Fracture analyses of walls under non-proportional loadings

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ABSTRACT: This paper starts with a brief review of existing crack models for concrete structures. The performance of fixed and non-fixed crack plane concepts in handling non-coaxiality, which often occurs in case of non-proportional loadings, is demonstrated through a proposed reversed shear-tension problem. For comparison, Willam's test for proportional shear-tension is also included. It is shown that in certain situations, simulation by non-fixed crack concepts may produce incorrect material response upon reversed loading. The consequences thereof are discussed for the case of a wall structure with existing initial cracks due to vertical loading, which is subsequently subjected to settlements.

Keywords: non-proportional loading, fracture, continuum crack model, fixed crack, rotating crack, tension-shear, wall

1 INTRODUCTION

Examples of non-proportional loadings applied to concrete structures can be clearly seen in any real-world situation. The dead weight of the structure is superimposed by the other loads. Old structures may experience certain degrees of long-term creep and in the past, cracking could be induced due to climate changes such as thermal fluctuation and shrinkage. Indeed, this loading history alters the initial condition of structures prior to subsequent plausible loadings such as wind loads, earthquakes or ground settlements. Reliable prediction of structural performance taking such aforementioned stress-strain history into account becomes essential for instance in case of historical buildings. This is particularly highlighted when the available observational data are scarce and engineering judgment should be done based on more detailed analyses by the finite element method.

One of the examples is the recent numerical analysis of settlement damage in structures e.g. Boonpichetvong & Rots (2003). From that study, it appeared that existing crack models showed a number of limitations. One of them is the difficulty in simulating the effect of *non-proportional* loadings. Initial cracks may be present due to live loads and dead load of the building. Upon applying the settlement trough, these initial cracks may be re-activated or they may be arrested while new cracks form under an angle with the initial cracks. The proper choice of crack model is essential to acquire physically meaningful results, as the initial cracks may be important upon non-proportional loading and trigger different failure modes depending on loading path and history.

In order to model cracking in quasi-brittle materials like concrete and masonry, several versions of crack models exist ranging from discontinuum to continuum crack models. For discontinua, predefined interface elements (e.g. Rots 1988), remeshing techniques (e.g. Ingraffea & Saouma 1985), embedded discontinuities based on incompatible strain modes (e.g. Oliver 1996) or cohesive zone models based on partition of unity methods (Wells 2001) can be used. For large-scale analysis, continuum models are often preferred. Here, engineers can choose between smeared cracking with decomposed strain based, total strain based, plasticity based models, damage models or other frameworks. A number of overview papers focused on theoretical development of continuum crack models e.g. Willam et al. (1987), Crisfield & Wills (1989), Weihe et al. (1998), Feenstra & Rots

(2001) and De Borst et al. (2003). Practical points of use when applying these models at structural level are discussed by e.g. Rots (2002). Nevertheless, up to the authors' knowledge, there are few contributions mentioning the robustness of each crack model for the fracture of concrete structures subjected to non-proportional loadings. This aspect is explored in this paper.

2 PLANE OF DEGRADATION AND CONTINUUM CRACK MODELS

When structures do not possess crack-like defects, they often behave in such a way that the principal stress direction remains coaxial with the principal strain direction at each material point. After cracking, the direction of principal stress may deviate from the direction of principal strain at that material point in particular in case of non-proportional loading. This is particularly significant if we have to simulate the stress-strain history and crack history of structures.

Continuum crack models may be classified into fixed crack plane concepts (three examples given in section 2.1 to 2.3) and non-fixed crack plane concepts (two examples given in section 2.4 and 2.5). Models belonging to the first framework are capable of memorizing crack planes and existing damage.

2.1 *Decomposed-strain based multi-directional fixed smeared crack model*

For this smeared crack model (e.g. de Borst & Nauta 1985, Rots 1988), the first crack plane is determined by the direction perpendicular to the principal tensile stress when its magnitude exceeds the material tensile strength. The strain is decomposed into a concrete part and a crack part. The crack strain can be sub-decomposed into the local crack strains of a number of cracks at different orientations. This makes it possible to handle non-orthogonal multi-directional cracking. However, the choice of the inter-crack threshold angle is subjective and the interplay between this threshold angle and the shear retention function selected can significantly affect the results.

