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MINIMUM SIZE OF CONCRETE SPECIMENS FOR LINEAR ELASTIC FRACTURE ANALYSIS

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Abstract
There exists a minimum size for fracture test of concrete specimens, d_{min}. When the specimen size $d < d_{min}$, the fracture property of the concrete specimen cannot be treated simply by linear elastic fracture mechanics (LEFM). d_{min} is also very important from practical point of view, if $d > d_{min}$, the concrete specimen is large enough for the application of LEFM, then, one-size and one-notch specimen will be adequate for testing fracture toughness of concrete. A prediction model for d_{min} is developed in terms of concrete composition parameters and specimen geometry factor. Any change in d_{min} due to a change in concrete mixing design and specimen geometry can be evaluated quantitatively.
Keywords: concrete, fracture, morphology, size effect

1 Introduction

One of the advantages of the size effect law (Bazant 1984; Bazant and Kazime, 1990) in evaluating fracture toughness of concrete is that it needs only the maximum load from concrete specimens, which is convenient to obtain in regular concrete laboratories. However, the original version of the size effect law requires concrete specimens of several different sizes.

This requirement makes it difficult to implement in the practice since testing of large size concrete specimens cannot be done in many laboratories simply due to the dimensional limitation of available testing machines. Recently, a one-size variable-notch method was proposed (Tang et al., 1996; Tang et al., 1998), which is a modified version of the original size effect law. This new method requires only one size of concrete specimen, but with different notch lengths. A question then arises: why we cannot use simply one-size and one-notch specimen just like the specimen used for other materials, such as metals. The answer to the question is that fracture properties of concrete does not always follow linear elastic fracture mechanics (LEFM) especially when the specimen size is small.

The motivation of this paper is to find out quantitatively how small the concrete specimen is small so that LEFM method cannot be applied and the size effect method (either the original version or the modified version) must be employed. On the other hand, it is a very important issue from practical point of view, because, if we know, before the fracture testing, that how large the concrete specimen is large enough for the application of LEFM method, then, one-size and one-notch specimen will be adequate for testing fracture toughness of concrete.

2 Fundamental considerations

In order to determine proper specimen size for a concrete fracture test, the influential parameters must be determined first. Considering fracture property of a composite material, there are three length scales that are very important:

(1) The internal length scale, l_c,
(2) The size of fracture process zone (FPZ), c_f
(3) The size of the specimen used to determine the fracture property, or characteristic length, d .

For metallic materials, l_c is in the range of lattice structure of metal atoms, which, compared with c_f and d, is extremely small. Therefore, only c_f and d are relevant. c_f is usually called plastic yielding zone for metals, and denoted as r_y. In order to evaluate fracture toughness of a metal and ensure that the measured fracture toughness is insensitive to specimen details, the ratio d/r_y must satisfy certain conditions, such as $d/r_y \geq 25$. This is obtained from the requirement $d \geq 2.5\left(K_{IC}/\sigma_y\right)^2$ and the size of yielding zone $r_y = 0.1(K_{IC}/\sigma_y)^2$ in which K_{IC} is the fracture toughness and σ_y is the yield stress (ASTM E399-83).

For cementitious materials, the situation is more complicated. First of all, the internal length scale depends on the gain size of cementitious materials, which is much larger than the grain size of metals, and thus, the effect of the internal length scale must be taken into account. This means that a model includes the internal length scale, l_c, the size of the fracture process zone, c_f, and the characteristic length, d, must be established.

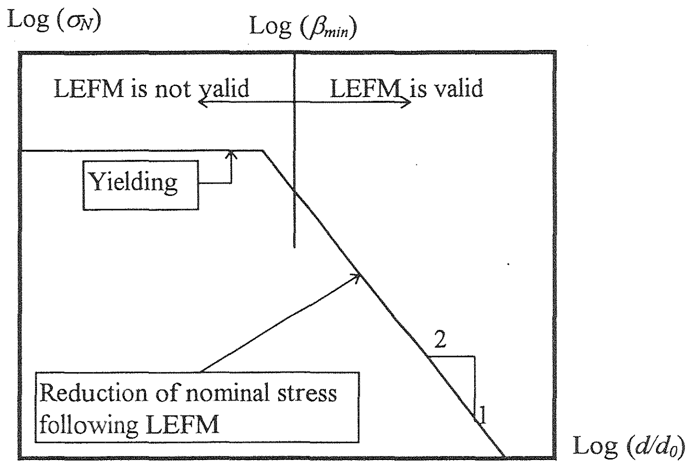


Fig. 1 Two asymptotic limiting regions: LEFM and yielding

Secondly, it has been experimentally observed that for geometrically similar specimens, the nominal stresses, σ_N, obtained from cement paste and mortar specimens are governed by linear elastic fracture mechanics (LEFM) with a slope of 1/2 (see Fig. 1), and the nominal stress from concrete specimens with large aggregates, on the other hand, does not follow LEFM, particularly when the specimen size is small. The test data of concrete are located typically in the transition zone between the horizontal line (yielding line) and the declined LEFM line (see Fig. 1). This indicates that grain size of cementitious materials plays an important role for their fracture properties.

