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Inelastic Model Combining Distributed Micro-cracking and Macro-cracking Induced by Strain Softening

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ABSTRACT: In this work, we present the theoretical formulation of a material model combining consistently two kinds of dissipations: the volume-proportional one which is accounted for by continuum damage or continuum plasticity model and the other one proportional to a (macro-) crack which is specified by the traction - separation constitutive model. The details pertinent to the numerical implementation of the proposed model within the framework of incompatible modes are also presented.

1 INTRODUCTION

Ever increasing demand to achieve a better understanding of the behavior of a particular structural system and achieve a more economical structural design requires that the latter be based upon the limit load computation. It is often the case, in an analysis of this kind, that certain parts of the structural systems are heavily damaged and thus enter a post-peak or softening phase before the limit load of the structure has been reached. It is well known by now (e.g. see de Borst (1999)) that the strain softening phenomena of this kind present a serious difficulty for classical continuum models for representing inelastic deformations (e.g. plasticity or damage models) in assuring that the numerical solutions computed with different finite element mesh grading all lead to the same post-peak softening response. It is much less known how to construct such models able to provide a correct measure of energy dissipation and to handle the strain softening in a very efficient manner. Namely, a number of remedies which have been proposed initially involve, in general, a fairly elaborate modification of the classical continuum models. For example, the non-local continuum formulation (e.g. see Pijaudier-Cabot & Bazant (1987)) imposes that the stress state in a single point depends not only on the total strain and internal variables at that point but also on the values of those variables in a predefined neighborhood. Similarly, the gradient plasticity and gradient damage model (e.g. see de Borst et al. (1992)) lead to a non-local characterization of the plastic multiplier or damage indicator. Each of these formulation imposes a significant increase in complexity of the corresponding numerical procedure, and introduces the notion of the characteristic length, or in other words, to quantify the dissipation as proportional to the volume. In order to circumvent the difficulty of having to identify the characteristic length, more recent works on strain-softening problems (e.g. see Oliver (1996), Armero (1999) or Jirasek & Zimmermann (2001)) have turned to using a discontinuity of the displacement field (effectively simulating a macro-crack) and re-interpolating the manner of computing the amount of energy dissipation by a chosen traction-separation softening diagram. The models of this kind only take into account two possible states purely elastic or a macro crack activation, and completely ignore the dissipation due to micro-cracking. The main idea of this work, relates to the fact that physics of the problem is the most closely represented by a model which is capable to take into account both micro- and macro-cracking and properly interpret the nature of associated inelastic energy dissipation. One such model is proposed in this paper. In Section 2, we present the governing equations (see Ibrahimbegovic et al. (1998)). For clarity of the ideas, this is done for one-dimensional setting. In Section 3, we discuss the corresponding details of numerical implementation which are set within the framework of incompatible modes (see Ibrahimbegovic & Wilson (1991)) and present a couple of illustrative examples. Some closing remarks are stated in Section 5.

2 THEORETICAL FORMULATION

We consider a one-dimensional problem of axial deformation of a truss-bar, supported at one end and loaded by both distributed loading $b(x,t)$ and traction $\bar{t}(t)$ applied at its free-end. (We note that t is the pseudo-time parameter used to describe a particular loading program). For any material model of the bar one can write the kinematics and equilibrium equation (under small displacement gradient hypothesis):

$$\varepsilon(x,t) = \frac{\partial u(x,t)}{\partial x} \quad (1)$$

$$\frac{\partial \sigma(x,t)}{\partial x} + b(x,t) = 0 \quad (2)$$

where ε is the infinitesimal strain and σ is the Cauchy stress.

The first model we focus upon is the plasticity model which assumes that there exist an elastic domain where no change of inelastic deformation would occur, i.e.

$$\phi(\sigma, q) = |\sigma| - (\sigma_y - q) \leq 0 \quad (3)$$

where σ_y is the yield stress, separating initially the elastic and plastic domains, whereas q is the hardening variable taking into account an eventual evolution of the elastic domain due to hardening/softening phenomena. In a plasticity model at small deformation it is usually assumed (e.g. see Lubliner (1990)) that the total deformation can be additively decomposed into elastic and plastic parts. Moreover, the stress depends only on the elastic deformation part, whereas the plastic flow is governed by normality rule which amount to:

$$\dot{\varepsilon}(x,t) = \frac{\dot{\sigma}}{E} + \dot{\gamma}(x,t) sign(\sigma) \quad (4)$$