2.2 *Total-strain fixed smeared crack model*

A total strain-based constitutive model (e.g. Cervenka 1970, Suidan & Schnobrich 1973, Feenstra et al. 1998) describes the stress as a function of the total strain. With the fixed version, the stress-strain relationships are evaluated in a co-ordinate system that is fixed upon cracking. Due to a fixed orthogonal co-ordinate system, only fixed orthogonal cracking is allowed in this model. For this fixed crack version, the shear retention factor is set explicitly.

2.3 *Microplane model*

In this model (e.g. Bažant & Prat 1988), unlike conventional tensorial models that relate the components of the stress tensor directly to the component of the strain tensor, normal and shear stresses across a fixed set of planes of various orientations are monitored. The basic constitutive laws are defined on the level of the microplane and must be transformed to the level of the material point using certain relations between the tensorial and vectorial components. The multiple planes suggest a similarity with the multi-directional smeared crack model. However, in the multi-directional smeared crack model of section 2.1 a set of planes is not predefined, but the crack planes emerge during the process, in the direction normal to the principal stress when this principal stress violates a tension cut-off.

2.4 *Total-strain rotating smeared crack model*

This crack model (e.g. Jirásek & Zimmermann 1998, Feenstra et al. 1998) again uses stress-total strain relations. It employs a co-rotational formulation involving an implicit shear term to enforce co-axiality between the rotating principal stress and strain (Bažant 1983). Due to the rotating orthogonal principal stress system, only orthogonal cracks can be modelled. Non-orthogonal multi-directional cracking cannot be included.

2.5 *Plasticity based or damage based crack models*

Based on these formulations, internal variables are driven either by a plasticity concept (e.g. Feenstra 1993, Meschke et al. 1998) or by damage concept (e.g. Mazars & Pijaudier-Cabot 1989). In the first concept, the plastic crack strain is determined according to a flow rule once the yield function in the stress-space is satisfied. For the damage model, damage growth is determined by a damage loading function in the strain-space.

3 NON-PROPORTIONAL LOADING TEST

To evaluate the fundamental differences between fixed and non-fixed crack models under non-proportional loading, two single element tests are performed. The elasticity-based crack models described in section 2.1 and 2.4 are employed.

3.1 *Revisited tension-shear model problem*

In the first test, an elementary problem originally proposed by Willam et al. (1987) is revisited. A bilinear plane-stress element with unit dimension is firstly loaded by tensile straining in the x-direction, accompanied by lateral contraction in the y-direction due to Poisson's effect. After the tensile strength has been reached by initial uniaxial loading, the element is then loaded in combined biaxial tension and shear strain (see Figure 1). This produces a continuous rotation of the principal strain axes after initiation of first cracking. The ratio between the strain component is taken as $\Delta\varepsilon_{xx}$: $\Delta\varepsilon_{yy}$: $\Delta\gamma_{xy}$ = 0.5: 0.75: 1. The referred plain concrete properties are given in Table 1.

Concrete Properties		
Young's modulus E_c	10,000	N/mm^2
Poisson's ratio ν	0.2	
Tensile strength f_t	1.0	N/mm^2
Mode I Fracture energy G_f	1.5×10^{-4}	N/mm

Table 1. Material properties in tension-shear model problem

For the fixed crack concept, the multiple fixed crack model with sub-step procedure (Rots 1988) is chosen. A linear tension-softening curve is used with constant shear retention factor equal to 0.1. This results in a constantly increasing shear stress with increasing shear strain. The threshold angle between the multi-directional cracks is set as 45 degrees although a former investigation by Rots (1988) with the same multi-directional crack model revealed that the shear response becomes softer with decreasing threshold angle. For the non-fixed crack concept, the rotating crack model (Feenstra et al. 1998) is employed also with linear tension softening curve. In both crack models, a crack band width of unit dimension is assumed.

In Figure 2, it can be seen that the normal stress-strain response in x-direction for the multiple fixed crack model directly represents the input tensile softening curve. This is obvious because of the alignment of the first crack plane with the normal x direction. This is also found for the y direction (see Figure 3) which defines the direction of the second crack plane for the multiple fixed crack model.

