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Process zone resolution by extended finite elements

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A new adaptive technique suitable for problems with strain localization is proposed. The material behavior is described by a nonlocal damage model. The nonlocal formulation serves as an efficient localization limiter and leads to objective results. However, fine grids are necessary to resolve the bands of localized strains, and stress oscillations often appear in low-order elements due to a mismatch between the interpolation of local and nonlocal quantities. This calls for innovative adaptative procedures that go beyond the commonly used h-adaptivity. The extended finite element method (X-FEM) is used to adaptively enrich the standard displacement approximation by incorporating special shape functions. A numerical example illustrating the new approach demonstrates the performance and accuracy of the method. It is shown that satisfactory results can be obtained even on a very coarse mesh with only a few added degrees of freedom corresponding to an enrichment by a regularized Heaviside function.

1 INTRODUCTION

Quasibrittle materials, such as concrete, rock, tough ceramics, or ice, are characterized by the development of nonlinear fracture process zones, which can be macroscopically described as regions of highly localized strains. Continuum-based modeling of the progressive growth of microcracks and their coalescence requires constitutive laws with strain softening. In the context of standard continuum mechanics, softening leads to serious mathematical and numerical difficulties. The boundary value problem becomes ill-posed, and the numerical solution exhibits a pathological sensitivity to the computational grid. The use of regularization techniques enforcing a mesh-independent profile of localized strain is needed.

A wide class of localization limiters is based on the concept of a nonlocal continuum (Pijaudier-Cabot and Bažant 1987; Bažant and Lin 1988; Tvergaard and Needleman 1995; Ožbolt and Bažant 1996; Strömberg and Ristinmaa 1996). An accurate resolution of bands of highly localized strain typically requires very fine computational grids (Huerta and Pijaudier-Cabot 1994). The efficiency of the analysis can be greatly increased by using adaptive techniques that adjust the mesh during the simulation, depending on the intermediate localization pattern and its evolution. The most common approach is based on h-adaptivity, i.e., on the adjustement of the element size, keeping the order of the elements constant (and usually low). However, low-order elements produce stress oscillations due to the mismatch between the interpolation of local and nonlocal quantities (Jirásek and Patzák 2000). As shown by Jirásek (2001), p-adaptive methods based on an increase of the polynomial order of finite element shape functions lead to only a partial improvement. The undesired stress oscillations are usually reduced around the center of the process zone but they remain appreciable around the boundary of the process zone surrounded by material that experiences unloading. This is caused by the poor ability of polynomial approximations to capture the non-smooth transition between the unloading region with almost constant strain and the softening process zone with fast strain increase in the direction perpendicular to the boundary. Consequently, there is a strong need for an innovative adaptive procedure that goes beyond the commonly used h or p-adaptivity.

The present methodology can be considered as a nonstandard mesh-adaptive technique specifically tailored for regularized continuum models, especially those based on nonlocal integral formulations. It draws inspiration from the recently emerged *extended finite element method* (X-FEM), which has been successfully applied to modeling of arbitrary branched discrete cracks (Dolbow 1999; Belytschko and Black 1999; Moës et al. 1999; Daux et al. 2000). The present contribution explores the application of this method to smeared crack models and continuum damage mechanics. The extended finite element method is used to adaptively enrich standard displacement approximation by special shape functions that make it possible to accurately capture the localized strain profile with only a few additional degrees of freedom, even on very coarse meshes.

2 FORMULATION

The essential feature of X-FEM consist in its ability to incorporate a suitable enrichment into the finite element formulation. The X-FEM takes the advantage of the partition of unity property of finite elements (Melenk and Babuška 1996), which allows a global enrichment to be incorporated locally. The standard approximation is enriched in a region of interest by a suitable global function multiplied by a linear combination of the standard shape functions associated with the nodes in that region. The enriched displacement approximation for a single process zone assumes the general form

$$\boldsymbol{u}(\boldsymbol{x}) = \sum_{i \in I} N_i(\boldsymbol{x})\boldsymbol{u}_i + \sum_{i \in L} N_i(\boldsymbol{x}) H(\boldsymbol{x}) \boldsymbol{u}_i^x \quad (1)$$

where I is the set of all element nodes, L is the subset of element nodes that are enriched, $\boldsymbol{u}_i$ are the standard displacement degrees of freedom, N_i are the corresponding shape functions, H is an enrichment function, and $\boldsymbol{u}_i^x$ are the added degrees of freedom associated with the enrichment.