Thirdly, how to select a proper parameter to describe the internal length scale is an important issue. With a given size of coarse aggregate, if the volume fraction of the aggregate is very small, there are only very few large particles dispersed within the cement paste, then, the cementitious composite can still be treated as if there is no coarse aggregate. Therefore, not only the size of the aggregate but also the volume fraction of the aggregate are important for fracture properties of concrete. As a matter of fact, the shape of the aggregate and surface smoothness of the aggregate, in addition to the size and the volume fraction, affect significantly the fracture properties of concrete. Therefore, a simple quantity, such as aggregate size, is not sufficient to characterize the effect of aggregate on fracture properties of concrete.

It becomes apparent from above discussion that there exists a minimum size for concrete specimens, d_{min}. When $d < d_{min}$, the fracture property of the concrete specimen cannot be treated simply by LEFM. To develop a fundamental criterion for the minimum specimen size of concrete fracture test, three basic steps will be involved. The first one is to define a proper quantity that can be used as an internal length scale for concrete. The internal length scale parameter should be expressed in terms of concrete composition parameters, such as water to cement ratio, aggregate content, aggregate size, and etc. The second is to develop a relationship between the internal length scale and the fracture process zone. In this way, the size of fracture process zone can be connected to concrete composition parameters. The third step is to use the characteristic size of specimen and the size of the fracture process zone in a proper way to identify the minimum size of concrete specimen for fracture testing. Our final goal is to develop a model for the minimum size of concrete fracture specimen in terms of concrete composition parameters. Any change in the minimum size due to a change in concrete mixing design (i.e. aggregate size or aggregate content) can be evaluated quantitatively. Then, before any fracture testing, one will be able to know whether the concrete specimen can be tested and analyzed by LEFM or not.

3 The parameter for concrete internal length scale

Recently, a morphological model, called mosaic pattern, for characterizing internal structure of concrete has been developed (Xi, 1996; Xi et al., 1996). There are two types of controlling parameters in the mosaic pattern, p_i and λ. For a two-phase composite, parameter p_i is the volume fraction of the constituent i, and λ is called coarseness of the spatial pattern. So, the parameter p_i controls area fractions of the constituents in 2D and volume fractions in 3D; while λ is a measure of the grain sizes of the mosaic, that is, the coarseness of the internal structure. Large λ corresponds to small sizes of the grains in the mosaic pattern, which may be called fine-grained. While small λ means large sizes of the grains in the mosaic pattern, called coarse-grained. The independence of λ with p_i

means that the grain size of mosaic pattern is controlled only by the density of the cell network in the mosaic pattern and has nothing to do with volume fractions of the constituents.

When concrete is considered as a two phase composite with aggregates as one phase and cement paste as another, the expression of coarseness λ is

$$\lambda = \frac{1}{D_{ave}\left(1 - V_a\right)} \tag{1}$$

in which D_{ave} is the mean value of the aggregate size distribution; and V_a is the volume fraction of the aggregate.

From Eq. (1), one can see that the parameter λ encompasses the effect of concrete mixing design parameters on the coarseness of the internal structure of concrete. The dimension of λ is 1/length. In this study, the coarseness λ was used as the internal length scale for concrete internal structures.

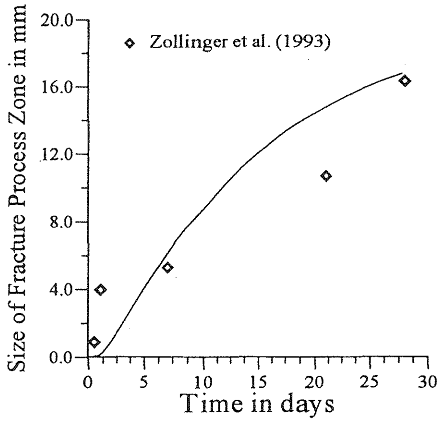
4 The model for fracture process zone

The size of fracture process zone, c_f, is definitely affected by the internal length scale λ. Moreover, c_f is also affected by other parameters. 1). Mechanical properties of the constituent phases, which may be characterized by the ratio of modulus of elasticity, E_m/E_a, where the subscripts m and a are for matrix and aggregate, respectively. 2). Interfacial bond between aggregate and matrix.

