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A predictive model for the peeling failure of plated RC beams

O.J.Gastebled
TNO Building and Construction Research, Delft, the Netherlands

I.M.May
Heriot-Watt University, Dep. of Civil and Offshore Engineering, Edinburgh, Scotland, UK

ABSTRACT: The plate bonding technique is a cost-effective method of strengthening reinforced concrete beams. Possible modes of failure can easily be predicted, except for the peeling failure for which no predictive model is available. Peeling is characterised by the debonding of the plate caused by a horizontal splitting crack propagating at the level of the inner reinforcement. Assuming that the ultimate load is reached when the splitting crack starts to propagate, a predictive model is developed using fracture mechanics. The critical load is obtained considering the energy balance of the system during splitting crack propagation. Simplifying assumptions are made and the empirical formula for the assessment of the fracture energy proposed by the CEB-FIP Model Code 1990 is used to obtain analytical formulae. This approach results in satisfactory agreement between experimental data and the predicted failure loads. It is concluded that a reliable design method could be developed from the proposed formulae.

1 INTRODUCTION

The plate bonding technique consists of strengthening existing structures by gluing steel or FRP plates to their tension face, L'Hermite & Bresson (1967) and Rostasy et al. (1992). It is now acknowledged to be a structurally sound and cost-effective method. The attractiveness of the method is such that its use is likely to increase in the future. However, there is still no broad consensus on design recommendations. While the flexural types of failure modes of plated beams can easily be predicted using engineers bending theory, there is no broadly accepted predictive model for the brittle peeling failure observed experimentally, Arduini & Nanni (1997). The development of a reliable predictive model for the premature peeling failure is essential to achieve safe and ductile design.

Peeling is characterised by horizontal fracture propagating at the level of the inner reinforcement, which causes the plate and the concrete cover to separate from the rest of the beam. Oehlers (1992) describes the mechanism of peeling in these terms: "The first crack to form was the diagonal crack [adjacent to the plate end]. [...] A further increase in the load induced a horizontal peeling crack at the level of the bottom reinforcement [...]. Shear failure of the beam propagated the peeling crack very rapidly, which, in turn, caused the plate to debond [...]. Shear failure of the reinforced concrete beam and debonding due to shear peeling coincided and it is worth emphasising that both shear failures were very rapid." These observations point out that the peeling failure coincides with the unstable propagation of a horizontal fracture plane at the level of the bottom reinforcement. After an extensive review of experimental studies available in the literature, this mechanism has been identified as splitting cracking associated with diagonal shear cracks, Gastebled (1999). It should be noted that this phenomenon is similar to the cracking mechanism observed in the flexural-shear failures of unplated beams without stirrups, Gastebled & May (2001).

The predictive model presented in this paper is based on the assumption that the peeling failure of plated beams is triggered by the unstable propagation of a splitting crack. The study of unstable crack evolution requires an energy approach. Using linear fracture mechanics, the ultimate load is worked out considering the energy balance of the reinforced concrete system.

2 FRACTURE MECHANICS MODELS

2.1 *Pure bending mode*

When peeling occurs in pure bending, the plate separates from the beam along with the concrete cover, thus releasing the inner reinforcement from its concrete encasement. This results in a dramatic decrease in the local flexural stiffness of the beam. If the energy balance of the system is considered, peeling occurs at a load for which exactly enough energy is released by the softening system to feed the fracture energy required by splitting. The advance of peeling does not change the topology of the problem. Therefore, the peeling occurs at constant load in a critical state of stability.

Consider an uncracked plated beam in the vicinity of the plate end, the flexural stiffness of the beam is discontinuous at the plate end, changing from an unplated cross-section to a plated cross-section. Both stiffnesses, however, are relatively high as initially no cracking is present and the inner reinforcement acts compositely with the surrounding concrete.

The propagation of the splitting crack by a length δe results in the plate being released for a length δe in the direction of the plate, resulting locally in a significant decrease in the flexural stiffness, to which only the steel bar and the concrete compression area contribute. Therefore, a hinge is formed locally with rotation occurring at constant load, with the incremental angle of rotation, $\delta\theta$, proportional to the crack extent, δe.

