*Fracture Mechanics of Concrete Structures, de Borst et al (eds) © 2001 Swets & Zeitlinger, Lisse, ISBN 90 2651 825 0*

# Using numerical simulations to determine the accuracy of the size-effect and two-parameter data reduction methods for fracture toughness tests of concrete

J.H.Hanson
*Bucknell University, Lewisburg, PA, USA*

A.R.Ingraffea
*Cornell University, Ithaca, NY, USA*

ABSTRACT: The size-effect and two-parameter data reduction methods were developed to extract size independent parameters from fracture toughness tests of concrete. However, the methods have limited ranges of applicability. This study uses simulated load versus crack mouth opening displacement graphs to investigate the accuracy of the fracture strength parameter obtained from the two data reduction methods. The cohesive zone properties, in the form of tension softening diagrams, used in the simulations represent tensile strengths from 2.2 to 9.3 MPa and fracture toughness values from 1.10 to 4.40 MPa√m. Each tension softening diagram has been used in simulations of three different sizes of single edge notched specimens loaded in bending based on the two RILEM recommendations of 1990. The results of the evaluation indicate that both data reduction methods underestimate the fracture strength parameter for most of the range of tension softening diagram properties considered.

## 1 INTRODUCTION

### 1.1 *Objectives*

The size-effect and two-parameter data reduction methods were developed to extract size independent parameters from fracture toughness tests of concrete. For most concretes, a laboratory-scale test specimen is too small to experience linear elastic fracture mechanics, LEFM, conditions. Therefore, these two methods were developed to account for the non-linear conditions that prevail at this scale. However, the methods have limited ranges of applicability. The objective of this study is to identify the accuracy of each of the data reduction methods for a range of concrete mixes.

Determining whether the fracture properties obtained for a particular test are truly size independent parameters is a difficult task when only experimental data is available. In order to show that a value is the same as one that would be obtained from an infinitely large specimen requires comparison of experimental test results from different geometries, specimen sizes, and data reduction methods. Only when the results agree can they be proclaimed to be the size independent values. However, numerical simulations based on cohesive cracking are an excellent tool for evaluating the range of applicability of the data reduction methods because the fracture toughness of the simulated material is known *a priori*. Therefore, cohesive crack simulations are used in this study to generate simulated load versus crack mouth opening displacement graphs for a range of simulated concrete mixes.

### 1.2 *Specimen geometry*

The specimen geometry chosen is the single edge loaded in bending, SE(B). The dimensions conform to the requirements of two proposed testing standards through the RILEM organization (1990a, b). The details of the specimen geometries are given in Figure 1. The total ligament length is the distance $W - a_o$, which equals $0.67W$ for all three specimen sizes. The supports were simulated as lines of constrained vertical displacement. The loaded region was 3.2 mm × 79 mm for all three specimen sizes.

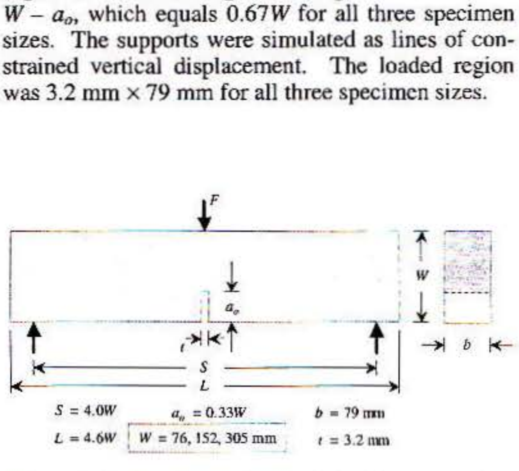


Figure 1. Single edge specimen loaded in bending. Dimensions are for the three simulated test specimen geometries.