The contribution from each influential parameter was determined first and then combined into an empirical model, which was calibrated based on available test data. The model for c_f is (Xi, 1998)

$$c_f = 534750\alpha_{Bond}\left(\frac{E_m}{E_a}\right)^{2.5}\left(1 - \frac{E_m}{E_a}\right)^{1.5}\left(\frac{2500}{1+\lambda^3} - 1.061\right) \tag{2}$$

where α_{Bond} represents bond conditions between the aggregate and surrounding cement paste. Available test results showed that the range of α_{Bond} could be from 1 to 5, for instance, α_{Bond} = 1 for river gravel and α_{Bond} = 5 for crushed limestone. E_m is a function of time due to cement hydration. E_m depends mainly on w/c and concrete age and therefore, can be estimated by using a suitable composite model combined with a model for cement hydration. In the present study, a multiphase and multiscale composite model is used for predicting E_m (Xi and Jennings 1997). λ is calculated by using Eq. (1).



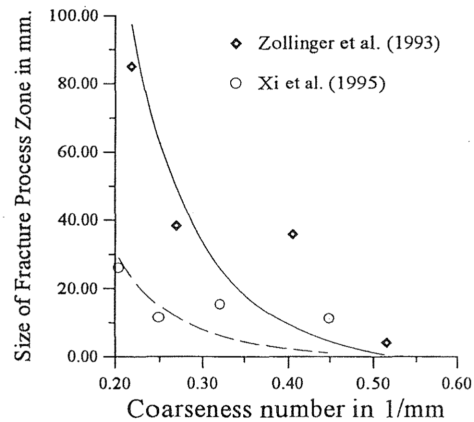


Fig. 2 Effect of concrete age on c_f ($\alpha_{Bond} = 4$)

Fig. 3 Effect of coarseness λ on c_f ($\alpha_{Bond} = 4$ for Zollinger et al.; $\alpha_{Bond} = 1$ for Xi et al.)

Fig. 2 shows the effect of concrete age on c_f and the comparison of test results with the prediction of Eq. (2). It is clear that c_f is quite small at early ages, and c_f increases with concrete aging. The ascending curve shown in Fig. 2 is valid for conventional concrete. The descending part of the curve (decreasing size of the process zone with increasing age of the concrete), as predicted by Eq. (2), occurs only for those very high strength concrete in which E_m is almost the same as E_a. Fig. 3 shows the test results of c_f with various coarseness numbers and the comparison of model predictions with the test results. From Fig. 3, one can see that c_f decreases with increasing λ, which represents fine-grained internal structures.

5 The brittleness number from the size effect law

The brittleness number, β, proposed by Bazant and Kazime (1990) was used as the basic criterion in the present study to distinguish whether the concrete specimen is governed by LEFM or not. Of course, there are several other brittleness numbers in the literature, and each of them has its own special features. In view of the present model developed for c_f, which was exclusively based on test results from the size effect law, it will be more consistent to use the brittleness number β defined in the same framework. The size effect law (Bazant, 1984) is

$$\sigma_N = \frac{Af_u}{\sqrt{1+d/d_0}} = \frac{Af_u}{\sqrt{1+\beta}} \tag{3}$$

where f_u is the tensile strength, d is the characteristic dimension, and A and d_0 are two constants which can be identified by linear regression of the test results from concrete specimens of different sizes. $\sigma_N = c_n P_u/(bd)$ = nominal strength of the specimen. c_n is a convenience factor, which can make the measured nominal stress match some formula for maximum stress, in a beam for example, the maximum elastic bending stress for a simply supported beam is matched by choosing $c_n = 1.5\ L/d$, where L is the span of the beam and L/d is a constant for geometrically similar specimens. P_u = maximum load; b = width of the beam. Based on Eq. (3), the brittleness number is defined as

$$\beta = \frac{d}{d_0} = \frac{g(\alpha_0)}{g'(\alpha_0)}\frac{d}{c_f} = \frac{D}{c_f} \tag{4}$$

in which $g(\alpha_0) = \pi\alpha_0 c_n^2 F^2(\alpha_0)$. $\alpha_0 = a_0/d$ and a_0 is the initial notch length. Function $F(\alpha_0)$ is called geometry factor, which is available in standard fracture mechanics handbooks for various specimen geometries. D in Eq. (4) is called effective structural dimension, and it takes into account the effect of structural geometry.