Due to the formation of the splitting crack, the variation, of the strain energy, which is due to the reduction in the flexural stiffness is given by:

$$\delta U_{el} = U_{el}^{sp} - U_{el}^{ini} \tag{1}$$

$$= \frac{1}{2} M^2 \left(\frac{1}{(EI)_{sp}} - \frac{1}{(EI)_{ini}} \right) \cdot \delta e \tag{2}$$

where M is the external bending moment, $(EI)_{ini}$ and $(EI)_{sp}$ are respectively the flexural stiffness before and after splitting, and δe is the increase in the extent of the splitting crack.

The work done by the external bending moment due to the incremental rotation, which results from the local softening of the structure is:

$$\delta W_{ext} = M \cdot \delta\theta \tag{3}$$

$$= M^2 \left(\frac{1}{(EI)_{sp}} - \frac{1}{(EI)_{ini}} \right) \cdot \delta e \tag{4}$$

The consumption of the fracture energy is proportional to the splitting crack extent:

$$\delta G = \Gamma \cdot \delta e \tag{5}$$

where Γ corresponds to the fracture energy required to propagate the splitting crack over one length.

Using the fundamental equation of fracture mechanics for splitting propagation, then the energy balance is:

$$\delta W_{ext} - \delta U_{el} - \delta G = 0 \tag{6}$$

This yields the following expression for the critical load level:

$$M_{cr} = \sqrt{\frac{2\Gamma}{\frac{1}{(EI)_{sp}} - \frac{1}{(EI)_{ini}}}} \tag{7}$$

Using the formula given by the CEB-FIP Model Code 1990 for the assessment of the fracture energy, assuming an aggregate size of 20 mm:

$$G_f = 34.76 \left(\frac{f_c}{10} \right)^{0.7} \quad \text{N.m/m}^2 \tag{8}$$

with the concrete compressive strength f_c in MPa.

Equation 7 becomes:

$$M_{cr} = 2.633 f_c^{0.35} \sqrt{\frac{a_f}{\frac{1}{(EI)_{sp}} - \frac{1}{(EI)_{ini}}}} \tag{9}$$

where M_{cr} is in kN.m, f_c in MPa, EI in kN.mm^2, E_s in GPa and a_f in mm, and where a_f is the assessment of the fractured area per unit length extension of the splitting crack.

2.2 *Flexural-shear mode*

In the flexural-shear mode, after a diagonal crack has formed in the member at the plate end, splitting occurs, propagating parallel to the plate, thus releasing the inner reinforcement. The release of the reinforcement allows the diagonal crack to further open and extend resulting in a rotation, $\delta\theta$, about the tip of the diagonal crack. A splitting crack of extent, δe, leads to an incremental rotation, $\delta\theta$, proportional to δe. Peeling occurs at a load such that just enough energy is released by the system during the transformation to feed the splitting crack extension.

The mechanism producing the external work is the rotation under constant load about the tip of the diagonal crack. In order to calculate the energy release, the rotational stiffness of the beam needs to be determined. To that aim, the bulk of uncracked concrete and the embedded reinforcement are considered to behave as a rigid body except for the concrete, above the diagonal crack, subjected to compression. The rotational stiffness depends on the axial stiffness of the stirrups and on the axial and the dowel stiffnesses of the longitudinal reinforcement, which depend on the extent to which the reinforcing bar has been released by splitting. The stiffness is worked out considering the free body diagram, Figure 1.

The discrete distribution of stirrups is smeared over the length of the beam, thus considering an area of stirrups of A_v/s per unit length, where A_v is the

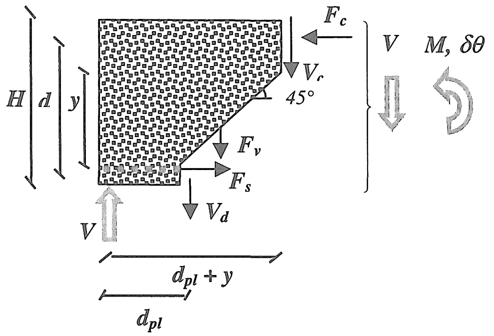

Figure 1. Free body diagram and notation.

area of one leg of a stirrup and s is the stirrup spacing. The opening of the diagonal crack is assumed to be linear, therefore the point of application of the stirrup forces, F_v, is at two-thirds of the crack length from the crack tip. In the following the angle of the diagonal crack is assumed to be 45 deg.

