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The Equivalent Reinforced Concrete model for simulating the behavior of shear walls under dynamic loading

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ABSTRACT: A new simplified modeling strategy for simulating the non-linear behavior of reinforced concrete structures submitted to severe dynamic shear is presented. The Equivalent Reinforced Concrete model (ERC) uses lattice meshes for concrete and reinforcement bars and uniaxial constitutive laws based on the principles of continuum damage mechanics and plasticity. Verification is provided through comparisons with the results of the NUPEC experimental program. ERC is a simplified method that intends to save computer time and to allow for parametrical studies. The proposed lattice model is promising and could be extended to 3D calculations or to simulate the behavior of plastic zones.

1 INTRODUCTION

Simulating the non-linear behavior of reinforced concrete structures submitted to severe dynamic shear is an important problem for the engineering community. Recent earthquakes in Kobe (Japan), Izmit (Turkey) and Athens (Greece) proved once more the need to study thoroughly the shear mechanism and to develop simplified tools for the everyday practice. Reinforced concrete squat columns and shear walls suffering from severe dynamic shear collapsed suddenly and lead to catastrophic failures.

Our research team in "Laboratoire de Mécanique et de Technologie" (LMT) has already proposed a simplified modeling strategy for bearing walls dominated by flexure. It consists on using multilayered 2D Bernoulli beam elements coupled with damage mechanics (Mazars 1998, Ragueneau 1999). The problem is however more complex when the effects of shear deformation are prevailing, as for example the case of shear walls (bearing walls that have small slenderness, usually less than 1.5). On this specific topic, the experimental program realized by the Japanese firm NUPEC - Nuclear Power Electric Corporation - gave valuable information on the seismic behavior of such structures (OECD 1996).

The purpose of this paper is to propose an original, simplified modeling strategy for reproducing the non-linear behavior of shear walls. We begin with the mathematical tools and concepts used in a simplified analysis, essential tool for parametric computations. The presentation of the new model, called Equivalent Reinforced Concrete model follows. The model, inspired by the truss models developed in the early 1900s (Ritter 1899, Mörsch 1909) is based on the Framework Method (Hrennikoff 1941) and uses a lattice type mesh coupled with modern constitutive laws. Verification is provided through comparison with the results of the NUPEC experimental program.

2 NUMERICAL TOOLS AND CONCEPTS FOR A SIMPLIFIED ANALYSIS

2.1 *Finite Element Code*

In order to perform nonlinear dynamic calculations the code EFICOS is developed at LMT. The code uses 2D Bernoulli multi-layered beam elements and uniaxial constitutive laws for the materials or 2D Timoshenko multi-layered beam elements with specific kinematic conditions (Dubé 1997, Ghavamian & Mazars 1998). The initial secant stiffness matrix algorithm is implemented where the nonlinear behavior appears in the second member of the equilibrium equation. For stability and precision reasons, a classical Newmark algorithm is used to solve the equation of motion.

The basic steps in order to model a typical structure are shown in Figure 1. The structure is discretised with 2D multi-layered beam elements and concentrated masses at specific points. A constitutive law is attributed at each layer and seismic loading is applied as an input motion at the base of the structure.

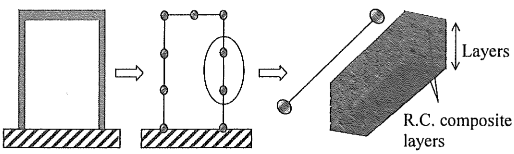

Figure 1. Spatial discretisation using EFICOS.

