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On the size-effect phenomenon in concrete structures

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ABSTRACT: A fresh interpretation of existing experimental data, combined with what one could aptly describe as numerical experiments, leads to a new explanation of the underlining causes for size effects in structural concrete. Such an outcome is based on the premise that one must interpret correctly physical phenomena before attempting to put forward mathematical theories which, as so often in the past, obfuscate rather than elucidate, the nature of the so-called size effect in certain types of concrete members. This is the result of recent collaborative research between the National Technical University, Athens (NTUA) and Imperial College, London (ICL), a collaboration which is currently being extended so as to also encompass the Nanyang Technological University, Singapore (NTUS).

1. INTRODUCTION

It is widely believed that the behaviour of structural concrete, as predicted by methods of analysis such as the finite-element (FE) method, is significantly affected by the size of the member analysed. This size effect on the predicted behaviour is generally attributed to inadequate modelling of the material properties and, thus, much of current research is focused on improving material modelling which is often based on sophisticated fracture-mechanics postulates. However, theoretical considerations can sometimes distract mechanicians from real issues and this would seem to be the case with the phenomenon under discussion. It would appear, therefore, that, instead of relying on hypothetical material characteristics that, somehow, are a function of the structure size, it is simpler and more rational to leave such properties unchanged and, instead, search for other – less involved and more plausible – causes for size effects.

The present article describes and summarizes recent research aimed at providing alternative explanations for the phenomenon under discussion. This can be subdivided into three stages. First, the effect of small, unintended eccentricities of the load applied to a member is examined. Then, the possibility of asymmetric cracking under symmetric loading is introduced in an attempt to allow for the effect of local material weaknesses which may exist as a result of the heterogeneous nature of concrete. Finally, current work on beams with small shear span-to-depth ratio is presented so as to enlarge the range of problems considered earlier.

2. EFFECT OF SMALL ECCENTRICITY OF APPLIED LOADING

Most of the research on size effects to date has been confined to conditions of implicit load symmetry. However, while such conditions can readily be imposed in numerical analysis, in real structures it is impossible to prevent the occurrence of out-of-plane actions. What is implicitly assumed is that, in a controlled experiment, any (unintended) eccentricity of the loading can be minimized so that the resulting out-of-plane actions will have a negligible effect on the structural response. And yet, it has recently been shown that ignoring small stresses – which arise from triaxial effects prior to failure – often leads to misinterpretations of the causes of observed structural concrete behaviour since the presence of such stresses usually has a significant effect on material strength (Kotsovos & Pavlovic 1995).

If a connection between size effects and unintended out-of-plane actions really exists, then, in cases where these actions are self-evidently negligible (e.g. slabs under transverse patch loading), the predicted behaviour should be unaffected by size effects. Similarly, the effect of secondary reinforcement ought to be sufficient to absorb the additional small stresses caused by the unintended eccentricity of the applied load (e.g.

reinforced concrete (RC) beams with stirrups). On the other hand, the absence of stirrups in RC beams may be expected to lead to size effects.

To test the above postulates, the nonlinear FE package for structural concrete described in Kotsovos & Pavlovic 1995 was used to predict the ultimate load-carrying capacities of members which, then, were compared to their experimental counterparts (Kotsovos & Pavlovic 1994, Kotsovos & Pavlovic 1995). The first investigation centred on two series of circular slabs (under concentric patch loading), the data for which was obtained from two different testing programmes (so as to cover a wide range of sizes) where all specimens failed in a brittle manner (see Kotsovos & Pavlovic 1994). The relevant details are contained in Table 1. It can be seem that the correlation between test results and the theoretical predictions is quite good, the error generally not exceeding 10%, which is within the tolerance level of accurate analytical predictions (Kotsovos & Pavlovic 1995). (A somewhat higher discrepancy, 15-20%, occurs for the more heavily reinforced specimens (ρ=0.0152): such slabs fail in a particularly brittle manner so that a larger scatter of experimental results could be expected if more tests were to be carried out.) The table also indicates that the correlation between the mean predicted and test values for the 200 mm deep slabs is as close as that for the 94 mm deep slabs: such a correlation leads one to conclude that the predicted slab behaviour is essentially independent of size effects. (It should also be pointed out that the analytical model overestimates the failure loads since no attempt has been made to allow for possible material heterogeneity – this will also be apparent from the results of the next two studies.)