### 1.3 *Simulation tools*

The simulation program used in this study was CohFRANC3D, which was developed and is maintained by the Cornell Fracture Group, *http://www.cfg.cornell.edu*. The program uses the boundary element method, BEM, to analyze fully three-dimensional models. The cohesive simulation capabilities of the program are based upon influence coefficients (Petersson 1981, Bittencourt 1993). The program has been used to generate simulated load versus crack mouth opening displacement, CMOD, responses of the test specimen. The simulation capabilities do not permit the cohesive zone to extend through the entire height of the simulated specimen; therefore, the extent of the simulated load versus CMOD response is limited. The program also utilizes the focal point model by Yankelevsky and Reinhardt (1989) to predict the unloading response of a specimen after the simulated process zone begins to grow. An example of a simulated response of a specimen with $W = 76$ mm is shown in Figure 2. Based on detailed studies (Hanson 2000), inaccuracies in the simulations due to errors in the BEM generated influence coefficients are in the range of 5-15%.

The cohesive properties of the material are characterized by the tension softening diagram, TSD (Fig. 3). The TSD is a constitutive relationship that governs the stress versus crack opening displacement behavior of the material. The CohFRANC3D program accepts TSD parameters for either linear or bi-linear softening. A bilinear TSD is characterized by four independent parameters, $f_t$, $f_{tr}$, $w_{tr}$, and $w_c$. The fracture energy, $G_{Ic}^{coh}$, is the resulting area under the TSD. Using the modulus of elasticity of the simulation model, the fracture energy can be converted to the fracture toughness, $K_{Ic}^{coh}$.

Rice (1968) proved that $G_{Ic}^{coh}$ is the fracture energy of a material at a point. Rice did not make any assertion that $G_{Ic}^{coh}$ is a material property, but he did show that if $G_{Ic}^{coh}$ is constant for a material it must be the size independent value, $G_{Ic}$. The authors make the assumption that $G_{Ic}^{coh}$ is a material property; therefore, $K_{Ic}^{coh}$ is the size independent fracture toughness of a simulated specimen. The implication of this statement is that the true fracture toughness of the simulated specimen is known. Therefore, the accuracy of fracture toughness values obtained using the two-parameter and size-effect data reduction methods on the simulated specimen responses can be determined.

## 2 DATA REDUCTION METHODS

### 2.1 *Two-parameter method*

The two-parameter method (Jenq & Shah 1985) asserts that the global response of a structure with a crack experiencing NLFM conditions can be reproduced by considering the structure to have an effective crack experiencing LEFM conditions. The effective critical crack length, $a_c^e$, is determined by measuring the unloading compliance of the specimen at or near peak load. The resulting measure of fracture toughness will be referred to as $K_{Ic}^{TP}$. The steps of the data reduction process are detailed in one of the RILEM recommended standards (1990a).

### 2.2 *Size-effect method*

The size-effect method (Bazant & Pfeiffer 1987) asserts that the nominal strength, $\sigma_N$, of geometrically similar specimens is only a function of one specimen dimension, say $W$. This non-linear function can be algebraically manipulated to obtain a linear relationship between $1/\sigma_N^2$ and $W$. Linear regression can be applied to the measured $1/\sigma_N^2$ and $W$ data to obtain the slope and intercept of the linear relationship. The measured fracture toughness, $K_{Ic}^{SZ}$, can be calculated from the square root of 1/slope. The steps of the data reduction process are detailed in another of the RILEM recommended standards (1990b).

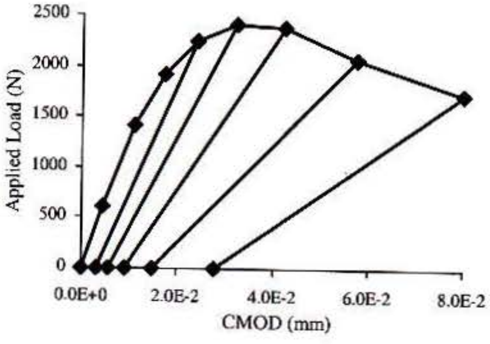


Figure 2. Simulated load versus CMOD response of a 76 mm deep SE(B) specimen including simulated unload cycles.