2.2 *Constitutive models*

Seismic loading, which includes cyclic aspects, produces micro-cracking in concrete. The major phenomena – decrease in material stiffness as the micro-cracks open, stiffness recovery as the cracks close (unilateral behavior of concrete) and inelastic strains concomitant to damage – have to be taken into account. The constitutive law used for concrete is based on the principles of damage mechanics (La Borderie 1991). The law, called "Unilateral damage law", is elaborated for the description of micro-cracks, involves two damage scalar variables one in tension and one in compression and the description of isotropic inelastic strains. The introduction of two damage scalars allows separating the mechanical effect of micro-cracking depending on the sign of the stress. The model is able to simulate the unilateral behavior of concrete via a recovery stiffness procedure at re-closure. The total strain in the 3D formulation of the law is given by:

$$\varepsilon=\varepsilon^{e}+\varepsilon^{in} \tag{1}$$

$$\varepsilon^{e}=\frac{\langle\sigma\rangle_{+}}{E(1-D_1)}+\frac{\langle\sigma\rangle_{-}}{E(1-D_2)}+\frac{\nu}{E}(\sigma-Tr(\sigma)\mathbf{1}) \tag{2}$$

$$\varepsilon^{in}=\frac{\beta_1 D_1}{E(1-D_1)}\frac{\partial f(\sigma)}{\partial\sigma}+\frac{\beta_2 D_2}{E(1-D_2)}\mathbf{1} \tag{3}$$

where ε^{e} = elastic strain tensor; ε^{in} = inelastic strain tensor; $\mathbf{1}$ = unit tensor; and $Tr(\sigma)=\sigma_{ii}$. Damage criteria are expressed as:

$$f_i=Y_i-Z_i \tag{4}$$

Y_i associates forces to damage; and Z_i threshold depending on the hardening variables.

The evolution law for damage takes the form:

$$D_i=1-\frac{1}{1+\left[A_i(Y_i-Y_{0i})\right]^{B_i}} \tag{5}$$

$$\begin{cases} Tr(\sigma)\in[0,+\infty] \rightarrow \dfrac{\partial f(\sigma)}{\partial\sigma}=1 \\ Tr(\sigma)\in[-\sigma_f,0] \rightarrow \dfrac{\partial f(\sigma)}{\partial\sigma}=\left(1-\dfrac{Tr(\sigma)}{\sigma_f}\right)\mathbf{1} \\ Tr(\sigma)\in[-\infty,-\sigma_f] \rightarrow \dfrac{\partial f(\sigma)}{\partial\sigma}=0.\mathbf{1} \end{cases} \tag{6}$$

$f(\sigma)$= crack closure function; σ_f= crack closure stress respectively; <>+ denotes the positive part of a tensor; E= initial Young's modulus; ν= Poisson ratio; D_1 and D_2 = damage variables for traction and compression respectively. Y_{0i}= initial elastic threshold ($Y_{0i}=Z_i(D_i=0)$) ; and A_i,B_i,β_1, β_2 = material constants.

A plasticity model with cinematic hardening is used for steel. Hardening can be linear or not depending on the information provided from the steel tensile strength tests. The stress-strain relation is given Figure 3.

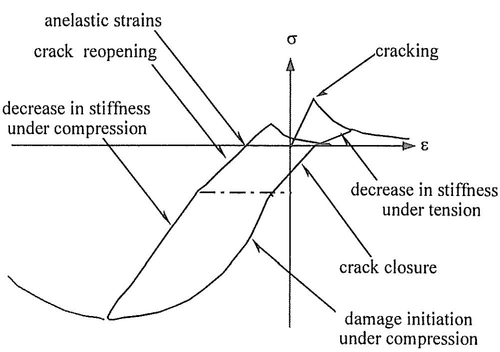

Figure 2. 1D "Unilateral damage law".

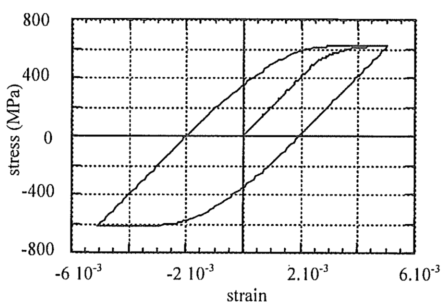

Figure 3. 1D steel constitutive law.

