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A study on R/C tension members under repeated load

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ABSTRACT: A numerical model to study tensile R/C elements under repeated load is proposed. This model is able to define stresses and strains in concrete and steel, taking into account the fracture mechanics and bond-slip behaviour. In particular, we focus on a fixed length block, whose end points are, respectively, the cracked cross-section (where the crack width w is known), and the Stage II cross-section (no tension in concrete). Computed steel strains are compared with the experimental ones. In this way it is possible to point out the role of cracking phenomenon and bond-slip behaviour during the unloading phase. When the unloading is complete, the values of the compressive strains in the concrete near a crack seem to depend only on the cohesive law.

1 INTRODUCTION

The loads applied to concrete structures change their magnitudes during the life of a generic building. This is true, for example, of tanks or reinforced concrete frames subjected to earthquake. Nevertheless, both the type of loads and the distance from the bearing capacity are different in the previous cases. Actions due to earthquake usually yield dynamic stresses close to Ultimate Limit States of the structure, while stresses and strains, due to repeated static loads, arise into Serviceability conditions.

Cyclic actions damage R/C members, even when they are far from their ultimate loads (CEB 1997). They may produce significant increases in crack width and in steel stress around the crack, the latter due to micro-damage in the concrete that is closer to the reinforcement. In other words, the stiffness of concrete in tension decreases (what is called a decrease in tension stiffening), causing an increase both in the curvatures and in the structural deflections.

These phenomena depend on the intensity and the history of applied repeated actions, on the cross-sections of structures and on the mechanical properties of concrete and steel. By means of mechanical models of R/C structures under cyclic actions, it is possible to understand the role of these parameters and to prevent large deflections and wide crack widths. That is very important for any reinforced concrete structure in the Serviceability state. A heavy crack pattern allows water and aggressive external agents to corrode the reinforcement, prejudicing the integrity of the structure and its performance.

The analytical definition of the crack pattern, observed in the first experimental analysis in R/C bending beams, appeared intricate even for monotonic loads (Clark 1956). In particular, the so-called tension stiffening was very difficult to evaluate. Therefore in several theoretical models, both in monotonic and repeated loads, uniform strain distribution is assumed for the concrete around the reinforcement. Through this hypothesis, the tension zones (concrete and steel) in a reinforced concrete bending beam are reproduced by a simpler tension member (Broms 1965). In the reinforced concrete elements in tension, numerical and experimental investigations allow to evaluate the effects produced both by fracture mechanics of concrete and by bond slip behaviour. In general, with tie elements is possible to analyse the complex phenomena shown in R/C structures, subjected to repeated loads. Moreover, reinforced concrete elements in tension can be also used in many situations where cracking and bond between steel and concrete have to be taken into account. In particular, they prove useful to model the hysteretic behaviour of R/C beam-column joints under cyclic excitations (Filippou et al. 1983).

2 REINFORCED CONCRETE TENSION MEMBERS UNDER REPEATED LOAD

In reinforced concrete elements subjected to cyclic actions, both experimental and theoretical observations have focused on the tension stiffening phenomenon. As known, its reduction, produced by the repeated actions, is due to the cracking process of concrete in tension and to the deterioration of the bond developed between the reinforcement and the concrete. Often, the pull-out tests investigated on the τ-s behaviour separately from the cracking.

The effects produced by both mechanisms on a cylindrical R/C elements have been analysed in the experimental tests of Bresler & Bertero (1968). The Authors, by measuring the steel strains along the reinforcement, show the residual strains near the crack at the end of the unloading phase of each cycle. In the same work, a mechanical model, based on the bi-dimensional finite element analysis, is also proposed. Since the cohesive stresses on the crack surface are ignored, the approach leads again to the simpler pull-out model.

In the one-dimensional model proposed in Morita & Kaku (1975), where a bond τ-slip s law takes the place of the boundary link elements, a similar approximation is present. In particular, a block between two consecutive cracks, without any tension, is analysed. As shown by the comparison with the experimental analyses, the model is not able to reproduce the stress-elongation diagram of steel when repeated loads are applied. At the end of the unloading phase, the computed residual stresses in the reinforcement near the crack appear lower than the measured ones.