The second study consists of RC beams with stirrups, with data drawn from three different sources which provide a very wide range of sizes (in the ratio (by volume) of 1:40:260) (Kotsovos & Pavlovic 1994). Table 2 summarizes the relevant information, and shows that the correlation between test results and analytical predictions is very close for all beams. As the beams differ in size quite considerably, such close correlation can only be interpreted as a proof of the negligible effect that beam size has on predicted structural behaviour when secondary reinforcement is present. Table 3 contains the details of the third case study, namely a series of RC beams without stirrups, the data for which was taken from the well-known Stuttgart shear tests (see Kotsovos & Pavlovic 1994), with two different sets of geometrically strictly similar specimens forming the basis of an experimental investigation into the problem of size effects in structural concrete. By comparing the reported experimental ultimate-load values as well as their analytical counterparts obtained presently, it is evident that, unlike the preceding two case studies, the correlation between test results and theoretical predictions is now satisfactory only for the case of the smaller beam D1. For all other beams, the predictions overestimate consistently the load-carrying capacity by a margin which gradually increases with size and finally appears to stabilize at a maximum value around 40% for series D and 50% for series C.

Such a size effect on the predicted behaviour is usually attributed to inadequacies in the modelling of the material properties, which form an essential part of any analytic approach. However, this view cannot explain the fact that the current analyses yield size-independent predictions for the first two case studies and size-dependent predictions for the third case study. In fact, it is significant that the size ratio (expressed as a volume ratio) of beams B1 and A1 (with stirrups), for which the predicted collapse loads are size independent, is approximately 1 to 260 which is similar to that of beams D1 and C4 (without stirrups), for which the predictions are size-dependent. Thus, it is considered that the cause of size-dependent predictions in the third case study relates to the development (during testing) of unintended out-of-plane eccentricities of the applied load which become more significant as the size of the specimen increases. Such unintended eccentricities create torsional effects which give rise to transverse tensile stresses within the specimens, leading to failure under a load which is lower than that expected to cause failure under in-plane loading conditions. On the other hand, components for which it is self-evident that small unintended eccentricities have an insignificant effect on structural response (e.g. slabs under concentric patch loading) or RC beams with secondary reinforcement (where the presence of stirrups is sufficient to withstand the additional tensile stresses caused by the small, unintended torsional actions) exhibit size-effect independent behaviour.

3. EFFECT OF LOCAL MATERIAL HETEROGENEITY

In addition to the above connection between small unintended eccentricities in applied loading and size effects, a further cause for the latter phenomenon could also be due to another source of out-of-plane actions, this time arising from non-symmetrical fracture processes in structural concrete. To

Table 1. RC circular slabs: data and comparison between experimental and predicted ultimate loads (P_u) (Note: d is effective depth; c is diameter of loaded area; a is slab span; ρ is tension-steel ratio)

Slab Reference	Dimensions: mm			Material properties: N/mm^2		ρ	P_u:kN		
	D	c	a	f_c	f_y		Test	Analysis	Analysis /Test
DKNT 05	200	250	2400	24.2	657	0.0080	603	691	1.15
DKNT 17	200	250	2400	25.4	668	0.0034	489	528	1.08
									Mean: 1.11
DREG 31	95	150	1370	23.2	494	0.0083	197	201	1.02
DREG33	95	150	1370	37.8	494	0.0083	214	226	1.06
DREG35	93	150	1370	26.8	494	0.0152	214	245	1.14
DREG36	93	150	1370	42.6	494	0.0152	248	301	1.21
									Mean: 1.11

explore this postulate, the three-dimensional nonlinear FE package described in Kotsovos & Pavlovic 1995 was employed to predict the behaviour of a number of geometrically strictly similar RC beams of various sizes, made of the same materials, with and without stirrups, which were divided into two groups: group A, in which the beams have been modelled so as to undergo non-symmetrical cracking under increasing loading; and group B, in which the beams in group A have been modelled so as to undergo symmetrical cracking under the same loading conditions. The ensuing conclusions were therefore based on a comparative study of the numerically predicted behaviour of the beams in groups A and B, with the aim to establish the connection between non-symmetrical cracking and beam size, as well as to investigate the influence of the stirrups on size effects (Kotsovos & Pavlovic 1997, Kotsovos & Pavlovic 1995).

