Fracture Mechanics of Concrete Structures, de Borst et al (eds) © 2001 Swets & Zeitlinger, Lisse, ISBN 90 2651 825 0

Friction and specimen slenderness influences on dissipated energy density of quasi-brittle materials in compression: an explanation based on fractal fragmentation theory

A.Carpinteri & N.Pugno
Department of Structural Engineering and Geotechnics, Politecnico di Torino, Torino, Italy

ABSTRACT: The influence of the slenderness of a specimen in compression and of the friction between it and the loading platens on the dissipated energy density (strain-softening response) is theoretically analyzed. An assumption based on fragmentation and comminution is herein presented, the energy dissipated during the process being proportional to the area of the free surface of the fragments.

Using the well-known empirical power-law for the frequency-size distribution of fragments describing the scale-invariant and fractal character of the phenomenon, the dissipated energy density is obtained in a structural element under compression. The influences of specimen slenderness and friction is captured by the proposed model in a satisfactory way.

1 INTRODUCTION

The study of the compressive mechanical behaviour of concrete, already analysed by several authors, does not present untill today a complete and systematic treatment, even if many salient aspects have been already emphasized. The most important of these aspects is represented by the phenomenon of strain-softening, that presents different characteristics by varying the test conditions. There are in fact several parameters to be taken into account, and two are the most important: the slenderness of the specimen and the friction between the specimen and the loading platens.

The investigations carried out by Carpinteri, Ciola and Pugno (2001) and Carpinteri, Ciola, Pugno, Ferrara and Gobbi (2001) emphasized these aspects both numerically and experimentally. The present investigation deals with this topic from a theoretical point of view based on the fractal fragmentation theory (Turcotte, 1986, 1989; Carpinteri and Pugno 2001a,b).

Several theoretical models have been proposed linking fractals (Mandelbrot, 1982; Feder, 1988) to fracture and fragmentation (Béla Beke, 1964). These models have been recently reviewed by Perfect (1997).

Carpinteri (1994), Carpinteri and Chiaia (1997) and Carpinteri et al. (1999) used the fractal and multifractal approaches to explain the scaling laws for strength and toughness in the breaking behaviour of disordered materials. Engleman et al. (1988) applied the maximum entropy method to show that the number-size distribution follows a fractal law for fragments that are not too large. By combining a fractal analysis of brittle fracture with energy balance principles, Nagahama (1993) and Yong and Hanson (1996) were able to derive a theoretical expression for the fragment size distribution as a function of energy density. Aharony et al. (1986) predicted the fragment size distribution from clusters of connected bonds in a cubic lattice using percolation theory.

On the other hand, only more recently the fragmentation theory has been applied to the study of compression (Momber, 2000).

2 FRACTAL FRAGMENTATION THEORY

After comminution or fragmentation (Turcotte, 1992), the *probability density function* $p(r)$, that times the interval amplitude dr represents the percentage of particles with radius comprised between r and r+dr, will be:

$$p(r)=D\frac{r_{min}^{D}}{r^{D+1}}, \tag{1}$$

where, for fragmentation under compression, the so-called fractal exponent D is experimentally close to two (Momber, 2000).

The total fracture surface area of fragments is obtained by integration $(r_{min} << r_{max})$:

$$A_f \propto N \int_{r_{min}}^{r_{max}} r^2 p(r)\,\mathrm{d}r \propto N r_{min}^2, \tag{2}$$

where N is the total number of particles.

On the other hand, the total volume of the particles is:

$$V_f \propto N \int_{r_{min}}^{r_{max}} r^3 p(r)\,\mathrm{d}r \propto N r_{min}^D r_{max}^{3-D}. \tag{3}$$

If we assume a material "quantum" of size r_{min}=constant (Novozhilov, 1969; Sammis, 1997; Carpinteri and Pugno 2000, 2001a,b), and make a hypothesis of selfsimilarity, i.e., $r_{max} \propto \sqrt[3]{V_f}$, (Carpinteri, 1986), the energy W dissipated to produce the new free surface in the comminution process, which is proportional to the total surface area A_f (Griffit 1921; Smekal 1937), is:

$$W \propto A_f \propto V_f^{D/3}, \tag{4}$$

and represents an extension of the Third Comminution Theory, where $W \propto V_f^{2.5}$ (Bond, 1952). The fundamental assumptions of material "quantum" and of self-similarity can be derived from the more general hypothesis that the energy dissipation must occur in a fractal domain comprised, in any case, between a surface and a volume (Carpinteri and Pugno 2001a,b). The extreme cases contemplated by eq. (4) are represented by D=2, surface theory (von Rittinger, 1867; Hönig, 1936), when the dissipation really occurs on a surface ($W \propto V_f^{2/3}$), and by D=3, volume theory (Kick, 1885; Hönig, 1936), when the dissipation occurs in a volume ($W \propto V_f$). The experimental cases of comminution are usually intermediate, as well as the size distribution for concrete aggregates due to Füller (Stroeven, 1991). On the other hand, concrete aggregates frequently are a product of natural fragmentation or artificial comminution. If the material to be fragmented is concrete, we have therefore a double reason to expect a fractal response.

