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Estimation of Fracture Energy from Basic Characteristics of Concrete

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ABSTRACT: Approximate formulae for predicting the concrete fracture energy and effective process zone size from basic design parameters of concrete are presented. The formulae are based on statistical analysis of 238 test series extracted from the literature. The coefficient of variation of the errors of fracture energy prediction is about 18%.

1. THE PREDICTION PROBLEM

This paper deals with the difficult problem of predicting the fracture energy of concrete and its other fracture parameters, such as the effective length of the fracture process zone, critical crack tip opening displacement and the fracture toughness, from simple characteristics of concrete.

Although it has already become clear that an accurate prediction cannot dispense with the testing of notched specimens of the given concrete, approximate (and admittedly crude) formulae predicting these parameters on the basis of standard compression strength, maximum aggregate size, water-cement ratio and aggregate type (river or crushed) can nevertheless be developed using several hundred test data that have by now been accumulated in the literature.

The fracture energy obtained by the size effect method and other methods using test data collected at peak load (i.e., the Jenq-Shah two- parameter method and Karihaloo's effective crack model) must be distinguished from the fracture energy obtained by the work-of- fracture method proposed by Hillerborg, which depends on the entire postpeak behavior.

2. APPROXIMATE PREDICTION FORMULAE

The test results obtained in one laboratory on one particular concrete have a much lower scatter and can be interpreted much more easily and unambiguously than the aggregate of the test results obtained at various laboratories on various concretes. However, the latter inevitably provides a far broader range, which is a great advantage for statistical studies.

The differences of the test results from their mean value reflect the differences between various concretes and cannot be regarded as the statistical scatter of random errors. To get a picture of the scatter of random errors in the huge data sets, including the test results from the literature obtained on many different concretes, one must first eliminate from the data their systematic (i.e., deterministic, mean) trends. In other words, one must first find the formulae optimally describing the mean trends of the data, and the statistical errors are then the deviations from these formulae.

A study by Bažant and Becq-Giraudon (2000) deals with this problem in detail. It shows that it is possible to use the standard compression strength, maximum aggregate size, water-cement ratio and aggregate type (river or crushed) to approximately predict the mean values of G_f and G_F, as well as the effective fracture process zone size c_f (from which further the fracture toughness K_c and δ_{CTOD} can be calculated).

A very large data base, consisting of 238 test series, was extracted from the literature and tabulated. Optimization of the fits of this data set led to new approximate prediction formulae, which read:

$$G_f = \alpha_0 \left(\frac{f'_c}{0.051}\right)^{0.46} \left(1 + \frac{d_a}{11.27}\right)^{0.22} \left(\frac{w}{c}\right)^{-0.30} \qquad \omega_{G_f} = 17.8\% \tag{1}$$

$$\ln c_f = \gamma_0 \left(\frac{f'_c}{0.022}\right)^{-0.019} \left(1 + \frac{d_a}{15.05}\right)^{0.72} \left(\frac{w}{c}\right)^{0.2} \qquad \omega_{c_f} = 47.6\% \tag{2}$$

$$G_F = 2.5 G_f \qquad \omega_{G_F} = 29.9\% \qquad (3)$$

where $\alpha_0 = \gamma_0 = 1$ for rounded aggregates, while $\alpha_0 = 1.44$ and $\gamma_0 = 1.12$ for crushed or angular aggregates; ω_{G_f} and ω_{G_F} are the coefficients of variation of the ratios G_f^{test}/G_f and G_F^{test}/G_F, for which a normal distribution may be assumed, and ω_{c_f} is the coefficient of variation of c_f^{test}/c_f, for which a lognormal distribution should be assumed.

The standard deviation of the errors of the new formula for fracture energy, compared to the 238 test series from the literature, is lower than that of the older formula in the 1990 CEB-FIP Model Code, which was of course developed from a much smaller data base.

It must be admitted that the aforementioned coefficients of variation of prediction errors, including that for G_f, are rather high. Therefore, a statistical approach to design is appropriate when these formulae are used.