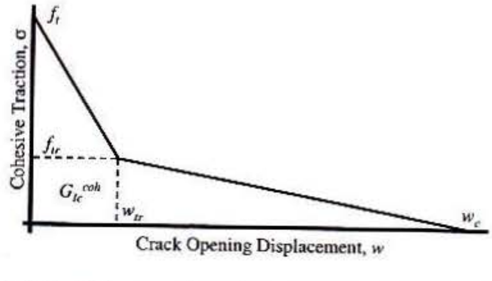


Figure 3. Bi-linear tension softening diagram used to govern the stress versus crack opening displacement behavior of a simulated specimen during a cohesive crack simulation.

## 3 RESULTS

### 3.1 *Properties of simulated concrete mixes*

In the course of this study, 17 different tension softening diagrams were used to simulate the load versus CMOD response of the three sizes of SE(B) specimens. The properties of the various TSD's are listed in Table 1. All of the models used a modulus of elasticity of 27.6 GPa. The $f_t$ values range from a value representative of normal strength concrete loaded in splitting tension or modulus of rupture tests, 2.2 MPa, to a very high value, 9.3 MPa. The $K_{Ic}^{coh}$ values range from a value commonly measured in laboratory tests using the two-parameter data reduction method, 1.1 MPa√m, to a very high value, 4.4 MPa√m. The TSD's with high $f_t$ values and low $K_{Ic}^{coh}$ values represent brittle fracture behavior. The TSD's with low $f_t$ values and high $K_{Ic}^{coh}$ values represent more ductile fracture behavior.

### 3.2 *Results using the two-parameter method*

The results from applying the two-parameter data reduction method to data from all three sizes of SE(B) specimens are presented in Table 2. The accuracy of the measured fracture toughness value is expressed as the percentage difference between the measured fracture toughness and the true fracture toughness for the particular TSD (Equation 1). The relative size of the process zone, PZ, when the peak load occurs is expressed as a percentage of the total ligament length (Equation 2).

### 3.3 *Results using the size-effect method*

The results from applying the size-effect data reduction method to the peak loads of all three sizes of SE(B) specimens are presented in Table 3. The accuracy of the measured fracture toughness was calculated using Equation 1. The ranges of relative size of the process zone are summarized from Table 2.

$$\%\,\mathrm{Diff} = \frac{K_{Ic}^{TP\ or\ SZ} - K_{Ic}^{coh}}{K_{Ic}^{coh}} \qquad (1)$$

$$\mathrm{PZ}\% = \frac{L_{Process\ Zone}}{0.67W} \qquad (2)$$

Table 1. Cohesive zone properties used to simulate a variety of concrete mixes

| TSD Series | $f_t$ (MPa) | $f_{tr}$ (MPa) | $w_{tr}$ (mm) | $w_c$ (mm) | $K_{Ic}^{coh}$ (MPa√m) |
|---|---|---|---|---|---|
| A | 2.17 | 0.22 | 1.34E-2 | 2.69E-1 | 1.10 |
| B | 2.17 | 1.09 | 3.67E-3 | 7.33E-2 | 1.10 |
| C | 2.17 | 0.22 | 3.36E-2 | 6.72E-2 | 1.10 |
| D | 2.17 | 1.09 | 2.02E-2 | 4.03E-2 | 1.10 |
| E | 2.17 | 0.22 | 2.15E-1 | 4.30E+0 | 4.40 |
| F | 2.17 | 1.09 | 5.86E-2 | 1.17E+0 | 4.40 |
| G | 2.17 | 0.22 | 5.38E-1 | 1.08E+0 | 4.40 |
| H | 2.17 | 1.09 | 3.23E-1 | 6.45E-1 | 4.40 |
| I | 9.31 | 0.93 | 3.14E-3 | 6.27E-2 | 1.10 |
| J | 9.31 | 4.65 | 8.59E-4 | 1.72E-2 | 1.10 |
| K | 9.31 | 0.93 | 7.85E-3 | 1.57E-2 | 1.10 |
| L | 9.31 | 4.65 | 4.72E-3 | 9.45E-3 | 1.10 |
| M | 9.31 | 0.93 | 5.02E-2 | 1.00E+0 | 4.40 |
| N | 9.31 | 4.65 | 1.37E-2 | 2.74E-1 | 4.40 |
| O | 9.31 | 0.93 | 1.25E-1 | 2.51E-1 | 4.40 |
| P | 9.31 | 4.65 | 7.53E-2 | 1.51E-1 | 4.40 |
| Q | 5.74 | 1.72 | 4.56E-2 | 1.66E-1 | 2.75 |