The smeared axial stiffness of the stirrups is determined assuming that the effective unbonded length of the stirrups is known, and is equal to δ_v:

$$k_v=\frac{A_vE_s}{s\delta_v} \qquad (10)$$

Consequently, the forces acting on the free-body diagram can be formulated as a function of θ, Figure 1. Assuming that the unbonded length of the longitudinal reinforcement is known to be equal to δ_s and that the angle of the diagonal crack, ϕ, is 45 deg.:

$$F_v=\int_0^y k_v 2t\theta\cdot dt=\frac{E_sA_v}{s\delta_v}y^2\theta \qquad (11)$$

$$F_s=\frac{E_sA_s}{\delta_s}\Delta u_s=\frac{E_sA_s}{\delta_s}y\theta \qquad (12)$$

$$V_d=\frac{G_s\Sigma_s}{\delta_s}\Delta v_s=\frac{9}{26}\frac{E_sA_s}{\delta_s}y\theta \qquad (13)$$

where y is the vertical distance between the diagonal crack tip and the bottom reinforcement, A_s is the area of the longitudinal reinforcement, E_s is the steel Young's modulus, G_s is the steel shear modulus, and Σ_s is the reduced cross-section of the longitudinal reinforcement, taken as $0.9A_s$.

The equilibrium of the free-body is satisfied in the horizontal direction, the vertical direction and about the tip of the diagonal crack, Figure 1:

$$F_s=F_c \qquad (14)$$

$$V_c+V_d+F_v=V \qquad (15)$$

$$V\cdot(d_{pl}+y)=V_d\cdot y+F_s\cdot d+F_v\cdot\frac{2}{3}y \qquad (16)$$

where y and d can be replaced using their ratios to the height of the beam, respectively βH and γH.

Substituting Equations 11 to 13 in Equation 16 results in the equation:

$$V\cdot(d_{pl}+y)=\left(\frac{9}{26}+\frac{\gamma}{\beta}+\frac{2}{3}\frac{A_v}{A_s}\frac{\delta_s}{\delta_v}\frac{\beta h}{s}\right)\frac{E_sA_s}{\delta_s}\beta^2H^2\theta \qquad (17)$$

If the expression for θ is deduced from Equation 17 and differentiated with respect to δ_s, considering that the variation of the unbonded length, δ_s, is equal to the extension of the splitting crack, δe, the following equation is obtained:

$$\delta\theta=\frac{V(d_{pl}+y)}{A_sE_s\left(\frac{9}{26}+\frac{\gamma}{\beta}\right)\beta^2H^2}\left(1+\frac{2}{3\left(\frac{9}{26}+\frac{\gamma}{\beta}\right)}\frac{A_v}{A_s}\frac{\delta_s}{\delta_v}\frac{\beta H}{s}\right)^{-2}\delta e \qquad (18)$$

The fundamental principal of fracture mechanics can now be applied to satisfy the energy balance of the transformation:

$$\delta W_{ext}=2\delta G \qquad (19)$$

$$(d_{pl}+y)V_{crit}\cdot\delta\vartheta=2\Gamma\cdot\delta e \qquad (20)$$

Substituting Equation 18 in Equation 20 and assuming that $\frac{\delta_s}{\delta_v}=\frac{\Phi_s}{\Phi_v}$ initially, where Φ_s and Φ_v are the diameter of the main reinforcement and the diameter of the stirrups respectively, it is possible to determine an expression for the critical shear force for a plated beam with stirrups:

$$V_{cr}=\sqrt{\frac{9}{13}+\frac{2\gamma}{\beta}}\left(1+\frac{2}{3\left(\frac{9}{26}+\frac{\gamma}{\beta}\right)}\frac{A_v}{A_s}\frac{\Phi_s}{\Phi_v}\frac{\beta H}{s}\right)\frac{\beta H}{d_{pl}+\beta H}\sqrt{A_sE_s\Gamma} \qquad (21)$$