Reinforcement in the beams is introduced with special layers, the behavior of which is a combination of those of concrete and of steel (Mazars 1998). Damping is introduced in the analysis through viscous forces generated by a global damping matrix taken as a linear combination of the global stiffness matrix and the mass matrix (Rayleigh damping). This damping matrix stays constant during the calculation.

3 EQUIVALENT REINFORCED CONCRETE MODEL

3.1 *Background*

Equivalent reinforced concrete model (ERC) uses lattice meshes for predicting the non-linear behavior of shear walls and is inspired on the Framework Method (Hrennikoff 1941). The basic idea of the Framework Method consists on replacing the continuous material of the elastic body under investigation by a framework of bars, arranged according to a definite pattern, whose elements are endowed with elastic properties suitable to the type of problem. The criterion of suitability of the framework pattern is equality in deformability of the framework and the solid material in elasticity. If the size of the pattern unit of such a framework is made infinitesimal the latter will present a complete mechanical model of the solid prototype, with identical displacements, strains and unit stresses. Some of the patterns proposed by Hrennikoff for plane stress elastic problems and homogenous material are shown in Figure 4. (The first pattern implies a Poisson's coefficient equal to ν=1/3).

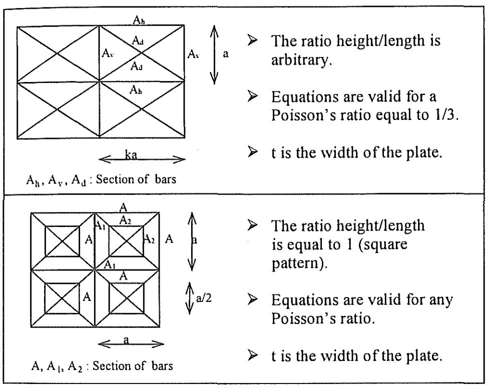

Figure 4. Some patterns of the Framework method for plane stress problems.

$$A_v=\frac{3}{8}\frac{3k^2-1}{k}at \qquad A=\frac{at}{1+\nu}$$

$$A_h=\frac{3}{8}(3-k^2)at \qquad A_1=\frac{at}{(1+\nu)\sqrt{2}} \qquad (7)$$

$$A_d=\frac{3}{16}\frac{(1+k^2)^{3/2}}{k}at \qquad A_2=\frac{3\nu-1}{2(1+\nu)(1-2\nu)}at$$

Pattern 1 Pattern 2

3.2 *Proposed lattice model*

The idea is to use the patterns proposed by Hrennikoff in a non-linear context and for a non-homogenous material. The new model is called Equivalent Reinforced Concrete model (ERC) and it is described hereafter:

- An elementary volume of reinforced concrete (EV) can be separated into a concrete element (C) and a horizontal and a vertical reinforcement bar (S_H and S_V respectively). Concrete and steel are then modeled separately using two different lattices (Fig. 5).

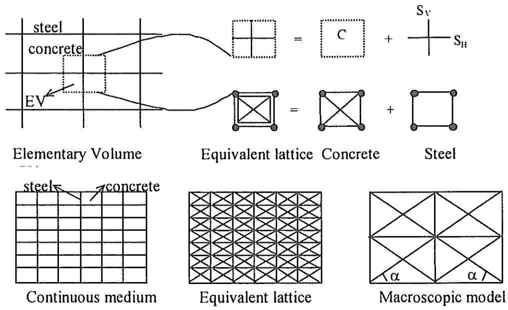

Figure 5. Principles of the ERC model.