The coefficients of variation of these predictions are nevertheless not higher than those in the widely used prediction formulae for concrete creep and shrinkage. But note that underestimation of fracture load is usually much more dangerous than underestimation of creep.

Therefore it cannot be overemphasized that the present simple prediction formulae are intended only for preliminary design, and only for structures of not too high fracture sensitivity.

The final analysis of important and sensitive structures should, of course, always be made on the basis of notched specimen tests performed on the local type of concrete used in the structure.

The statistical information available in literature on fracture energy of concrete, which has by now become quite extensive, is compiled and presented as a data bank in Bažant and and Becq-Giraudon (2000). This data bank should facilitate further studies.

Some of the statistical evaluations of the proposed formulae are shown in Fig. 1–4.

A statistical comparison with the G_F prediction formula given in CEB-FIP Model Code 1990 has also been made, and an improvement of the coefficient of variation of the prediction errors from 33.3% to 29.9% has been found. An improvement over the prediction formula given by Bažant and Oh (1983) has also been demonstrated.

3. RESULTS AND CONCLUSIONS

1. Although the statistical variations of the fracture energy of concrete and the effective size of the fracture process exhibit high random scatter, some clear statistical trends can be discerned.

2. Approximate statistical prediction of the fracture energy and of the order of magnitude of the effective length of the fracture process zone can be based on the standard compression strength of concrete, the maximum aggregate size, and the water-cement ratio. Among these parameters, the first appears the most important for the fracture energy, and the last the least. In the case of c_f, the maximum aggregate size appears to be by far the most important parameter, and the compressive strength the least. However, this is true only if the statistical correlation among these two parameters are ignored.

3. Formulae predicting the mean fracture energy G_f or G_F and the mean effective length c_f of the fracture process zone have been established. These formulae should be used in the statistical sense, taking into account the established coefficients of variation of G_f or c_f. Structural designs should be made for a certain specified probability cutoff based on assuming a normal or Weibull distribution for the fracture energy, and lognormal distribution for the effective length of the fracture process zone. The corresponding values of the critical crack tip opening displacement and of fracture toughness can be deduced from well-known formulae.

4. If all the important influencing parameters were known, it would have to be possible to be cast the prediction formulae in a dimensionless form. At present, however, this does not seem possible. It follows that not all the relevant parameters are known and further research is needed.

5. The coefficient of the data deviations from the mean prediction formula is much higher (1.67 $\times$ higher) for the fracture energy G_F measured by the work-of-fracture method than it is for the fracture energy G_f measured by the size effect method. The reason can be either that the work-of-fracture method, per se, has a higher degree of uncertainty, or that predicting the mean for G_F is harder than it is for G_f. Although it remains to clarify which is the main reason, it is likely that the first possible reason is valid at least to some extent, because the tail of the softening stress-separation curve of the cohesive crack model is more uncertain than the initial tangent of this curve (the large

errors in G_F could doubtless be reduced by including some factors accounting for the effects of size and shape on the G_F, but that would be tantamount to admitting that G_F is not a material parameter, and thus not generally usable).

6. The high scatter of the existing test results suggests that future efforts should examine the possibility of using further parameters of concrete composition and microstructure. Separate formulae may have to be developed for high strength concrete and lightweight concrete.