where E is Young's modulus and γ is the plastic multiplier. If the displacement field is allowed to have a discontinuity (e.g. a crack -opening) it can be written as

$$u(x,t) = \bar{u}(x,t) + [\![\bar{u}]\!](t) H_{\bar{x}}(x) \quad (5)$$

where $\bar{u}(x,t)$ is a smooth part and $H_{\bar{x}}(x)$ is the Heaviside function and $[\![\bar{u}]\!](t)$ is the crack opening. The strain field obtained from (1) and (5) can be written as:

$$\varepsilon(x,t) = \frac{\partial u(x,t)}{\partial x} + [\![\bar{u}]\!](t)\delta_{\bar{x}}(x) \quad (6)$$

where $\delta_{\bar{x}}(x)$ is the Dirac delta function. By introducing the result (6) into (4) and appealing to the smoothness of the stress field (consider for example a statically determined problem where stress field can be obtained from equilibrium equation only) one concludes that it must hold

$$\gamma(x,t) = \bar{\gamma}(x,t) + \bar{\bar{\gamma}}\delta_{\bar{x}}(x) \quad (7)$$

We can thus recover that

$$\dot{\sigma}(x,t) = E[\, \frac{\partial \dot{\bar{u}}(x,t)}{\partial x} - \bar{\gamma}(x,t) sign(\sigma)] \quad (8)$$

$$\bar{\bar{\gamma}}(t) = |[\![\dot{\bar{u}}]\!]| \quad (9)$$

The consistency condition leads from (3) to

$$\begin{aligned} 0 = \dot{\phi}(\sigma, q) = \; & sign(\sigma) E \tfrac{\partial \dot{\bar{u}}}{\partial x} - \bar{\gamma}(x,t) E[\, sign(\sigma)]^2 \\ & + \dot{q} + E \underbrace{\left([\![\bar{u}]\!] sign(\sigma)_{\bar{x}} - \bar{\bar{\gamma}}\right)}_{=0} \delta_{\bar{x}}(x) \end{aligned} \quad (10)$$

Assuming that the micro and macro-cracking would occur sequentially, we obtain for the first phase

$$\bar{\gamma}(x,t) = \frac{sign(\sigma) E \frac{\partial \dot{\bar{u}}}{\partial x}}{E[\, sign(\sigma)]^2 + K} \quad (11)$$

Subsequently, the non-local form of the consistency condition would lead to

$$\dot{q} = -sign(\sigma)\dot{\sigma} \quad (12)$$

which implies that the hardening stress-like variable must remain bounded. Finally, by taking into account that $\dot{q} = -K\dot{\xi}$ and $\dot{\xi} = \gamma$ we can conclude in view of (9) that

$$\bar{\bar{\gamma}}\delta_{\bar{x}}(x) = |[\![\bar{u}]\!]|\delta_{\bar{x}}(x) = -K^{-1}\dot{q} \quad (13)$$

which further implies that

$$K^{-1} = \bar{K}^{-1}\delta_{\bar{x}}(x) \quad (14)$$

Quite the same formalism as the one presented herein for 1D case can be used in a higher dimensional setting if one considers the Rankine yield criteria

$$\phi(\sigma_i, q) = \sigma_i - \sigma_y h(q) \quad (15)$$

where σ_i are the principal values of stress, and $h(q)$ is a particular form of the hardening/softening law.

Considering this model enables to describe quite efficiently mode I failure in concrete structures.

3 NUMERICAL IMPLEMENTATION AND SIMULATIONS

The non-classical plasticity model presented in the previous section is only a part of a successful development capable of handling the localization phenomena. Another ingredient, equally if not even more important is the chosen numerical implementation. In this work we build upon the simplest type of 2D finite element, a 3-node triangle previously used by Larsson et al. (1993), Oliver (1996) or Armero (1999).

The potential macro-crack is placed in the center of element and the displacement interpolation is constructed as shown in Figure 1 .