Table 2. Results from applying the two-parameter data reduction method to simulated results from three sizes of SE(B) specimens using a variety of concrete mixes.

| TSD Series | $K_{Ic}^{coh}$ Input (MPa√m) | $K_{Ic}^{TP}$ (MPa√m) 76 mm | 152 mm | 305 mm | Difference Percentage (Eqn 1) 76 mm | 152 mm | 305 mm | Relative Length of PZ (Eqn 2) 76 mm | 152 mm | 305 mm |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 1.10 | 0.51 | 0.59 | 0.69 | -54% | -46% | -38% | 40% | 30% | 20% |
| B | 1.10 | 0.51 | 0.68 | 0.82 | -54% | -38% | -26% | 50% | 50% | 40% |
| C | 1.10 | 0.67 | 0.80 | 0.95 | -39% | -28% | -13% | 50% | 40% | 30% |
| D | 1.10 | 0.67 | 0.81 | 0.97 | -39% | -27% | -12% | 50% | 40% | 30% |
| E | 4.40 | 1.21* | 1.70* | 1.92 | -73% | -61% | -56% | ≥75% | ≥75% | 60% |
| F | 4.40 | 1.12* | 1.27 | 1.49 | -74% | -71% | -66% | ≥75% | 75% | 50% |
| G | 4.40 | 1.27* | 1.85* | 2.74* | -71% | -58% | -38% | ≥75% | ≥75% | ≥75% |
| H | 4.40 | 1.29* | 1.86* | 1.81* | -71% | -58% | -59% | ≥75% | ≥75% | ≥75% |
| I | 1.10 | 0.67 | 0.91 | 1.34 | -39% | -17% | 22% | 10% | 10% | 10% |
| J | 1.10 | 0.96 | 1.10 | 1.43 | -13% | 1% | 30% | 20% | 10% | 5% |
| K | 1.10 | 0.95 | 1.12 | 1.30 | -14% | 2% | 18% | 10% | 10% | 5% |
| L | 1.10 | 0.96 | 1.14 | 1.27 | -12% | 4% | 15% | 10% | 5% | 5% |
| M | 4.40 | 2.13 | 2.46 | 2.80 | -51% | -44% | -36% | 40% | 30% | 20% |
| N | 4.40 | 2.12 | 2.84 | 3.44 | -52% | -35% | -22% | 50% | 50% | 40% |
| O | 4.40 | 2.81 | 3.34 | 3.98 | -36% | -24% | -9% | 50% | 40% | 30% |
| P | 4.40 | 2.84 | 3.38 | 4.04 | -35% | -23% | -8% | 50% | 40% | 30% |
| Q | 2.75 | 1.66 | 1.94 | 2.28 | -40% | -30% | -17% | 50% | 40% | 30% |

* The $K_{Ic}^{TP}$ value is based on the last simulated data point since it had the largest applied load. Since the peak load might have not yet been simulated, the $K_{Ic}^{TP}$ value presented might be low.

Table 3. Results from applying the size-effect data reduction method to simulated results from three sizes of SE(B) specimens using a variety of concrete mixes.

| TSD Series | $K_{Ic}^{coh}$ (MPa√m) | $K_{Ic}^{SZ}$ (MPa√m) | % Diff (Eqn 1) | Rel. Length of PZ |
|---|---|---|---|---|
| A | 1.10 | 0.77 | -30 % | 20 - 40 % |
| B | 1.10 | 1.08 | -2 % | 40 - 50 % |
| C | 1.10 | 1.18 | 7 % | 30 - 50 % |
| D | 1.10 | 1.21 | 10 % | 30 - 50 % |
| E | 4.40 | 3.03 | -31 % | 60 - 75 % |
| F | 4.40 | 2.03 | -54 % | 50 - 75 % |
| G | 4.40 | unknown * | N/A | ≥75 % |
| H | 4.40 | unknown * | N/A | ≥75 % |
| I | 1.10 | 3.96 ** | 260 % | 10 % |
| J | 1.10 | 1.68 ** | 53 % | 10 - 20 % |
| K | 1.10 | 1.22 | 11 % | 5 - 10 % |
| L | 1.10 | 1.12 | 2 % | 5 - 10 % |
| M | 4.40 | 3.04 | -31 % | 20 - 40 % |
| N | 4.40 | 4.47 | 2 % | 40 - 50 % |
| O | 4.40 | 4.74 | 8 % | 30 - 50 % |
| P | 4.40 | 4.91 | 12 % | 30 - 50 % |
| Q | 2.75 | 2.51 | -9 % | 30 - 50 % |