It is possible to make a certain number of assumptions without losing much of the generality of the formula:

$$d=\gamma.H=0.9\,H$$
$$y=\beta.H=0.8\,H$$

Equation 21 then becomes:

$$V_{cr}=1.372\cdot\left(1+0.363\frac{A_v}{A_s}\frac{\Phi_s}{\Phi_v}\frac{H}{s}\right)\frac{H}{d_{pl}+0.8H}\sqrt{A_sE_s\Gamma} \qquad (22)$$

Using the formula given by the CEB-FIP Model Code for the assessment of the fracture energy, assuming an aggregate size of 20 mm, Equation 22 becomes:

$$V_{cr} = 3.613 \cdot \left(1 + 0.363 \frac{A_v}{A_s} \frac{\Phi_s}{\Phi_v} \frac{H}{s}\right) \frac{H}{d_{pl} + 0.8H} f_c^{0.35} \sqrt{A_s E_s a_f} \tag{23}$$

where V_{cr} is in kN, f_c in MPa, A_s in m^2, E_s in GPa and a_f in mm, and where a_f is the assessment of the fractured area per unit length extension of the splitting crack.

3 EMPIRICAL DETERMINATION OF FRACTURED AREA

For the prediction of the failure load, Equations 9 and 23, to be complete, the fractured area caused by the extension of the splitting crack by a unit length, a_f, needs to be assessed. Two types of cracking are associated with splitting, i.e. the progression of the horizontal splitting crack itself and the shifting of the set of conical cracks ensuring shear transfer near an unbonded section of a ribbed bar. The total fractured area can then be divided into a known part corresponding to the horizontal splitting crack and an empirically assessed part corresponding to conical cracking. This results in the following expression:

$$a_f = b + \alpha \frac{A_{cc}}{r_s} \tag{24}$$

Where b is the beam width, α is a correction factor, r_s is the bar rib spacing and A_{cc} is an assessed conical crack area. Assuming conical cracks radiating at an angle of 40 degrees up to a radius of 15 mm outside the reinforcing bar, A_{cc}= 54 n (Φ_s + 15) mm, where n is the number of bars.

The difficulty resides in the assessment of the factor α. It is believed that α, as a measure of the level of damaged caused by conical cracking that occurs around the reinforcing bars due to bond stresses, is influenced by several parameters. By considering the mechanics of the conical crack formation, it is believed that the two major parameters influencing α are the concrete cover and the tensile strain of the steel bar. The concrete cover controls the distance to which the conical crack can radiate from the bar, and the tensile strain in the steel bars controls the crack mouth opening of the conical cracks. The assumption that α equals zero is a safe assumption for design purposes. However, the adoption of this assumption results in predictions that severely underestimate the failure loads measured experimentally, Gastebled (1999). It is therefore desirable to develop an empirical evolution law for the factor α. In the following, α is assumed to depend only on the steel strain, ε_s, and a multi-linear evolution law is deduced from experimental data.

3.1 *Bending mode*

When the plated beam is reported to fail in the bending mode, i.e. after the occurrence of a flexural crack near the plate end, ε_s is calculated using cross-sectional analysis at the plate end:

$$\varepsilon_s = \frac{V_{exp} d_{pl}}{(EI)_{cr}} (H - c - y_{cr}) \tag{25}$$

where V_{exp} is the experimental ultimate shear load, $(EI)_{cr}$ is the flexural stiffness of the cracked unplated beam, i.e. equal to $(EI)_{sp}$, c is the concrete cover and y_{cr} is the depth of the neutral axis of a cracked unplated beam.