When the ultimate-strength level is reached (i.e. the conditions for macrocracking are satisfied) in the FE model, a microcrack is allowed to form only at the location where the stress conditions are the most critical (Kotsovos & Pavlovic 1995). Now, owing to the "double-precision accuracy" used in the package, it is unlikely that the main decimal digits associated with the stress values at nominally symmetric locations will exactly coincide (unless symmetry is imposed in the analysis, as in group B beams); thus, the stress conditions may even be found to be more critical in only one of two symmetrical locations within the structure (i.e. when the whole structure is analysed (neglecting geometric and loading symmetries which might permit only one half of the cross-sectional width to be considered), as in group A beams). Adopting such a procedure to describe the fracture process may therefore lead to non-symmetrical macrocracking even for the case of symmetrical

Table 2. RC beams with stirrups: data and comparison between experimental and predicted ultimate loads (P_u). (Note: b is width; d is effective depth; a_v is shear span; L is span; ρ is tension-steel ratio; A_{sv} is stirrup area; s is stirrup spacing)

Beam Reference	Dimensions: mm				Material properties: N/mm^2		P	A_{sv}/s	P_u:kN		
	b	d	a_v	L	f_c	f_y			Test	Analysis	Analysis/Test
B1	50	90	300	900	37.0	417	0.0126	0.322	13.6	14.0	1.03
B150-11-3	152	298	1067	2134	69.5	448	0.0335	0.579	323.0	360.0	1.11
A-1	310	461	1827	3658	24.1	555	0.0184	0.306	467.0	450.0	0.96

Table 3. RC beams without stirrups: data and comparison between experimental and predicted ultimate loads (P_u). (Note: b is width; d is effective depth: a_v is shear span; L is span; ρ is tension-steel ratio)

Beam Reference	Dimensions: mm				Material properties: N/mm^2		ρ	P_u:kN		
	b	d	a_v	L	F_c	F_y		Test	Analysis	Analysis/Test
D1	50	70	210	520	38.0	460	0.0162	14.8	15.3	1.03
D2	100	140	420	1040	38.2	435	0.0162	44.4	49.0	1.10
D3	150	210	630	1560	39.5	421	0.0162	89.2	125.0	1.40
D4	200	280	840	2080	36.1	448	0.0162	148.0	204.0	1.38
C1	100	150	450	1000	40.0	433	0.0134	44.0	48.0	1.09
C2	150	300	900	2000	40.0	433	0.0134	132.5	156.0	1.18
C3	200	450	1350	3000	40.0	433	0.0134	202.0	324.0	1.60
C4	225	600	1800	4000	40.0	433	0.0134	310.0	450.0	1.45

structural forms subject to symmetrical loading.

Tables 4-7 contain the details of the simply supported beams, with and without stirrups, adopted in the present investigation. The beams were selected so as to be similar in geometry and main reinforcement to those among the Stuttgart shear tests also used in the previous section. All members were analysed both with and without stirrups, the latter being designated by adding the ending "s" to the description of the beams without stirrups. Each beam was analysed twice: first, by considering that the FE meshes represent half the specimens (symmetry with respect to the cross-section at midspan of the beams); and then by considering that the same meshes represent one-quarter of the beams (two-fold symmetry with respect to the vertical (longitudinal and transverse) bisectors of the members). In the former case, the beams have been designated as group A beams (consisting of beams D and C, without (D1, ..., D4, C1, ..., C4) and with (D1s, ..., D4s, C1s, ..., C4s) stirrups), while in the latter case, as group B beams (also consisting of beams D and C, without (D11, ..., D44, C11, ..., C44) and with (D11s, ..., D44s, C11s, ..., C44s) stirrups). Clearly, whereas for group B symmetry (including symmetrical cracking) is imposed by the prescribed boundary conditions which prevent displacements across the two planes of symmetry, for group A the sole prevention of the longitudinal displacements at midspan is insufficient to prevent non-symmetrical cracking.

The predicted values in Tables 4-7 lead to the following observations. First, note how the group A beams without stirrups, where non-symmetrical cracking mimics local material weakness, shows a much closer correlation with the experimental data (Table 4) than was the case earlier, when symmetrical cracking was implicit in the analysis (Table 3). Secondly, the introduction of secondary (transverse) reinforcement is, once again, sufficient to eliminate size effects since the local weakness imposed by material heterogeneity (group A) is absorbed by the stirrups so that the beam strengths are essentially the same as those of their group B counterparts (these are listed in Table 7), as can be seen in Table 5. Thirdly, as soon as stirrups are removed, size effects arising from material heterogeneity reappear: this is evident by reference to Table 6 where non-symmetrical cracking weakens the strength of the beams with respect to the corresponding collapse values obtained by imposing symmetric cracking.