Günter & Mehlhorn (1989) proposed a more reliable model, where the average steel strain ε_{sm} in a R/C tension element due to normal force N is evaluated with the following formula:

$$\varepsilon_{sm} = \frac{N}{A_s \cdot E_s}\sqrt{1-\left(\frac{N_{cr}}{N}\right)^k} \qquad (1)$$

where E_s and A_s are respectively the modulus of elasticity and the area of steel, N_{cr} is the normal force when the first crack occurs and k is a factor that depends on the loads. Equation (1), and the similar expression of the Eurocode 2 (1994) model, must be used when the stabilized crack pattern appears. In these situations, the dashed lines in Figure 1 represent, respectively, equation (1) with $k = 2$ (short terms load) and $k = 5$ (long term and/or cycling load). The same picture also shows the unloading and reloading branches measured in the experimental investigation. After a certain number of cycles, if the maximum normal force N_1 is constant, both branches stabilise and the crack pattern does not grow up. The stress-strain behaviour of the specimen shown in Figure 1 clearly shows that the average strains in the reinforcement ε_{sm} can exceed those of the bar alone (Stage II, with linear elastic behaviour of the steel). Furthermore, when the unloading is complete, the specimen keeps a constant residual stress in the reinforcement. A good agreement between equation (1) and experimental measurement appears only when the steel stress is computed in correspondence of the maximum value N_1. In other words, the equation (1) model, unable to compute strains during the unloading and reloading phases, is turned on the evaluation of the effects produced by long term loads.

Shima & Tamai (1987) went into the details of the unloading phase, by means of a suitable definition of the stress field in the concrete. In particular, tension stiffening is due to bond slip behaviour, modelled with cosine function, and to the contact stresses in the cracks surfaces. During the unloading, in the concrete

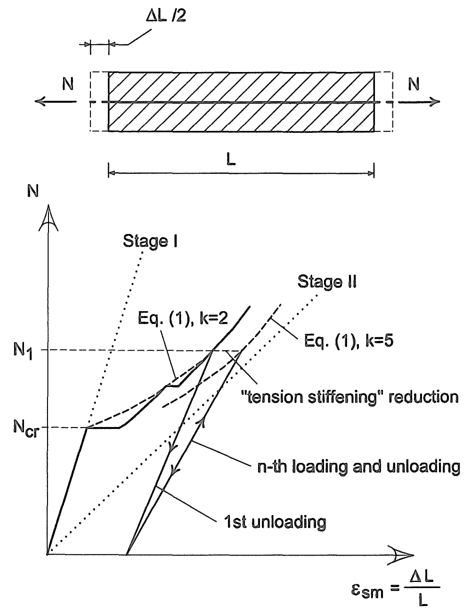


Figure 1. N-ε_{sm} diagram of R/C members under monotonic and repeated loads (Günter & Mehlhorn 1989).

near the cracks residual compressive stresses appear, while the average steel strain ε_{sm} increases and exceeds the strain of Stage II. Nevertheless, in this approach increase in steel strains, produced by repeated actions, falls on the contact mechanism of the crack surface, whereas the bond-slip is kept constant. On the contrary, Yannopoulus & Tassios (1991) shows that, under repeated loads, appropriate cyclic constitutive relationships must be used not only for the concrete, but also for local bond between steel and concrete. To be more precise, it is also necessary to adopt cyclic law both for concrete and τ-s when monotonic loads affect the R/C structures. As pointed out by Hognestad (1962), cracking phenomenon causes unloading in the concrete on tension surface, where compressive strains could be present. It is possible to explain the nature of these compressive strains, considering the growth of new cracks and the concomitant closing of the previous cracks (Fantilli &Vallini 2000).

3 PROPOSED MODEL

The stress-strain behaviour of a R/C tension element subjected to repeated load, seems to depend on the cracking mechanism of tensile concrete and bond slip behaviour. Therefore, a one-dimensional block model, able to define stresses and strains in concrete and steel, taking into account both phenomena, is proposed. In particular, we refer to the R/C element depicted in Figure 2a, which is similar to the tension member with a single crack investigated by Bresler &

Bertero (1968). An analogous model was introduced to define the crack pattern in reinforced concrete tendons subjected to monotonic loads (Fantilli &Vallini 2000). However, in our case, the problem is quite different: we focus on a fixed length block, whose end points are, respectively, the cracked cross-section (where the crack width w is known), and the Stage II cross-section (no tension in concrete).