- The sections of the bars simulating concrete are derived directly from the Framework Method. The first motif of the Framework method is used because of its simplicity and the small number of elements required. This pattern is accurate for a Poisson's ratio equal to 1/3, obviously not the case for reinforced concrete. This choice is however justified by the fact that the problem is highly non linear (collapse of the specimens) and Poisson's ratio is significantly changing.
- A crucial parameter for the success of a non-linear simulation (especially for the case of lightly reinforced shear walls) is the angle α that the diagonals of the concrete lattice form with the horizontal bars (Kotronis 2000). It depends on the reinforcement ratios, the loading and the boundary conditions. It has to be between 30° and 60° in order to avoid negative values for the sections of the bars calculated according to the first motif of the Framework Method. For the non linear calculations presented afterwards, the value of the angle has been calibrated in order to reproduce correctly the monotonic experimental curve (end of the linear regime, ultimate strength). This value is used for the prediction of the non linear dynamic behavior.
- The "Unilateral damage model" in its 1D formulation is used for simulating the non-linear behavior of concrete. Tests on reinforced concrete elements demonstrated however that even after extensive cracking, tensile stresses still existed in the cracked concrete and that they significantly increased the ability of the cracked concrete to resist shear stresses (ASCE-ACI 1998). Adjusting the post pick behavior of the "Unilateral damage

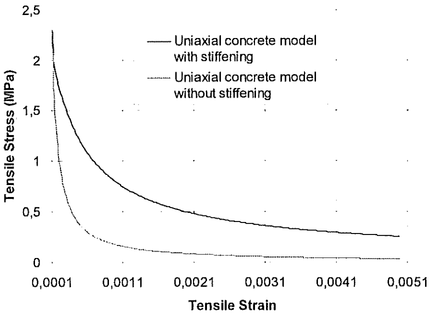

Figure 6. Taking into account the "stiffening" phenomenon.

model" we can simulate this phenomenon known as the " tension stiffening phenomenon".

- A lattice composed by horizontal and vertical bars coupled with the uniaxial plasticity model presented in Figure 3 simulates steel. The section and position of the bars coincide with the actual section and position of the reinforcement. In order to simplify the mesh and to gain computer time we can also use the method of distribution for the reinforcement, where the sections of bars are defined proportional to a corresponding surface area. For at least the type of structure tested here, where the stress field is quite homogeneous, the number of elements that simulate concrete or steel doesn't seem to have a great influence on the result (Kotronis 2000). Two meshes with the same angle α and different number of bars give practically the same results. A 'macro' model can so be used instead of the 'equivalent lattice' (Fig. 5).
- The influence of the angle decrease when the ratio of reinforcement in the two directions increases (Kotronis 2000).
- Perfect bond slip is assumed between concrete and steel.
- Symmetry of the pattern is required for cyclic and transient dynamic calculations.
- ERC can be used for structures where the stress field is quite homogeneous and the angle α doesn't change a lot during the loading. Otherwise, re-meshing strategies or other type of models - like the "strut-and-tie" ones (Schlaich et al. 1987) - can be used.

4 VERIFICATION OF THE ERC MODEL THROUGH A DYNAMIC TEST

The objectives of the experimental program organized by NUPEC were to comprehend the response characteristics of seismic shear walls at levels ranging from the elastic state to the elasto-plastic ultimate state and to provide data for computer code improvement by comparing the results of computer analysis with the test results (OECD 1996). NUPEC gave the data of the test to member countries of Organization for Economic Cooperation and Development (OECD) as an exercise of the International Standard Problem (ISP). Its proposal was officially approved as the 'Seismic Shear Wall International Standard Problem' (SSWISP) at the Committee on the Safety of Nuclear Installations (CSNI) annual meeting in December 1993.

The main characteristics of the NUPEC specimen are presented in Figure 7 and Table 1. The dynamic test was performed at NUPEC's large-scale shaking table at the Tadotsu Engineering Laboratory. The specimen was excited only in the x. The vibration test steps were determined aiming at different levels of responses (from 0.5 m/sec^2 peak ground acceleration up to 12 m/sec^2 – Figure 8). Measurements were carried out for global and local quantities (displacements, forces and strains) throughout the test.