Thus, the predicted values in Tables 4 to 7 clearly demonstrate that the load-carrying capacity of group A beams, without stirrups, decreases as a percentage of that of their group B counterparts with increasing beam size. The graphical representation of these results, depicted in Figure 1, shows that the above reduction in structural-strength capacity exhibits a trend qualitatively similar to those of size effects as the latter are

Table 4. Group A of RC beams without stirrups: data and comparison between experimental and predicted ultimate loads (P_u). (Note: b is width; d is effective depth; a_v is shear span; L is span; ρ is tension-steel ratio)

Beam Reference	Dimensions: mm				Material properties: N/mm^2		ρ	P_u:kN		
	b	d	a_v	L	f_c	f_y		Test	Analysis	Analysis/Test
D1	50	70	210	520	38.0	460	0.0162	14.8	15.3	1.03
D2	100	140	420	1040	38.2	435	0.0162	44.4	36.0	0.81
D3	150	210	630	1560	39.5	421	0.0162	89.2	90.0	1.01
D4	200	280	840	2080	36.1	448	0.0162	148.0	144.0	0.97
C1	100	150	450	1000	40.0	433	0.0134	44.0	48.0	1.09
C2	150	300	900	2000	40.0	433	0.0134	132.5	156.0	1.18
C3	200	450	1350	3000	40.0	433	0.0134	202.0	270.0	1.34
C4	225	600	1800	4000	40.0	433	0.0134	310.0	330.0	1.06

Table 5. Group A of RC beams with stirrups: data and comparison of predicted ultimate loads (P_u) with their group B counterparts. (Note: b is width; d is effective depth; a_v is shear span; L is span; f_{yv} is stirrup yield strength; ρ, ρ_v are tension-steel ratio, stirrup-steel ratio respectively; * denotes predicted values for group B beams from Table 7)

Beam Refe-Rence	Dimensions: mm				Material properties: N/mm^2			ρ	ρ_v	P_u:kN	
	B	d	a_v	L	f_c	f_y	f_{yv}			Ana-lysis	Group A / Group B*
D1s	50	70	210	520	38.0	460	439	0.0162	0.0025	15.3	1.0
D2s	100	140	420	1040	38.2	435	439	0.0162	0.0025	57.6	1.0
D3s	150	210	630	1560	39.5	421	439	0.0162	0.0025	126.0	1.0
D4s	200	280	840	2080	36.1	448	439	0.0162	0.0025	234.0	1.0
C1s	100	150	450	1000	40.0	433	424	0.0134	0.0017	63.0	1.08
C2s	150	300	900	2000	40.0	433	424	0.0134	0.0017	168.0	0.93
C3s	200	450	1350	3000	40.0	433	424	0.0134	0.0017	384.0	1.07
C4s	225	600	1800	4000	40.0	433	424	0.0134	0.0017	540	1.07

Table 6. Group B of RC beams without stirrups: data and comparison of predicted ultimate loads (P_u) with their group A counterparts. (Note: b is width; d is effective depth; a_v is shear span; L is span; ρ is tension-steel ratio; * denotes predicted values for group A beams from Table 4)

Beam	Dimensions mm				Material properties: N/mm^2		ρ	P_u:kN	
Refe-rence	b	d	a_v	L	f_c	f_y		Analysis	Group A* / Group B
D11	50	70	210	520	38.0	460	0.162	15.3	1.0
D22	100	140	420	1040	38.2	435	0.0162	49.0	0.73
D33	150	210	630	1560	39.5	421	0.162	125.0	0.72
D44	200	280	840	2080	36.1	448	0.162	204.0	0.71
C11	100	150	450	1000	40.0	433	0.0134	48.0	1.0
C22	150	300	900	2000	40.0	433	0.0134	156.0	1.0
C33	200	450	1350	3000	40.0	433	0.134	324.0	0.83
C44	225	600	1800	4000	40.0	433	0.0134	450.0	0.73

Table 7. Group B of RC beams with stirrups: data and predicted ultimate loads (P_u). (Note: b is width; d is effective depth; a_v is shear span; L is span; ρ, ρ_v are tension-steel ratio, stirrup-steel ratio respectively)

Beam	Dimensions mm				Material properties: N/mm^2			ρ	ρ_v	P_u:kN
Refe-rence	b	D	a_v	L	f_c	F_y	f_y			Analysis
D11s	50	70	210	520	38.0	460	439	0.0162	0.0025	15.3
D22s	100	140	420	1040	38.2	435	439	0.0162	0.0025	57.6
D33s	150	210	630	1560	39.5	421	439	0.0162	0.0025	126.0
D44s	200	280	840	2080	36.1	448	439	0.0162	0.0025	234.0
C11s	100	150	450	1000	40.0	433	424	0.0134	0.0017	58.5
C22s	150	300	900	2000	40.0	433	424	0.0134	0.0017	180.0
C33s	200	450	1350	3000	40.0	433	424	0.0134	0.0017	360.0
C44s	225	600	1800	4000	40.0	433	424	0.0134	0.0017	504.0