3.1 Equilibrium and compatibility equations

It is possible to evaluate the response of the R/C tendon showed in Figure 2a by solving the equilibrium and compatibility equations, as in a classical structural problem. Exactly, in a generic cross section of the element, the following equilibrium equation must be satisfied (Fig. 2b):

$$\sigma_c \cdot A_c + \sigma_s \cdot A_s = N \tag{2}$$

where σ_c, σ_s and A_c, A_s are the stresses and areas of concrete and steel respectively and N is the applied normal force. Static conditions in the interface between the two materials are also needed. In particular, for the infinitesimal length dz of the reinforcement can be written (Fig. 2c):

$$\frac{d\sigma_s}{dz} = \frac{p_s}{A_s} \cdot \tau(s(z)) = f_1(s(z)) \tag{3}$$

where p_s is the perimeter of steel bar and $\tau(s(z))$ is the bond stress.

Compatibility conditions are applied both on the surface between steel and concrete and in every cross-section, where strain profile is assumed to be plane (Fig. 2d). For the same infinitesimal length dz of the element, it is possible to define the slip $s(z)$ as the difference of the displacements between two initially overlapping points of steel and concrete. The derivative of the $s(z)$ furnishes the following compatibility equation:

$$\frac{ds}{dz} = \varepsilon_s(z) - \varepsilon_c(z) = f_2(z). \tag{4}$$

3.2 Numerical solution

If the constitutive laws for concrete and steel and the bond-slip relationship are known, the problem can be solved through the numerical integration of the equations (2-4). That is possible by means of explicit methods, since the functions f_1 and f_2 are unknown. However, by each step, to improve the numerical results obtained with the explicit method (predictor), a new implicit integration (corrector) is needed. In the element depicted in Figure 2a, the loading process is controlled by the crack width w, so in the cracked section ($z = 0$) the slip ($s = w/2$) is known. This value decreases when moving from the crack to the end of the block, where negative slips are detected. In the same block, due to bond effects, the tensile stresses move from concrete to reinforcement (Fig. 2a). In particular, in the cross-section at the end of the element there are no stresses in concrete (Stage II cross-section). To obtain the stresses and the strains in concrete and steel in each cross-section of the block, as well as the slip s and the bond stress τ, for a given value o crack width w the corresponding normal force N must be calculated. That is possible by means of the trial and error procedure showed in Figure 3.

The average deformation of the steel bar can be evaluated with the following equation:

$$\varepsilon_{sm} = \frac{\Delta L}{L} = \frac{1}{L}\int_L \varepsilon_s \, dz \tag{5}$$

where L is the length of the reinforcement and ΔL its elongation (Fig. 1).

4 THE CONSTITUTIVE LAWS ADOPTED

To solve the problem, it is necessary to put together the equilibrium and compatibility equations (eq. 2-4), the constitutive laws of both materials and the bond-slip relationship τ-s.

Mechanical behaviour of steel and concrete is modelled with linear elastic stress-strain relationships, whose elastic modules are respectively E_s (Fig. 4a) and E_c (Fig. 4b). This assumption is correct for the reinforcement, because in the Serviceability

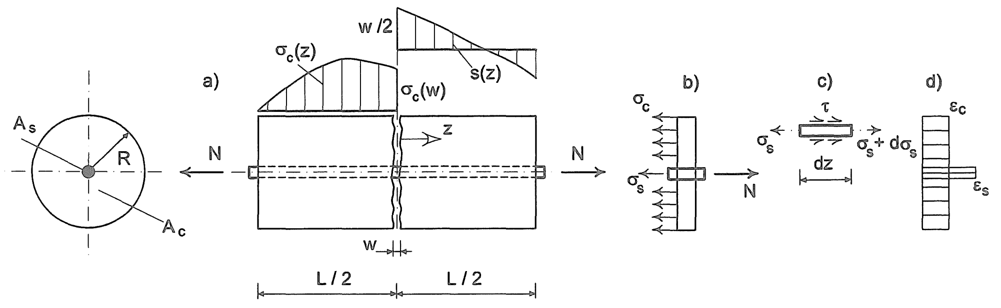


Figure 2. R/C tension member with a single crack: a) geometrical dimensions; b) equilibrium in a cross-section; c) equilibrium of an infinitesimal length dz of the reinforcement; d) compatibility condition in a cross-section.