The physical meaning of the brittleness number is shown in Fig. 1, in which a minimum brittelness number, β_{min}, has assumed, and Log (β_{min}) serves as a dividing line. When the brittleness number of a concrete specimen is larger than β_{min}, it can be treated by using LEFM, which means one-size and one-notch specimen may be adequately used to evaluate the fracture toughness. When the brittleness number of the concrete specimen is smaller than β_{min}, the fracture toughness must be determined by using the size effect law or other proper procedures.

6 The minimum size of concrete specimen for fracture test

Combining the concept of the minimum brittleness number and the present model for the size of fracture process zone, c_f, a criterion for the minimum size of concrete specimen can be established. The major progress made by the present study is that the model for c_f is expressed in terms of concrete composition parameters, therefore, the minimum size of concrete specimen for fracture test can be determined before fracture testing. From Eq. (4), the minimum size is

$$d_{\min} = \beta_{\min} \frac{g'(\alpha_0)}{g(\alpha_0)} c_f \tag{5}$$

Now, let us analyze two limit cases for d_{min}. The first limit case, without any detailed analysis on the values of β_{min}, $g(\alpha_0)$ and $g'(\alpha_0)$, simply following the formula used for metals as discussed earlier, $d/r_y \geq 25$ (ASTM E399-83), we have $d_{\min} = 25c_f$, from Fig. 2, taking the maximum value of c_f = 0.65 inch at 28 days, d_{min} = 15 inch, which is quite large for material testing in the laboratory; taking the minimum value of c_f = 0.05 inch at 1 or 2 days, d_{min} = 1.25 inch, which is very small. From Fig. 3, although c_f varies in a quite large range, but when coarseness number λ > 9, c_f tends to be smaller than 1 inch. One can see clearly in this limit case that if $d_{\min} = 25c_f$ is really valid for concrete, then most of concrete specimens with a broad range of mixing parameters could be tested and analyzed by LEFM with a proper specimen size which can be handled in most of laboratories.

The second limit case, for the same specimen geometry, the geometry factor $g(\alpha_0)/g'(\alpha_0)$ = constant. Specifically, for a three-point-bend concrete beam with span to depth ratio of 2.5 and α_0 = 0.25 (a commonly used specimen geometry), $g(\alpha_0)/g'(\alpha_0)$ = 0.2 (Tang et al. 1998) and thus $g'(\alpha_0)/g(\alpha_0)$ = 5. On the other hand, according to a conservative estimation by Bazant and Kazime (1990), when β > 10, the failure of concrete specimen may be analyzed according to LEFM. When β < 0.1, the failure may be analyzed on the basis of the strength criterion or plastic limit analysis. For 0.1< β < 10, nonlinear fracture analysis is required. Substituting these values into Eq. (5) gives $d_{\min} = 50c_f$. If c_f is around 2.5 cm, then, $d_{mim} \cong$ 125 cm, which, apparently, is too large to handle in the regular laboratory. Therefore, if $d_{\min} = 50c_f$ is really valid for concrete, then d_{min} for most of concrete specimens would be very large except for the concrete at early ages. This means that one-size and one-notch specimen and LEFM method cannot be used directly to most of the concrete. The size effect law and other proper methods must be employed.

In reality, most of concrete specimens are in the range between these two limit cases. For instance, the smallest specimen used in the reinforced concrete beam test (Bazant and Kazemi, 1990; Bazant and Xi, 1991) had a value of $\beta = 5$. Although the brittleness number is much smaller than the one specified by Bazant and Kazemi, $\beta > 10$, the test data was already in the range of LEFM, which means that those test data obtained from reinforced concrete beams can be analyzed simply by using LEFM.

The change of fracture property of concrete from the region of LEFM to the region of nonlinear fracture mechanics is not a sudden jump, but a smooth transition. Therefore, determination of the value for β_{min} depends, to a large extent, on the level of tolerance that we allow to deviate from LEFM. As another option, more than one value of β can be specified, together with the level of the error associated with each β. This will be a very interesting research topic in the future.

7 Conclusions

1. There exists a minimum size for concrete specimens, d_{min}. When the characteristic dimension $d < d_{min}$, the fracture property of the concrete specimen cannot be treated and analyzed by LEFM. More importantly, when $d > d_{min}$, the fracture property of the concrete can be tested by one-size and one-notch specimen and analyzed by LEFM.

2. A prediction model for d_{min} is developed and two limit cases were analyzed. Comparing with available test data, it was found that most of concrete specimens are in the region between the two limit cases. Since the change of fracture property of concrete from the region of LEFM to the region of nonlinear fracture mechanics is not a sudden jump, but a smooth transition. Therefore, determination of the value for the minimum size of fracture specimen depends, to a large extent, on the level of tolerance that we allow to deviate from LEFM.

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