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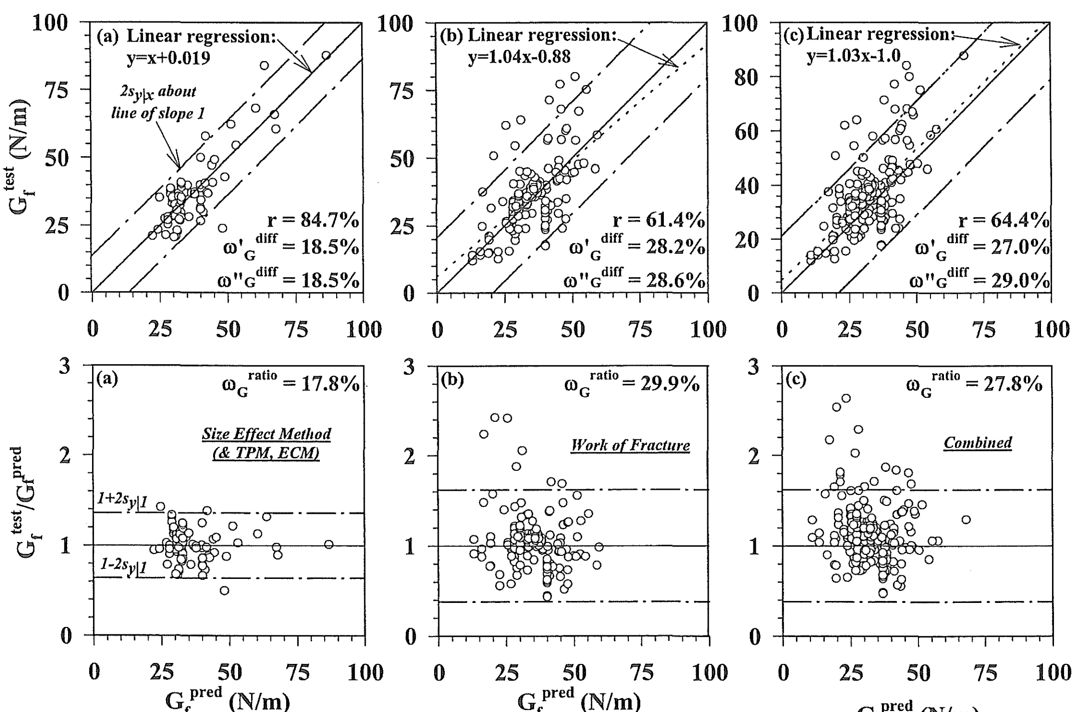

Figure 1. Plots of measured versus predicted values of fracture energy G_f or G_F, obtained for (a) SEM, TPM, ECM (set I, 77 data); (b) Work of fracture (set II, 161 data); (c) SEM, TPM, ECM and work of fracture combined (set III, 238 data). *Note:* $s_{y|x}$: standard deviation of vertical differences of data from line of slope 1; $s_{y|1}$: standard deviation of the differences of G_f^{test}/G_f^{pred} from 1.

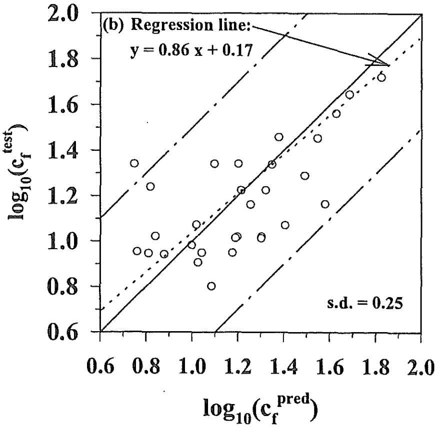

Figure 2. Plot of measured versus predicted values of $\log c_f$, for size effect and Jenq-Shah methods.