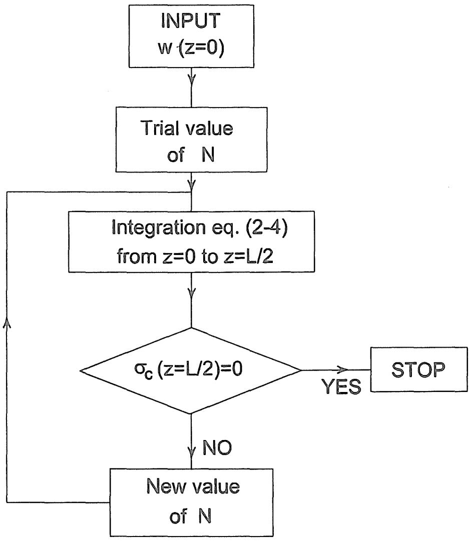


Figure 3. Flow-chart of the numerical procedure.

stage the stresses in the steel are far from the yielding strength f_y .On the contrary, when the concrete tension reaches the tensile strength f_{ct} , a crack develops in a section of the analysed block.

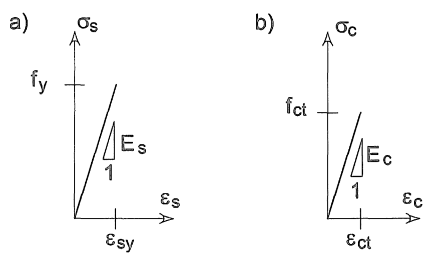


Figure 4. The adopted constitutive laws: a) σ - ε for the reinforcement; b) σ - ε for the concrete.

On the crack surfaces, according to the cohesive model depicted in Figure 5 (Hordijk 1991), the stresses in concrete must be considered, also in the case of repeated actions. For sake of simplicity, in the adopted σ-w relationship, both the unloading and the reloading curves are defined with the following equations:

$$\sigma = \sigma_p + \frac{f_{ct}}{3\frac{w_p}{w_c} + \beta}\left[0.014\left(log\frac{w}{w_p}\right)^5 - 0.57\sqrt[4]{\frac{w}{w_p}}\right] \quad (6)$$

where σ_p and w_p locate the point A on the envelope curve where the unloading branch starts and $\beta = 0.4$ is a constant value.

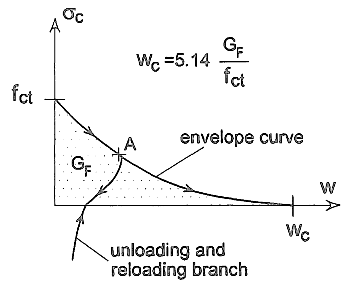


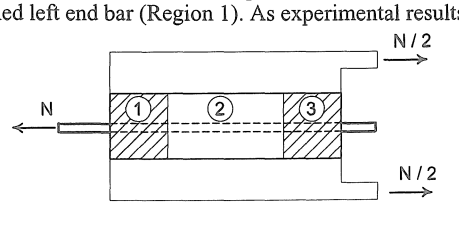
Figure 5. The continuous function model (Hordjik 1991).

4.1 Bond slip relationship

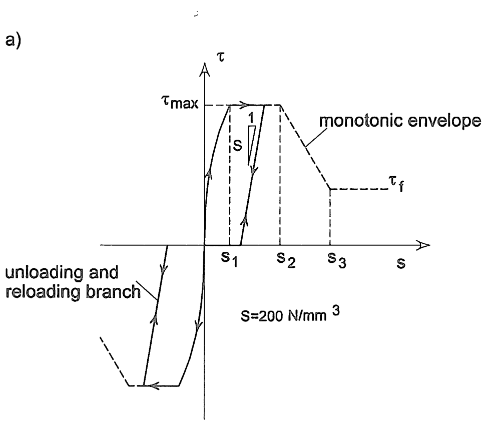
To model the bond-slip behaviour between steel and concrete, according to the experimental results of Ciampi et al. (1981), Ciampi et al. (1982) and Eligehausen et al. (1983), the bond-slip model represented in Figure 6a is used (CEB, 1991). For monotonic loads, the τ-s relationship is composed by four branches, whose parameters, valid for ribbed reinforcing bars, are also shown in Figure 6b. The experimental investigations were conducted to evaluate the bond stress relationship under cyclic excitations, therefore the function depicted in Figure 6a represents only the envelope curve. Reduced envelope curves are obtained from the monotonic envelope by decreasing τ_{max} and τ_f by means of a reduction factor (Ciampi et al. 1982), which is a function of the number of cycles and the maximum bond stress reached in each cycle.