defined in practical structural design. Within this context, size effects are associated with shear capacity and they are expressed in an empirical form such as, for example, the relation recommended by the British Code BS 8110

$$s.e. = (400/d)^{1/4} \quad (1)$$

or that contained in the CEB-FIP Model Code for Concrete Structures

$$s.e = 1.6\text{-}d \quad (2)$$

where *s.e.* is not smaller than *1* and *d* is the depth of the critical cross-section. The graphical representation of expressions (1) & (2) normalized with respect to their values of beam D1 (d=70 mm) is also included in Figure 1, which indicates that the correlation between the code and the analysis predictions is satisfactory, considering both the approximate nature of the code expressions and the random triggering of early failure through double-precision numerics (Kotsovos & Pavlovic 1997).

4. BEAMS WITH SMALL SHEAR SPAN-TO-DEPTH RATIOS

So far, the nonlinear FE model was used in investigations of size effects in RC beams with a shear span-to-depth ratio (a_v/d) larger than 2. Currently, the scope of this work is being extended so as to include the case of beams with an a_v/d smaller than 1.15. The full details of the findings are presently being prepared for publication and

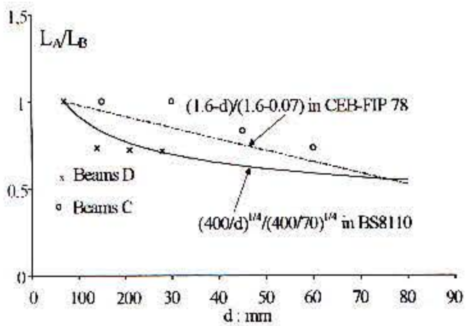

Figure 1 Variation of predicted load-carrying capacity (L_A) for group A (non-symmetrical cracking) beams without stirrups, normalized with respect to that (L_B) of their group B (symmetrical cracking) counterparts, with the size of the beams.

only the main observations will be summarized in what follows.

It was found that, unlike the beams with $a_v/d>2$, girders with $a_v/d<1.15$ appear to be essentially size-effect independent. Three observations lead to such a conclusion. First, although the largest deviation of the predicted ultimate loads from the test-data values is of the order of 30%, most predictions exhibit a significantly smaller deviation from their experimental counterparts. Secondly, whereas the dependence of the behaviour of a structural member on size effects is usually manifested by predicted values of load-carrying capacity which overestimate their experimental counterparts, the opposite is found to be the case when $a_v/d<1.15$: namely, the experimental ultimate capacities are consistently larger than the corresponding predicted ones. Thirdly, while the difference between the two sets of values is essentially independent of member size, the larger discrepancies occur mainly in the case of the smaller beams which is incompatible with the trend associated with "size effects.

The above difference in behaviour regarding "size effects" exhibited between beams with $a_v/d\leq2$ and beams with $a_v/d>2$ reflects the wider differences in behaviour characterizing RC beams with such values of a_v/d (Kotsovos & Pavlovic 1998). As described in Kotsovos & Pavlovic 1998, the causes of failure of RC beams are associated with the development of tensile stresses normal to the compressive-stress trajectories. For beams with $a_v/d>2$, such stresses develop within the compressive zone, which, due to its small volume, does not have the capacity to absorb – through redistribution – the effect of any additional such stresses caused by out-of-plane actions. As a result, the beam's load-carrying capacity is very sensitive to the development of the above additional stresses, which eventually lead to "premature" failure. On the other hand, for beams with smaller values of a_v/d, as defined in this section, the tensile stresses develop within the beam web. The beam web, in contrast to the compressive zone, is sufficiently large to provide the space required for the redistribution of any additional tensile stresses due to out-of-plane actions in a manner that their effect on load-carrying capacity is negligible.

5. CONCLUSIONS

The phenomenon of size effects in structural concrete represents a challenge to mechanicians. In the search for a simple and rational explanation, collaborative work between NTUA and ICL suggests that such effects reflect the dependence of load-carrying capacity on small unintended eccentricities of the applied load and/or load-induced anisotropy rather than, as widely considered, on fracture-mechanics characteristics. Such size effects disappear when the additional unintended stresses/actions are self-evidently negligible (e.g. slabs under patch loading), when these additional transverse tensile stresses are absorbed by secondary reinforcement (e.g. beams with stirrups), or when such tensile stresses develop in the (large) beam web rather than in the (small) compressive zone (e.g. deep beams).

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