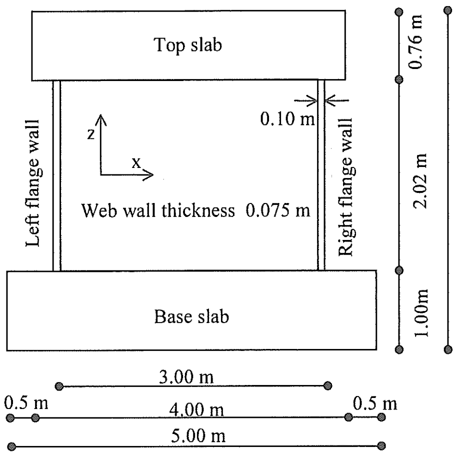

Figure 7. Geometry of the NUPEC specimen.

Tableau 1. Main characteristics of the NUPEC specimen.

Type of test	Dynamic	
Boundary conditions	Rot. free at the top	
Height / Length	0.67	
Section of web wall	m^2	0.225
Section of flanges	m^2	0.596
Horizontal reinforcement	%	1.2
Vertical reinforcement	%	1.2
Comp. strength of concrete	MPa	28.6
Tension strength of concrete	MPa	2.3
Young modulus of concrete	MPa	22960
Yield strength of steel	MPa	384
Young modulus of steel	MPa	188000
Normal stress at the base	MPa	1.5
Mass	Kg	122000

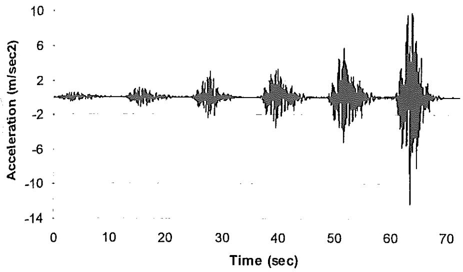

Figure 8. NUPEC – Loading sequences.

4.1 *ERC model - NUPEC specimen*

As mentioned before, the angle α that the diagonals of the concrete lattice form with the horizontal bars has been calibrated with the experimental results. It is found approximately equal to 45°. Each flange is represented by eight multi-layered beams (Euler - Bernoulli hypothesis) to account for possible flexure. The width of those beams equals the actual length of the flange (2.98 m). Six very stiff beams, free to rotate, simulate the top slab. Distributed masses are introduced at the top of the wall by three multi-layered beams. Base slab is not simulated. The section of the bars has been calculated according to the Framework method.

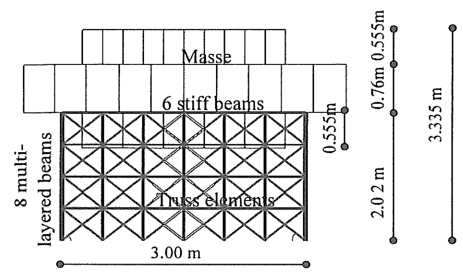

Figure 9. NUPEC - Concrete mesh.

Horizontal and vertical bars simulate horizontal and vertical reinforcement. Their sections and positions have been found using the distribution method. Reinforcement in flanges is introduced through special mixed layers in the beams.
Specific values used for the materials are the one already reported in Table 1.

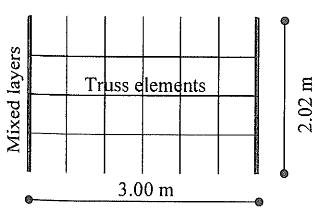

Figure 10. NUPEC - Steel mesh.

4.2 *Numerical results*

The modal analysis predicted quite well the first frequency of the mock-up (13.1 Hz for the test and 12.9 Hz for the simulation). For the dynamic analysis the Rayleigh damping coefficients have been adjusted to ensure 4% damping on the two first modes. For the lower levels not significant differences between numerical and experimental results appear, as the structure stays nearly in the elastic region. A zoom to the last two sequences show that the ERC model predicts well the global behavior of the structure even under severe loading (just before collapse). No shifting between the numerical and experimental curves appears (Fig. 11).

The importance of the "tension stiffening phenomenon" is shown in Figure 12. If not taken into account, the global behavior of the mock-up is not well reproduced and shifting appears between the curves.