In this work, the role of cracking phenomenon and bond-slip behaviour during the first and the second cycle are considered. According to Ciampi et al. (1981), when the specimen is subjected to load, unload and reload, in the τ-s law damage effects have to be excluded. Therefore only the monotonic envelope curve and the unloading - reloading branch can be considered (Fig. 6a). In particular, the unloading branch is linear, with a slope of 200 N/mm^3, and it is independent from the value of the slip reached on the envelope curve. In the unloading phase, when the bond stress vanishes, any frictional branches are excluded.

In the τ-s relationship, some deleterious effects on the bond stresses, due to the partial and the splitting cracking, are included. The decrease in bond stresses also depends both on degree of confinement and on the stress in the concrete surrounding the reinforcement. Referring to the static situation of Figure 7, the minimum bond resistance is present near the unconfined left end bar (Region 1). As experimental results



a)



b)

	2	3	4	5
	Unconfined concrete *		Confined concrete **	
	Bond conditions		Bond conditions	
	Good	All other cases	Good	All other cases
s_1	0.6 mm	0.6 mm	1.0 mm	
s_2	0.6 mm	0.6 mm	3.0 mm	
s_3	1.0 mm	2.5 mm	clear rib spacing	
α	0.4		0.4	
τ_{max}	$2.0\sqrt{f_c}$	$1.0\sqrt{f_c}$	$2.5\sqrt{f_c}$	$1.25\sqrt{f_c}$
τ_f	$0.15\ \tau_{max}$		$0.40\ \tau_{max}$	

* failure by splitting of the concrete

** failure by shearing of the concrete between the ribs

Figure 6. The bond slip relationship (CEB 1991): a) the model adopted; b) parameters of the τ-s for ribbed reinforcing bars.

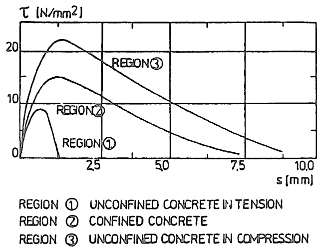


Figure 7. τ-s relationship in the cases of different confinement (Ciampi et al. 1982).

show (Ciampi et al. 1982), that is due to the formation of a concrete cone, which separates from the main block. On the contrary, around the unconfined right end bar (Region 3), the compressive stress in the concrete increases the degree of confinement compared to the middle sections (Region 2). From results given in Ciampi et al. (1982) and Eligehausen et al. (1983), the length of the Region 1 and Region 3 can be estimated about 5 Φ, where Φ is the bar diameter. As shown in Figure 7, within the Region 1 the bond stresses increase moving from the left end bar to the tip of the concrete cone, where the Region 2 begins.

Near the crack of the tensile R/C member (Fig. 2a), and in the Region 1 alike, a lower degree of confinement must be considered. In other words, within the length $l_a = 5\ \Phi$ from the next crack, the assumed τ-s relationship has to take into account a reduction in bond strength according to Eligehausen et al. (1997) (Fig. 8).

In the proposed model, where the crack width w was imposed, the applied actions cause a gradual growth in the length l_a around the crack. In particular, l_a, which is a linear function of w, is equal to 5 Φ

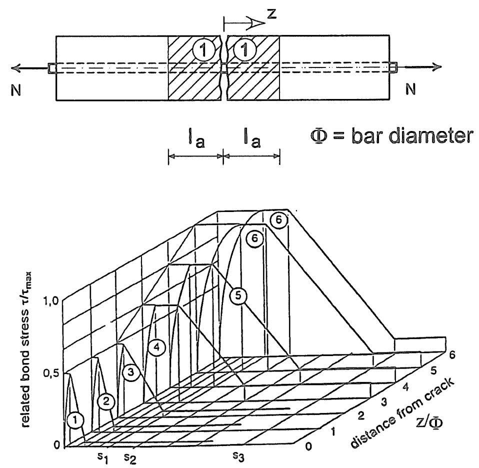


Figure 8. Bond reduction near the crack according to Eligehausen et al. (1997).

when the crack width reaches the w_c of the adopted cohesive model (Fig. 5). In this way, when the element is subjected to repeated loads, a reduction in l_a is possible because, due to a reduction in w, the compressive stresses on the crack surface increase the degree of confinement.