Classical fracture mechanics represents the crack by a discontinuity in the displacement field. Nonlocal damage models provide a more refined description of the fracture process zone. The displacement discontinuity is smeared over a finite width. The proposed technique describes the displacement field inside the fracture process zone as a linear combination of standard finite element approximations and a regularized Heaviside function in the direction normal to the crack. In this paper, the regularized Heaviside function, depending on the coordinate r normal to the crack direction, is assumed to have the form

$$H(r) = \begin{cases} 0 & \text{if } r < -R \\ \dfrac{15}{16}\left(\dfrac{r}{R} - \dfrac{2r^3}{3R^3} + \dfrac{r^5}{5R^5} + \dfrac{1}{2}\right) & \text{if } |r| \le R \\ 1 & \text{if } r > R \end{cases} \quad (2)$$

where R is the width of the process zone. The derivative of this function is the bell-shaped function, widely used for nonlocal averaging:

$$D(r) = \frac{\mathrm{d}H(r)}{\mathrm{d}r} = \begin{cases} \dfrac{15}{16R}\left(1 - \dfrac{r^2}{R^2}\right)^2 & \text{if } |r| \le R \\ 0 & \text{if } |r| \ge R \end{cases} \quad (3)$$

The displacement field is enriched only in the direction normal to the centerline of the process zone. For a single process zone, there is only one additional scalar coefficient u^x associated with the enrichment, representing the generalized displacement in the normal direction. The enriched displacement approximation for a single process zone in two dimensions has the form

$$u = \sum_{i \in I} N_i u_i + \sum_{i \in L} N_i H^u u_i^x \quad (4)$$

$$v = \sum_{i \in I} N_i u_i + \sum_{i \in L} N_i H^v u_i^x \quad (5)$$

where $H^u = H n_1$, $H^v = H n_2$, and $\boldsymbol{n} = \{n_1, n_2\}^T$ is the unit normal vector to the centerline of the process zone constructed at the closest point of the idealized crack (see Fig. 1).

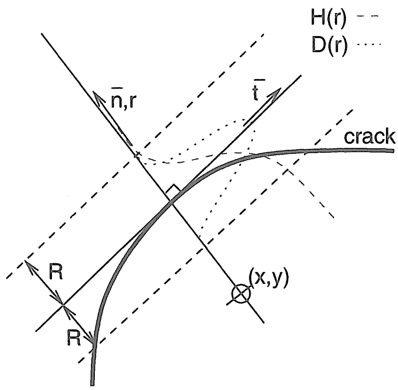

Figure 1: Centerline of the process zone (idealized crack) and regularized Heaviside function.

To assemble the strain-displacement matrix of the enriched element, the partial derivatives of H^u and H^v are needed. By application of the chain rule, one obtains

$$\begin{aligned} \frac{\partial H^u}{\partial x} &= \frac{\partial}{\partial x}(H(r(x,y))n_1) = \frac{\mathrm{d}H}{\mathrm{d}r}\frac{\partial r}{\partial x} n_1 = D\, n_1^2 \\ \frac{\partial H^v}{\partial y} &= \frac{\partial}{\partial y}(H(r(x,y))n_2) = \frac{\mathrm{d}H}{\mathrm{d}r}\frac{\partial r}{\partial y} n_2 = D\, n_2^2 \\ \frac{\partial H^u}{\partial y} &= \frac{\partial}{\partial y}(H(r(x,y))n_1) = D\, n_2 n_1 = \frac{\partial H^v}{\partial x} \end{aligned} \quad (6)$$

The introduction of the global enrichment function H requires to track the position of the centerline of the process zone (idealized crack), i.e., the set of points at which $r = 0$. The development of the methodology for tracking general curved cracks is the subject of ongoing research. At present, a fixed and straight crack trajectory is assumed, and the enrichment is allowed to grow only in the prescribed direction.

A truly adaptive technique dynamically introduces the additional degrees of freedom during the analysis. In the present approach, the analysis starts with all elements in a "standard mode", with standard shape functions and with no extensions introduced. After reaching the equilibrium state for a given load incre-

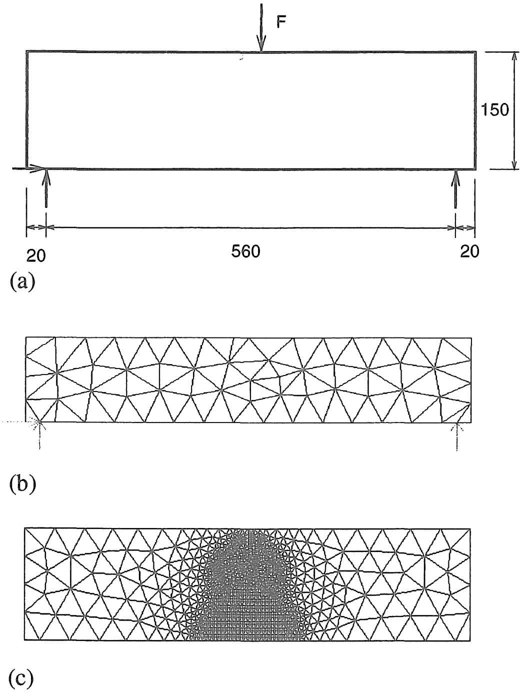

Figure 2: Three-point bending test: (a) specimen geometry, (b) coarse mesh, (c) fine mesh.

ment, the list of newly damaged elements is created. For each element in the list, the element nodes are checked and the new additional degrees of freedom representing the generalized displacement in the normal crack direction are introduced (if not previously present) and initialized to zero, while keeping the values of the previously existing degrees of freedom. After all elements in the list have been processed, the residual associated with the new degrees of freedom is evaluated and global equilibrium is restored by additional iterations, which provides the initial values of the new degrees of freedom.