The energy dissipation occurs on a two-dimensional surface according to Griffith, rather than on a morphologically fractal set. On the other hand, the distribution of particle size follows a power-law, the number of infinitesimal particles tending to infinity.

The fragmented volume $V_f = l_f^3$ and the volume of the specimen under compression $V = l^3$ are not necessary coincident, so that:

$$l_f \propto l^{\beta}. \tag{5}$$

In the extreme cases the fragmented volume is independent of the specimen volume and the exponent β is equal to zero or they are directly proportional and β is equal to one.

The exponent β permits to model the friction between loading platens and specimen. Only when there is not any friction, the more intuitive hypothesis of direct proportionality between fragmented volume and specimen volume (β close to one) can be assumed. As a matter of fact, the frictional shearing stresses acting at the interface produce triaxially-confined regions near the bases where a multitude of microcracks propagate (Carpinteri, Ciola and Pugno, 2001; Carpinteri, Ciola, Pugno, Ferrara and Gobbi, 2001). In other words, the micro-cracked confined region, or the fragmented volume, will be constant varying the slenderness (β close to zero, Fig. 1).

Noting that the slenderness (specimen-height over side-length) can be obtained as $\lambda = \frac{V}{V_{\lambda=1}}$, from eqs. (4) and (5) we can evaluate the relative dissipated strain energy density $\Psi = \frac{W}{V}$ during the compression of the specimen as a function of its slenderness:

$$\frac{\Psi}{\Psi_{\lambda=1}} = \lambda^{\beta D/3-1}, \tag{6}$$

where D is close to two and β is close to zero or one respectively for friction tests and frictionless ones.

3 COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS

In this section, a comparison between the experimental (Carpinteri, Ciola and Pugno, 2001;

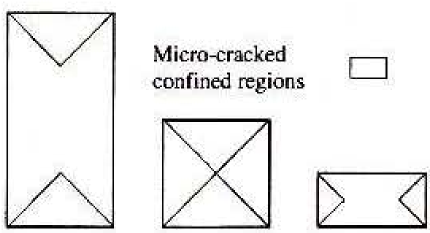

Figure 1. Microcracked confined regions for friction tests.

Carpinteri, Ciola, Pugno, Ferrara and Gobbi, 2001) and the theoretical predictions obtained from the very simple law of eq. (6) is presented. The comparison regards prismatic concrete specimens with a square base (50 x 50, 100 x 100, 150 x 150 mm^2) and with three different slendernesses (0.5, 1.0, 2.0), with and without friction between the specimen and the loading platens, for a total of eighteen cases.

The friction condition is represented by the direct contact between specimen and platens, since the shearing stresses at the interface arise in opposition to the lateral expansion of the specimen. On the other hand, the introduction of teflon layers between the specimen and the loading platens allows for the lateral expansion of the material; as a consequence, the shearing stresses at the interface become negligible (the friction coefficient in that case is close to 0.01).

Eq. (6), described by a straight line in a bi-logarithmic diagram, is experimentally confirmed (Figures 2 and 3). The slope of the straight line, as theoretically predicted by eq. (6), is larger for friction tests (β close to zero, Figure 2) than for frictionless ones (β close to one, Figure 3).