5 COMPARISON WITH THE EXPERIMENTAL RESULTS

The R/C tension member with a single crack shown in Figure 2a, was also investigated in Bresler & Bertero (1968). The specimen consists in a concrete cylinder reinforced axially with one steel bar, whose ends are loaded by means of normal force N. In Figure 9, the geometrical dimensions of the element, the mechanical properties of both materials and the magnitude of applied loads during the first and second cycle, are represented. The experimental investigation, for a given value of N, furnishes the strains in the reinforcement in many points of the bar. These measurements had been made possible by several strain-gauges placed within the reinforcing bar. Since the strains in the steel are lower than the yielding strain, the linear elastic law for the reinforcement can be used (Fig. 4a).

In Figure 10, the numerical results obtained with the proposed model both with (CASE 2) and without (CASE 1) the bond reductions are compared with the experimental ones, during the unloading and reloading phases. It is possible to notice that the CASE 1 is closer to the experimental measurements, despite the strains ε_s in the reinforcement near the crack, at the complete unloading, are not correctly evaluated. In particular, the computed steel strains are lower respect to the experimental ones, because the stresses on the crack surface are not correctly defined during the unloading. In other words, during this phase the unloading branch of the fictitious crack model plays an important role. If $\beta = 0$ is assumed in the equation 6 of the cohesive model (i.e. there is an increase in the slope of the unloading branch), there will be a good agreement between the computed and the measured strains in the steel (CASE 3 in Fig. 11).

6 CONCLUSIONS

A numerical model to study the cracking phenomenon in tensile R/C elements with a single crack, and subjected to repeated loads, is proposed. According to the experimental analyses, this model is able to define stresses and strains in concrete and steel, taking into account the fracture mechanics and bond-slip behaviour. In particular, it is possible to measure the residual steel strains at the crack due to the cyclic action, like in the experimental tests performed in Bresler & Bertero (1968). When the unloading stage is complete, the importance of the adopted cyclic relationships τ-s and σ-w is pointed out. In this phase, the structural response of the element is rather dependent on the unloading branch of the cohesive model, than on the reduction in bond stress near the crack.

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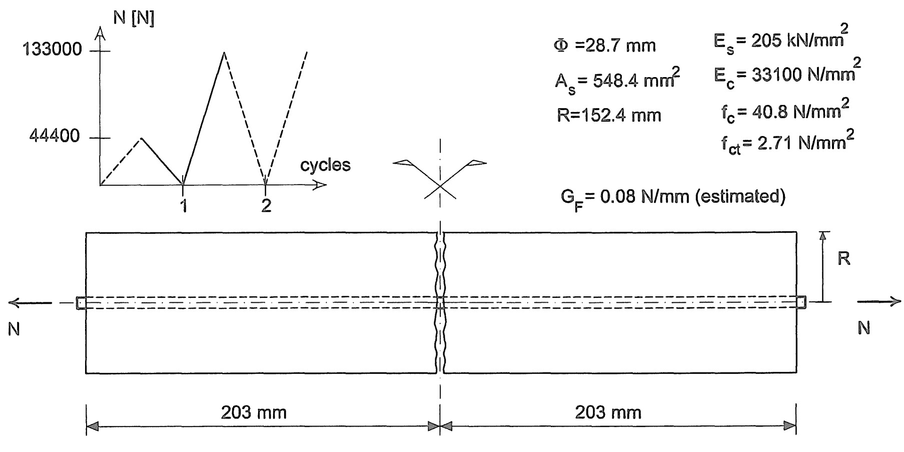

Figure 9. Reinforced concrete tension member tested in Bresler & Bertero (1968)