* The peak load was likely under predicted for all three sizes of specimens. As a result, the peak loads resulted in a slope value that was negative. Therefore, a $K_{Ic}^{SZ}$ value could not be determined.

** The relatively large $K_{Ic}^{SZ}$ value is likely a result of lack of accuracy in the simulated peak loads due to the relatively large crack face node spacing compared to the simulated process zone size.

## 4 CONCLUSIONS

The results of the evaluation indicate that both data reduction methods underestimate the fracture strength parameter for most of the range of tension softening diagram properties considered. This implies that the size-effect and two-parameter data reduction methods are not, in general, producing size independent fracture parameters from laboratory-scale tests of concrete.

The two-parameter data reduction method assumes that the process zone is sufficiently small at peak load that the behavior of the specimen can be expressed in terms of an effectively longer crack experiencing LEFM conditions. From the simulation results in this study, it appears that the relative process zone size must be less than approximately 15% of the ligament length to produce a $K_{Ic}^{TP}$ value within a few percent of the size independent value, $K_{Ic}$.

The size-effect data reduction method assumes that the nominal strength-specimen size relationship for a material is unaffected by the relative size of the process zone. The validity of this assumption is difficult to show through direct observation of the process zone. However, when the measured $K_{Ic}^{SZ}$ value is within a few percent of $K_{Ic}$, the assumption must be satisfied. From the various simulation results in this study, it appears that the total relative process zone size at peak load is *NOT* an indication of the accuracy of $K_{Ic}^{SZ}$. The appropriate guideline should probably be based upon the relative process zone size considering only the initial softening portion of the tension softening diagram.

## 5 REFERENCES


Bazant, Z.P. & Pfeiffer, P.A., 1987. Determination of Fracture Energy from Size Effect and Brittleness Number. *ACI Materials Journal* 84(6): 463-480.

Bittencourt, T.N. 1993. Computer Simulation of Linear and Nonlinear Crack Propagation in Cementitious Materials. Ph.D. Dissertation. Department of Civil and Environmental Engineering: Cornell University.

Hanson, J.H. 2000. An Experimental-Computational Evaluation of the Accuracy of Fracture Toughness Tests on Concrete. Ph.D. Dissertation. School of Civil and Environmental Engineering: Cornell University.

Jenq, Y.S. & Shah, S.P., 1985. A Fracture Toughness Criterion for Concrete. *Engineering Fracture Mechanics* 21(5): 1055-1069.

Petersson, P-E. 1981. Crack Growth and Development of Fracture Zones in Plain Concrete and Similar Materials. Report TVBM-1006/1-174. Division of Building Materials: Lund Institute of Technology.

Rice, J.R. 1968. Mathematical Analysis in the Mechanics of Fracture. In H. Liebowitz (ed.), *Fracture: An Advanced Treatise* New York: Academic.

RILEM, 1990a. Determination of Fracture Parameters ($K^S_{Ic}$ and $CTOD_c$) of Plain Concrete Using Three-Point Bend Tests. *Materials and Structures* 23: 457-460.

RILEM, 1990b. Size-Effect Method for Determining Fracture Energy and Process Zone Size of Concrete. *Materials and Structures* 23: 461-465.

Yankelevsky, D.Z. & Reinhardt, H.W. 1989. Uniaxial Behavior of Concrete in Cyclic Tension. *Journal of Structural Engineering* 115(1): 166-182.