For elements with one or more extended degrees of freedom, the evaluation of equivalent nodal forces and stiffness matrix at the element level requires high-order integration rules, An integration scheme with a high number of equally distributed integration points is used. The adaptive switching from one integration rule to another requires the mapping of internal variables. In the present study, the internal variables corresponding to new integration points are transferred from the closest integration point of the old integration scheme. The order of integration must be increased not only in the newly damaged elements but also in the neighboring elements connected to the nodes with extended degrees of freedom.

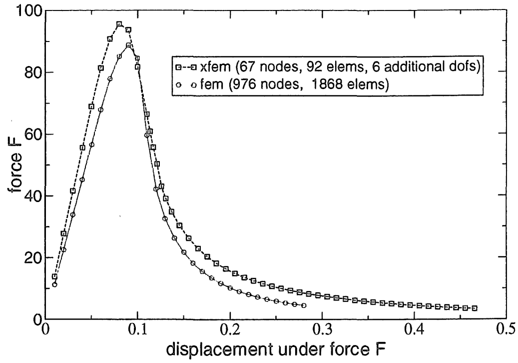

Figure 3: Three-point bending test: load-displacement diagrams produced by extended finite elements on a coarse mesh and by standard finite elements on a fine mesh.

3 EXAMPLE

To illustrate the proposed approach and its capabilities, a three-point bending test has been simulated using a nonlocal isotropic damage model with a Rankine-like equivalent strain measure (Jirásek and Zimmermann 1998), which takes into account only damage due to tension. The geometry of the specimen is shown in Fig. 2a. The material properties are as follows: Young's modulus $E = 20$ GPa, Poisson's ratio $\nu = 0.2$, equivalent strain at the onset of softening $\varepsilon_0 = 0.0001$, equivalent strain corresponding to complete damage $\varepsilon_f = 0.0008$, and radius of nonlocal interaction $R = 30$ mm. Linear softening law and a bell-shaped weight function have been used.

The propagating crack geometry has been assumed as a growing straight segment placed on the axis of symmetry of the specimen. The mesh used in this example is shown in Fig. 2b. It consist of 67 nodes and 92 constant-strain elements. During the adaptive analysis only 6 additional generalized displacement degrees of freedom have been introduced.

Fig. 3 compares the resulting load-displacement diagram with the diagram obtained using the standard finite element interpolation on a fine mesh, containing 976 nodes and 1868 constant-strain elements (Fig. 2c). A good agreement has been achieved, showing the capabilities of the proposed method. The main advantage is the ability to accurately capture the evolution of the fracture process zone even on very coarse meshes. This can be illustrated using the obtained profiles of local strain (Fig. 4) and damage (Fig. 5).

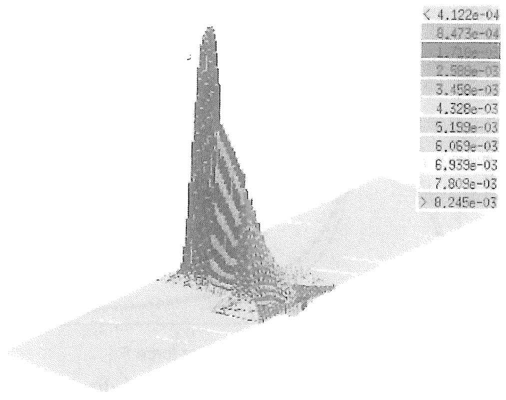

Figure 4: Three-point bending test: strain profile.

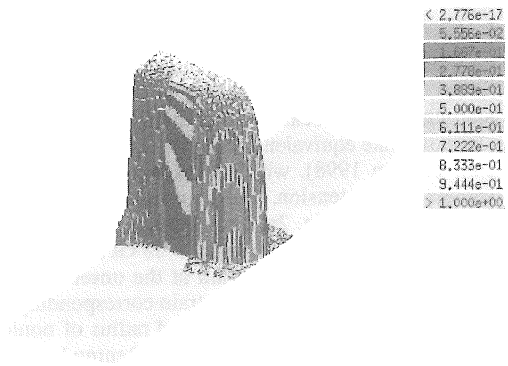

Figure 5: Three-point bending test: damage profile.

4 CONCLUSIONS

The extended finite element method has been successfully applied to the simulation of a propagating fracture process zone using a nonlocal material model. The displacement field has been adaptively extended by incorporating a special enrichment corresponding to a regularized Heaviside function. This allows to accurately capture the localized nature of the solution even on very coarse grids. Although tested only for the case when the crack trajectory is straight and known in advance, the methodology seems to be promising. The generalization to arbitrary curved crack trajectories and to three dimensions is the subject of ongoing research.

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