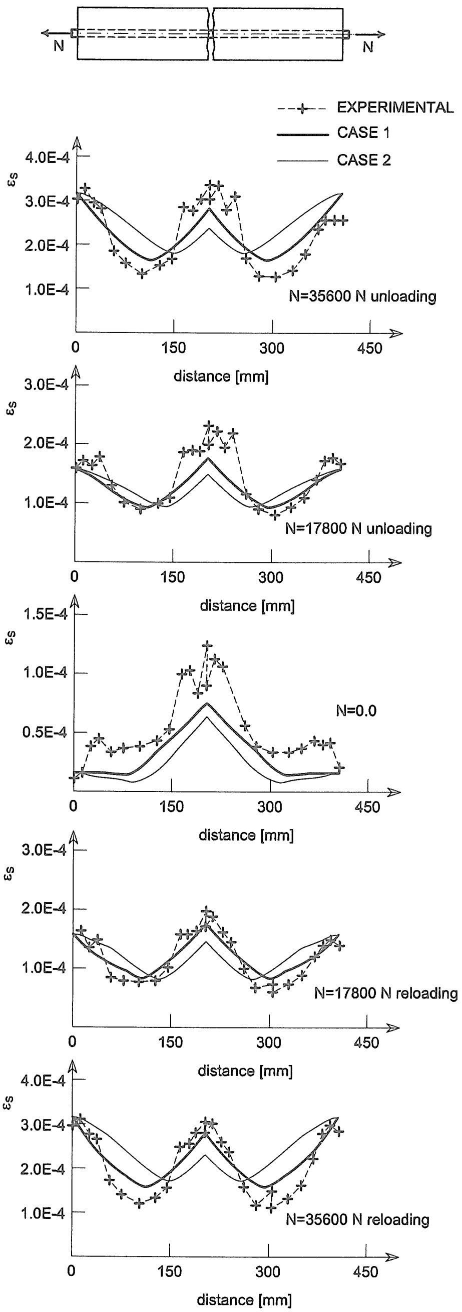

Figure 10. Comparison between experimental analysis of Bresler & Bertero (1968) and the computational results: CASE 1 without bond reduction and $\beta = 0.4$; CASE 2 with bond reduction and $\beta = 0.4$.

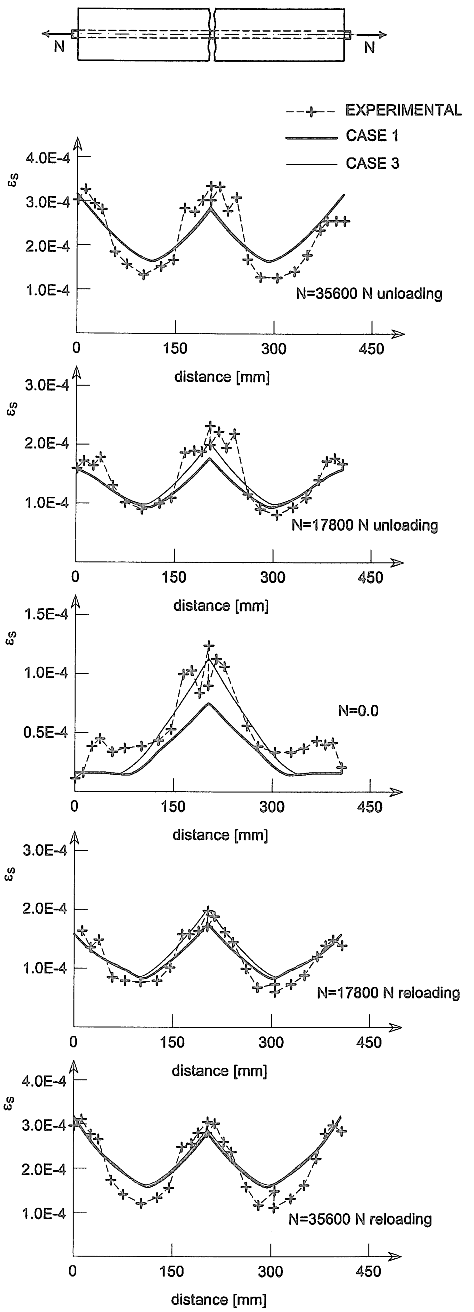

Figure 11. Comparison between experimental analysis of Bresler & Bertero (1968) and the computational results: CASE 1 without bond reduction and $\beta = 0.4$; CASE 3 without bond reduction and $\beta = 0$.

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