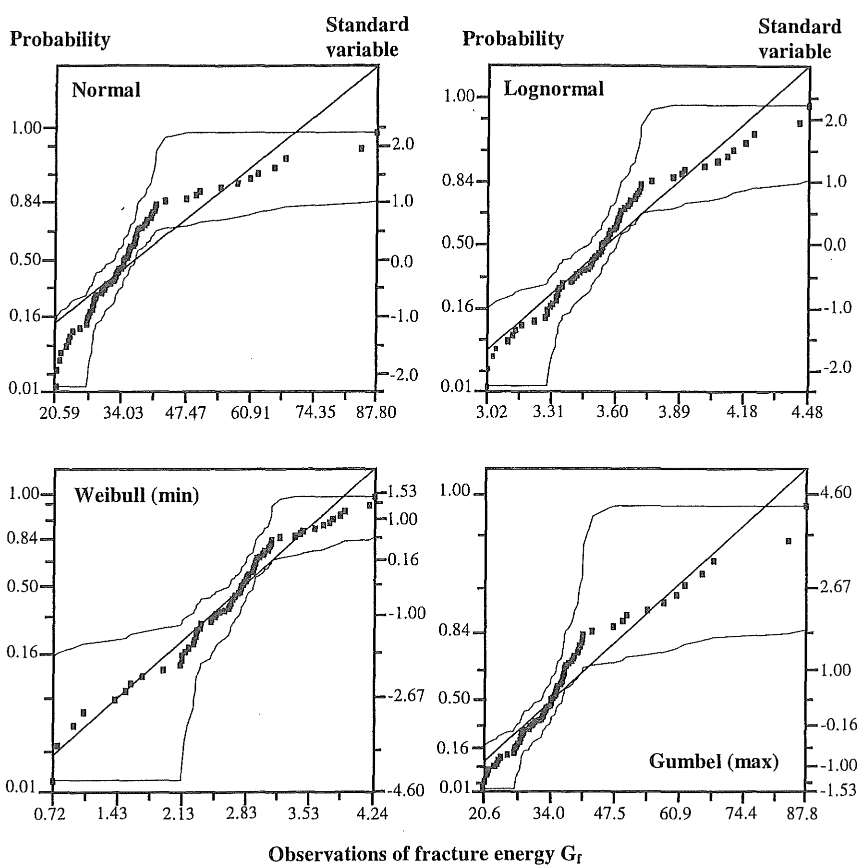

Figure 3. Cumulative frequency plots of G_f on various probability papers, for data measured by size effect, Jenq-Shah and Karihaloo method.

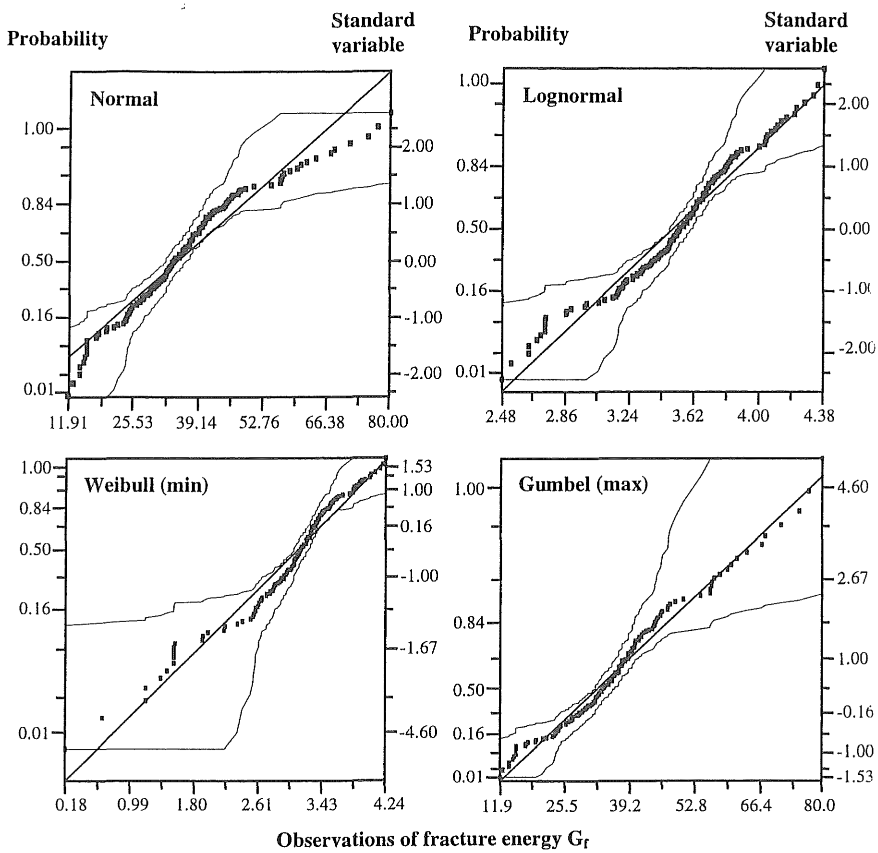

Figure 4. Cumulative frequency plots of c_f on various probability papers, for data obtained by size effect and Jenq-Shah methods.