The corresponding α_{exp} is assessed from the experimental ultimate load, V_{exp}, and the theoretical failure load, M_0, obtained using Equation 9 with $\alpha = 0$:

$$\alpha_{exp} = \frac{b r_s}{A_{cc}} \left(\frac{V_{exp}^2 d_{pl}^2}{M_0^2} - 1 \right) \tag{26}$$

3.2 *Flexural-shear mode*

When the plated beam is reported to fail in the flexural-shear mode, the force in the steel at the location of the diagonal crack is assessed using the free body diagram, Figure 1. This results in the following expression for ε_s:

$$\varepsilon_s = \frac{V_{exp}(d_{pl} + \beta H)}{\gamma H \cdot E_s A_s} \tag{27}$$

The corresponding α_{exp} is assessed from the experimental ultimate load, V_{exp}, and the theoretical failure load, V_0, obtained using Equation 23 with $\alpha = 0$:

$$\alpha_{exp} = \frac{b r_s}{A_{cc}} \left(\frac{V_{exp}^2}{V_0^2} - 1 \right) \tag{28}$$

3.3 *Empirical evolution law*

Experimental results for plated beams reported to have failed in peeling, either in the bending mode or the flexural shear mode, have been collected in the literature. In total, 32 experimental results have been extracted from Oehlers (1992), Ziraba et al (1994) and Swamy et al (1995).

Equations 25 and 26 or 27 and 28 are used to plot the experimental α against the experimental steel strain at failure, Figure 2. The experimental points appear to lie on a straight line crossing the axis of zero α at a steel strain of 5x10^{-4}.

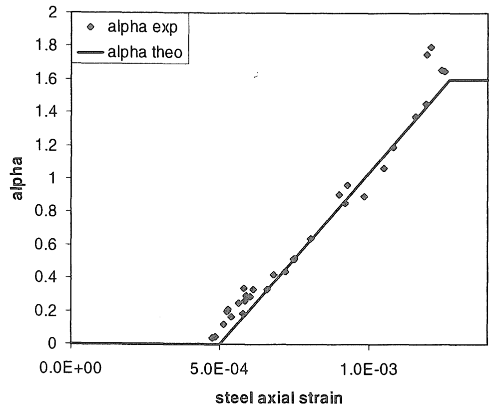

Figure 2. Empirical evolution law of α.

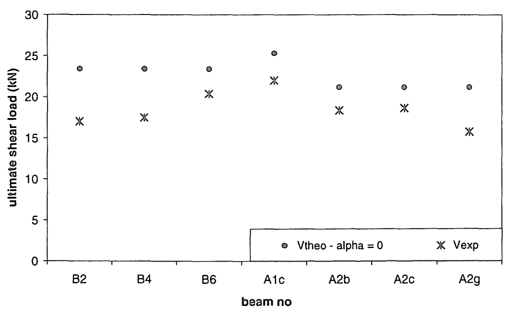

Figure 3. Ultimate shear loads from Quantrill et al (1996).

Below $\varepsilon_s = 5\text{x}10^{-4}$, the tensile strain appears to be too small to cause conical cracking, i.e. $\alpha = 0$. This results can be checked assuming a concrete Young's modulus of 25 GPa and an cracked-unbonded to uncracked-bonded steel stress ratio of 6, which yields a concrete tensile stress of 2 MPa at $\varepsilon_s = 5\text{x}10^{-4}$. This is consistent with usual concrete tensile strengths.

Above $\varepsilon_s = 13\text{x}10^{-4}$, no experimental points are available. Thus the predicted value of α has been limited to 1.6 in order to avoid uncontrolled divergence of the iterative procedure described further, Gastebled (1999).

Using linear regression analysis, the slope of the best-fit line between $\varepsilon_s = 5\text{x}10^{-4}$ and $\varepsilon_s = 13\text{x}10^{-4}$ is calculated. This results in the following definition of the multi-linear evolution law of α:

$\alpha = 0$ for $\varepsilon_s < 5\text{x}10^{-4}$ (29)

$\alpha = 1.035 + 2.07\ \text{x}10^{-3}\ \varepsilon_s$ for $5\text{x}10^{-4} < \varepsilon_s < 13\text{x}10^{-4}$ (30)

$\alpha = 1.6$ for $\varepsilon_s > 13\text{x}10^{-4}$ (31)

The empirical evolution law for α can now be used to predict peeling failure of plated beams. Because the steel strain at failure is not known a priori, the following iterative procedure is required:

1- set α to zero and calculate a_f according to Equation 24.
2- Calculate the failure loads in the bending mode and in the flexural shear mode according to Equations 9 and 23.
3- Use Equation 25 and 27 to calculate the corresponding steel strains at failure.
4- Use equations 29 to 31 and Equation 24 to update α and a_f.
5- Go back to step 2 until convergence of the predicted failure loads.
6- Compare the obtained failure loads for bending and flexural shear. Chose the smaller of the two as predicted ultimate load.

