peak behavior does not appear to be much dominant. It only affects the initial slope and as such its contribution to total unit fracture energy is less prominent in mode I type fracture. The most dominant single material parameter appears to be the tensile strength. A comparative study of Table 1 and Fig. 4 suggests that there is a direct effect of tensile strength on fracture energy as expected. Hence, how the concrete develops its tensile strength with age is an important aspect of concrete properties. According to Eqn. (10) ultimate damage κ_u have a direct effect on G_f. However, its suggested values in Table 1 implies that it is not as important as f_t since its magnitude of variation is not as high as f_t. In non-local formulation, characteristic length l is an important parameter, which determines the width of fracture process zone. As such its influence on fracture energy is also significant. If the magnitude of $l(\kappa)$ (Eqn. 9) does not vary too much with increase of damage then its effect on unit fracture energy may be directly calculated via w where $w \cong 2\pi l$ and using it in Eqn. (8). For the present simulation, l varied from 2 mm to 0.9 mm for different ages and a trend of decreasing l with increasing age can be observed. This is quite logical in the sense that as the concrete gets matured it becomes more brittle and the width of the fracture process zone gets narrower as the brittleness increases. However, the relationship of l with age of concrete cannot be linear because in that case l will become zero at some later age. This is not practical because zero l means the width of the fracture process zone will become zero. A more logical variation would be something like that shown in Fig.5. To establish a definite relationship of the internal length factor, l, with age or some other material parameters more study, both experimental and numerical, is needed.

6 Conclusions

A softening function to describe the fracture process of concrete is proposed and it is shown that the softening rule can be applied to simulate the fracture process of early age concrete as well as hardened using the non-local gradient dependent continuum. The characteristics of the softening rule and gradient influence factor, which should be considered for successful application of gradient plasticity approach is enumerated. Incorporating the effect of g and η, a modified equation to calculate fracture energy is proposed. It has been shown that with the use of the proposed strength degradation rule it is possible to calculate fracture energy once the appropriate material parameters are chosen or if the fracture energy is given then it can be used in conjunction with the numerical FEM analysis to estimate the trial values of parameters κ_u and l (since c_1 and c_2 can be considered fixed at 2.0 and 7.0 respectively, E, ν and f_t can be determined from experimental results). The calculated fracture energy closely approximates the experimentally obtained values.

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