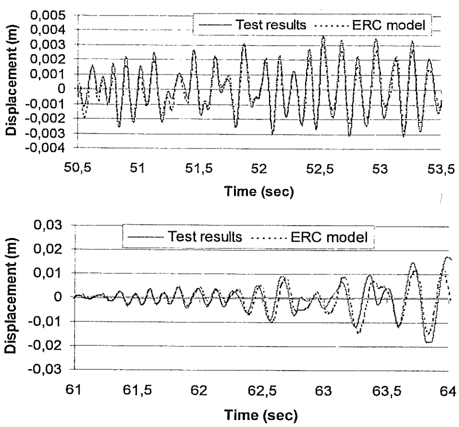

Figure 11. NUPEC - Displacement history analysis for the last loading sequences.

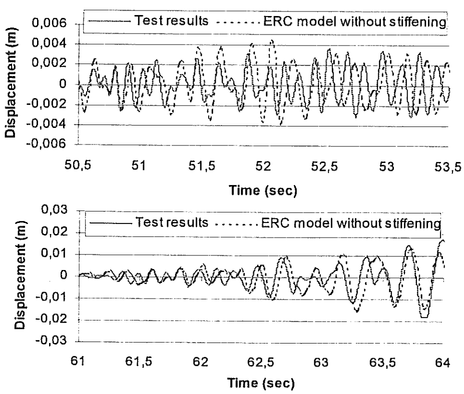

Figure 12. NUPEC - Displacement history analysis for the last loading sequences without taking into account the "tension stiffening phenomenon".

5 DISCUSSION ON THE ANGLE α

The angle α between the diagonal and the horizontal bars of the mesh is crucial for the success of the simulations. In this paper, its value has been calibrated from the monotonic experimental curve.

The sensibility study presented in Mazars et al. 2001 shows that for normally reinforced concrete structures the results of the simulation don't change a lot with the angle. A value between 35° and 45° reproduces correctly the global behaviour of the mock-ups. A way to calculate approximately the angle α would be to consider it equal to the direction ϕ of the principal stress at the end of the linear regime.

For the specific case of lightly reinforced structures with important normal stress however, the value of the angle influences significantly the results. An accurate estimation of the angle is then necessary using non-linear methods in order to calculate the variation of the angle ϕ during the loading.

This variation can be calculated using again simplified approaches. Assuming that the model reproduces the Ritter - Mörsch scheme, ϕ is derived from the equilibrium equations of the corresponding truss and the stress-strain relationships of concrete and steel (Collins & Mitchell 1980):

$$\tan^2\phi = \frac{\varepsilon_l + \varepsilon_d}{\varepsilon_t + \varepsilon_d} \qquad (8)$$

where ε_l = strain of the horizontal reinforcement; ε_t = strain of the vertical reinforcement; and ε_d = strain of the concrete struts. The stress-strain relationships for the materials are given from the Compression Field Theory (Collins & Mitchell 1980) or the Rotating angle Softened truss model theory (Hsu 1988).

6 CONCLUSIONS

A new modeling strategy for predicting the non-linear behavior of reinforced concrete shear walls submitted to dynamic shear has been presented in this paper. ERC is based on the Framework Method and uses a lattice type mesh to simulate the walls. The use of simplified structural elements and 1D constitutive laws for the materials makes the method straightforward and not time consuming, allowing for parametrical studies. The success of the simulation depends on the value of the angle that the diagonal compressive trusses form with the horizontal ones. Some ways for predicting this value have been discussed, but research is still going on. The influence of the « tension stiffening phenomenon » has also been identified.

The proposed lattice model is very promising and could be used to simulate the non-linear behaviour of plastic zones developed at beam-columns joints, base of bearing walls and extremities of beams. It can be introduced to commercial codes used by civil engineers that are more familiar with elements such as trusses and beams. Finally, 3D applications of the model, taking into account out of plane flexure and torsion phenomena are also possible.

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