4 BENCHMARKING

Benchmarking of the predictive formulae is described below. Experimental tests carried out by Quandrill et al (1996) on FRP plated beams with plain reinforcing bars, which all failed in the flexural-shear mode, allow checking the analytical formula, Equation 23. Experimental tests carried out by Swamy et al (1987) on steel plated beams with ribbed reinforcing bars allow checking the semi-empirical approach presented in section 3.3.

4.1 *Peeling with plain reinforcing bars*

Quandrill et al (1996), using plated beams reinforced with plain bars, observed peeling failure in seven of the tested beams. Since plain bars achieve shear transfer without producing conical cracks, the corresponding factor α can then be taken equal to zero. It is therefore possible to directly use Equation 23 in order to predict the failure load of Quandrill's beams. The comparison of the experimental results and the predicted failure loads is shown in Figure 3.

Statistical analysis of the results yields a relative mean value of $\mu_{exp/theo}$ = 124 % and a relative standard deviation of $\sigma_{exp/theo}$ = 28 %. It can be noted that the analytical formula consistently over-predicts the failure load by an average 24%, which accounts for most of the observed standard deviation. Such a discrepancy is standard in the prediction of the failure of reinforced concrete structures and can be put down to the uncertainty, which belong to the measured or assessed material properties.

4.2 *Peeling with ribbed reinforcing bars*

Swamy et al (1987) tested steel plated beams reinforced with ribbed bars. Thirteen of the beams failed in

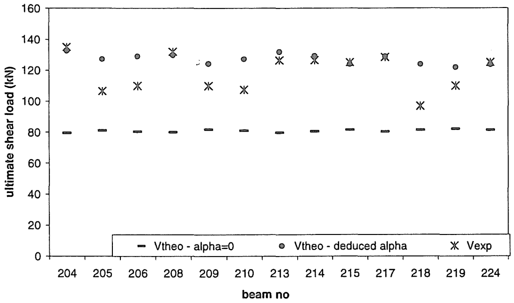

Figure 4. Ultimate shear loads from Swamy et al (1987).

peeling. The iterative procedure proposed in section 3.3 is used to predict the failure loads of these beams. The comparison of the experimental results and the predicted failure loads is shown in Figure 4.

Statistical analysis of the results yields a relative mean value of $\mu_{exp/theo}$ = 108 % and a relative standard deviation of $\sigma_{exp/theo}$ = 13 %. Noting that nine of the thirteen beams are practically identical, the inherent experimental scatter is assessed using the Khi squared distribution. This results in an inherent experimental standard deviation comprised between 4 % and 24 % with a level of confidence of 90 %. Since $\sigma_{exp/theo}$ lies in the center of this range, it is confirmed that the observed scatter is essentially due to the uncertainty inherent to the flexural-shear peeling failure and not the predictive method itself.

5 CONCLUSIONS

Peeling failure is recognized as the most critical failure mode of externally plated beams. However, there is currently no broadly accepted predictive model available. In this paper, two predictive formulae based on fracture mechanics and an empirical factor are proposed and benchmarked.

Few simplifying assumptions are used to derive the analytical formulae and only two parameters are derived from empirical formulae, namely the specific fracture energy of concrete and a correction factor α for the assessment of the fracture area associated with conical cracks.

The benchmarking of the formulae against experimental data resulted in a satisfactory scatter of the experimental results around the predictions. This confirms the validity of the physical model and the choice of the simplifying assumptions. It is concluded that these formulae ca be reliably used to predict the peeling failure plated beams, both with steel plates and FRP plates. It is believed that a design method could easily developed from this approach.

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