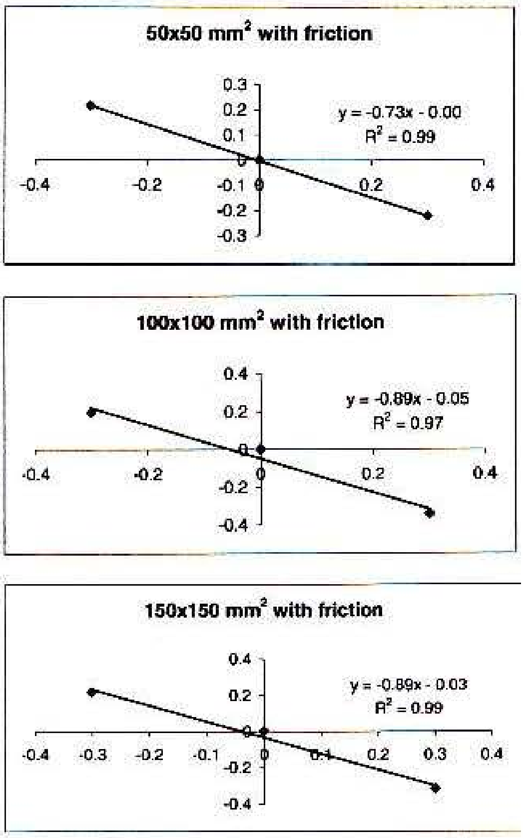

Figure 2. Dissipated energy density $y = \log \Psi / \Psi_{\lambda=1}$ vs specimen slenderness $x = \log \lambda$. Comparison between theoretical straight line (eq. (6)) and experimental data (bi-logarithmic diagrams for friction tests).

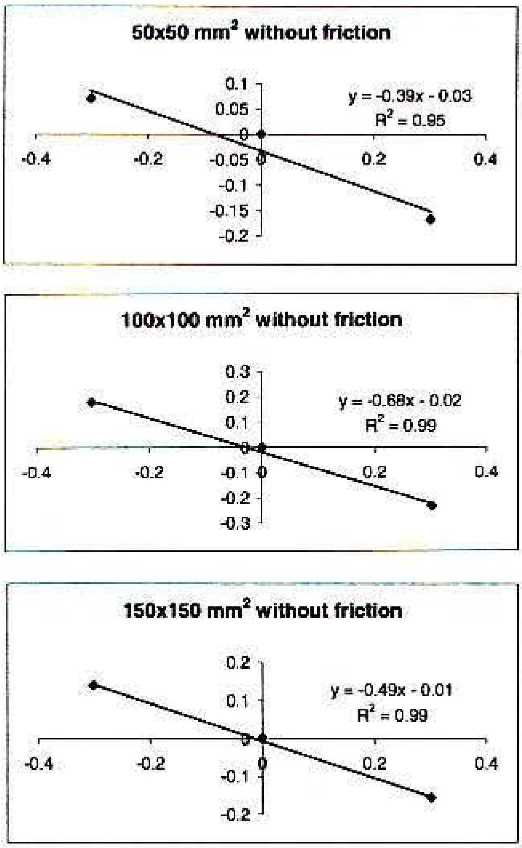

Figure 3. Dissipated energy density $y = \log \Psi / \Psi_{\lambda=1}$ vs specimen slenderness $x = \log \lambda$. Comparison between theoretical straight line (eq. (6)) and experimental data (bi-logarithmic diagrams for frictionless tests).

The experimental average of the slope for friction tests is –0.84 and for frictionless ones –0.52. If we consider $\beta \cong 0.2$ for friction tests and $\beta \cong 0.8$ for frictionless ones, eq. (6) predicts (with D=2) respectively –0.87 and –0.47.

4 CONCLUSIONS

The analysis of the results presented in the paper shows a satisfactory correspondence between the theoretical predictions and the experimental data. The experimental ductility increases with the specimen slenderness decrease and the friction influence is theoretically captured.

As a consequence, the very simple law of eq. (6), based on the developed fractal fragmentation theory, can be used to predict the slenderness and friction influences on the dissipated energy density of quasi-brittle materials in compression.

ACKNOWLEDGEMENTS

The present research was carried out with the financial support of Ministry of University and

Scientific Research (MURST), National Research Council (CNR) and EC-TMR Contract N° ERBFMRXCT960062.

REFERENCES

Aharony A., Levi A., Englman R., Jaeger Z. 1986. Percolation model calculations of fragment properties. *Ann. Israel Phys. Soc.* **8**, 112-119.

Béla Beke D. 1964. *Principles of Comminution*. Publishing House of the Hungarian Academy of Sciences, 90-92.

Bond F. C. 1952. The third theory of comminution. *Mining Engineering* **193**, 484-494.

Carpinteri A. 1994. Scaling laws and renormalization groups for strength and toughness of disordered materials. *Int. J. Solids and Structures* **31**, 291-302.

Carpinteri A. and Chiaia B. 1997. Multifractal scaling laws in the breaking behaviour of disordered materials. *Chaos, Solitons and Fractals* **8**, 135-150.

Carpinteri A. and Pugno N. 2000. Structural elements with re-entrant corners. In *XV National Congress of the Italian Group of Fracture*, 3-5 May 2000, Bari, Italy, 391-398.

Carpinteri A. and Pugno N. 2001a. A Fractal comminution approach to evaluate the drilling energy dissipation. To appear.