The real strain field is constructed as

$$\varepsilon(x,t) = \sum_{a=1}^{3} \nabla^s N_a(x) d_a + G\alpha \quad (16)$$

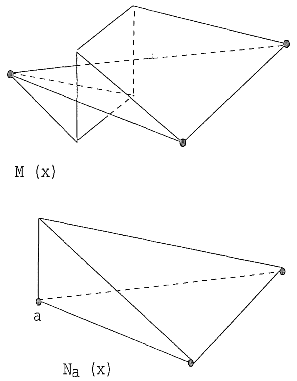

Figure 1: Interpolation functions of a 3-node triangle with discontinuity.

where G is produced from the chosen displacement interpolation by applying the same differential operator to smooth and discontinuous part. Contrary to last result, the interpolation of the virtual strain is constructed with

$$\delta\varepsilon(x,t) = \sum_{a=1}^{3} \nabla^s N_a(x) w_a + \hat{G}\beta \tag{17}$$

where $\hat{G}$ is a modified incompatible mode strain-displacement matrix with a zero-mean, which should guarantee the satisfaction of the patch test condition (e.g. see Ibrahimbegovic & Wilson (1991)). With such a choice of virtual strain interpolations, the discrete problem takes a standard format of the equations of incompatible mode method, where the set of global equilibrium equations is supplemented by element-based local equilibrium equations

$$\begin{aligned} &\mathop{\mathrm{A}}_{e=1}^{e=N_{elem}} \left(f^{e,int} - f^{e,ext} \right) = 0 \\ &f^{e,int} = \textstyle\int_{\Omega^e} \nabla^s N(x) \sigma d\Omega \end{aligned} \tag{18}$$

$$h^e = \int_{\Omega^e} \hat{G}\sigma d\Omega = 0, \; \forall e \in [1, N_{elem}] \tag{19}$$

where $\mathop{\mathrm{A}}_{e=1}^{e=N_{elem}}$ denotes the finite element assembly procedure. We note that the choice of the element and $\hat{G}$ implies the automatic satisfaction of (19).

3.1 *One-dimensional model of simple tension test*

In the first example, we take a simple one dimensional representation of the standard traction test, where the specimen is driven to the ultimate load and its post-peak response is explored under displacement control. In that sense, we need to combine the initial inelastic response under distributed micro-cracking and post-peak response governed by traction-separation of the macro-crack. The finite element models used to carry out this computation consider different number of truss-bar elements of ever decreasing size. The results obtained by the model developed herein remain insensitive to the finite element mesh grading, and all the computed force-displacement diagrams coincide. See figure 2.

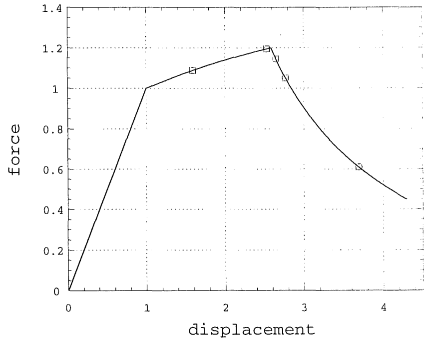

Figure 2: Force-displacement diagram in a simple tension test.

3.2 *Two-dimensional of simple tension test*

In this second analysis, we take a two-dimensional representation of the previous problem. 3-node triangles with discontinuity are used to mesh the geometry. The deformed mesh at the end of the analysis is depicted in Figure 3. For simplicity, the analysis is carried out considering a J2 plasticity model under plane strain conditions where 45 shear band is the dominant mode of failure (see Figure 3). Similar analysis is under way for Rankine model.

4 ACKNOWLEDGMENTS

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5 CONCLUSIONS

In this work we presented an inelastic constitutive model which is capable of combining the phenomena of micro-cracking, typically produced in initial damage phase, and macro-cracking, which is often dominant in the final post-peak phase. We have shown that both theoretical formulation and numerical implementation have to be addressed in order to guarantee the robustness of such a model. The model of this kind offers possibility to construct a more realistic physical basis of total energy dissipation, between one proportional to damaged volume as opposed to the one proportional to crack surface.

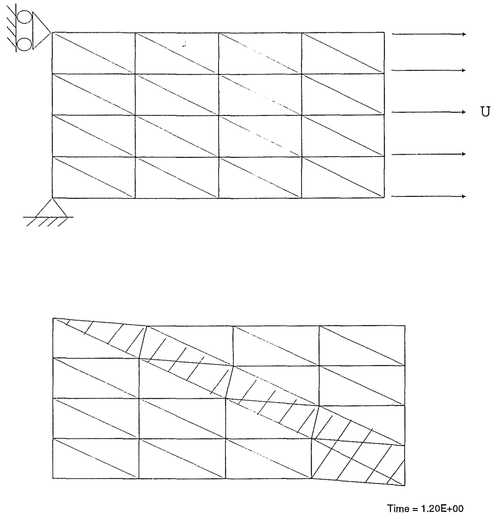

Figure 3: Tension test simulation : numerical results.

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