Carpinteri A. and Pugno N. 2001b. A Multifractal comminution approach for drilling scaling laws. To appear.

Carpinteri A. and Sih G. C. 1984. Damage accumulation and crack growth in bilinear materials with softening: application of strain energy density theory. *Theoretical and Applied Fracture Mechanics* **1**, 145-160.

Carpinteri A., Ciola F. and Pugno N. 1998. Numerical methods for the strain-softening response of concrete in uniaxial compression. In *Damage and Fracture Mechanics*, Eds. A. Carpinteri & C. Brebbia, Computational Mechanics Publications, 297-306, Bologna, Italy.

Carpinteri A., Ciola F. and Pugno N. 2001. Boundary element method for the strain-softening response of quasi-brittle materials in compression. *Computers & Structures* **79**, 389-401.

Carpinteri A., Ciola F., Pugno N., Ferrara G. and Gobbi M. E. 1998. Applications of the Boundary Elements Method to the compressive strain-softening behaviour of concrete. In *Fracture Mechanics of Concrete Structures*, Eds. H. Mihashi & K. Rokugo, Aedificatio Publishers, 1949-1962, Gifu, Japan.

Carpinteri A., Ciola F., Pugno N., Ferrara G. and Gobbi M. E. 2001. Size-scale and slenderness influence on the compressive strain-softening behaviour of concrete: experimental and numerical analysis. *Fatigue & Fracture of Engineering Materials and Structures*. In print.

Carpinteri A., Ferro G. and Monetto I. 1999. Scale effects in uniaxially compressed concrete specimens. *Magazine of Concrete Research* **51**, 217-225.

Engleman R., Rivier N., Jaeger Z. 1988. Size distribution in sudden breakage by the use of entropy maximization. *J. Appl. Phys.* **63**, 4766-4768.

Feder J. 1988. *Fractals*. Plenum Publishing Corporation, New York.

Ferrara G. and Gobbi M.E. 1995. Strain softening of concrete under compression. *Report to RILEM Committee 148 SSC*, ENEL-CRIS Laboratory, Milano, Italy.

Hönig F. 1936. Grundgesetze der Zerkleinerung. *VDI Forschungsheft* **378**, Berlin.

Kick F. 1885. *Das Gesetz der Proportionalen Widerstände*, Leipzig.

Mandelbrot B.B. 1982. *The Fractal Geometry of Nature*. Freeman, NY.

McDowell G.R., Bolton M.D., Robertson D. 1996. The fractal crushing of granular materials. *J. Mech. Phys. Solids* **44**, 2079-2102.

Momber A.W. 2000. The fragmentation of standard concrete cylinders under compression: the role of the secondary fracture debris. *Engineering Fracture Mechanics* **67**, 445-459.

Nagahama H. 1993. Fractal fragment size distribution for brittle rocks. *Int. J. Rock Miner Sci. Geomech. Abstr.* **30**, 469-471.

Novozhilov V. 1969. On a necessary and sufficient criterion for brittle strength. *Prik. Mat. Mek.* **33**, 212-222.

Perfect E. 1997. Fractal models for the fragmentation of rocks and soils: a review. *Engineering Geology* **48**, 185-198.

Report of the Round Robin Test carried out by RILEM TC 148-SSC 1997. Strain softening of concrete in uniaxial compression. *Materials & Structures* **30**, 195-209.

Rittinger P.R. 1867. *Lehrbuch der Aufbereitungskunde*, Berlin.

Smekal A. 1937. Physikalisches und technisches Arbeitsgesetz der Zerkleinerung. *Zeitschr.* VDI, Beiheft Verfahrenstechnik, 159-161.

Stroeven P. 1991. Fractal and fractography in concrete technology. *International Symposium on Brittle Matrix Composites III*, Warsaw, Poland, 1-10.

Turcotte D.L. 1986. Fractal and fragmentation. *J. Geophys. Res.* **91**, 1921-1926.

Turcotte D.L. 1989. Fractals in geology and geophysics. *Pure Appl. Geoph.* **131**, 171-196.

Turcotte D.L. 1992. *Fractals and Chaos in Geology and Geophysics*. Cambridge University Press, 20-32.

Van Mier J.G.M. 1984. *Strain Softening of Concrete under Multiaxial Compression*. Ph. D. Thesis, Eindhoven University of Technology, Eindhoven, the Netherlands.

Yong Z., Hanson M.T. 1996. A rotational source of plane fractals and its application to fragmentation analysis of thin plates. *Chaos, Solitons and Fractals* **7**, 31-40.