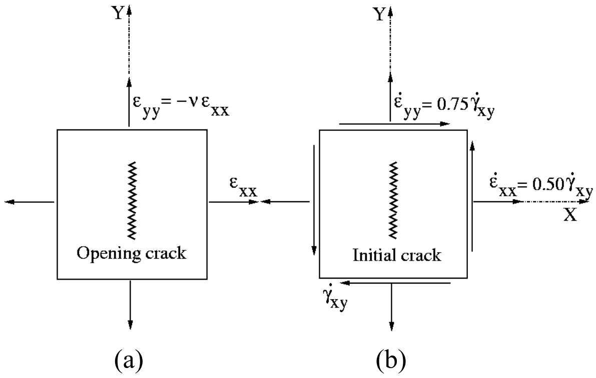

Figure 1. Tension-shear model problem (a) uniaxial up to first cracking (b) continuous biaxial tension with shear after first cracking

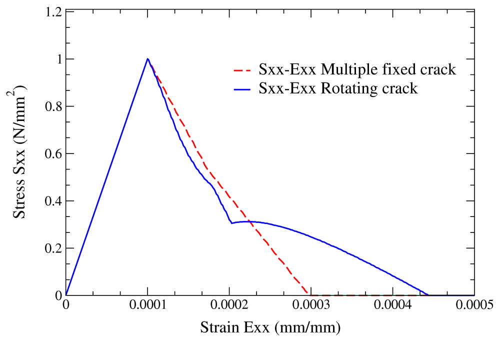

Figure 2. Compared σ_{xx}-ε_{xx} response, multiple fixed vs rotating crack in tension-shear model

For the rotating crack model, the normal stress-strain response in y direction shows a gradual degradation of the strength and stiffness. The tail of diagrams for rotating crack model is affected for two reasons. The first is by implicit shear softening behavior (see Figure 4) and the second is by the consideration of lateral effects due to Poisson's ratio in the formulation (see Feenstra et al. 1998).

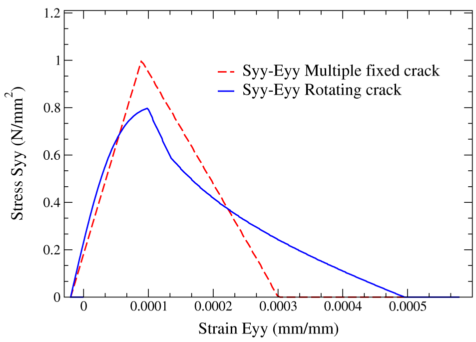

Figure 3. Compared σ_{yy}–ε_{yy} response, multiple fixed vs rotating crack in tension-shear model

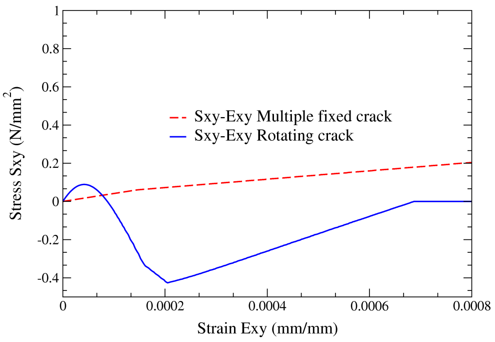

Figure 4. Compared σ_{xy}–γ_{xy} response, multiple fixed vs rotating crack in tension-shear model

An important issue pointed out by Willam et al. (1987) is the superior nature of the rotating crack model compared to the fixed crack model in controlling the maximum tensile stress. In addition, it was proved also that the rotating crack model exhibits less stress locking (see Rots 1988) when applied to structures. Due to non-coaxiality of principal stress and strain in the multiple fixed crack model, a built-up of shear stress is observed when a certain value of shear retention is used (see Figure 4). The use of a zero or low shear retention factor (e.g. β=0.001) is necessary to circumvent such shear stress built-up. Figure 5 shows the development of the primary principal tensile stress versus the principal tensile strain. After initiation of the first crack, the secondary principal tensile stress eventually leads to a second crack orthogonal to the first crack. An inclined crack does not occur for this case with this shear retention factor, as the inclined principal tensile stress obviously does not exceed the tensile strength.

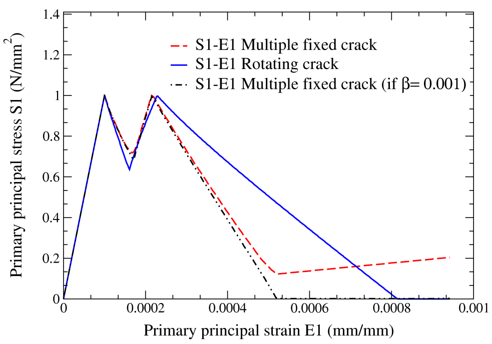

Figure 5. Compared σ_1–ε_1 response, multiple fixed vs rotating crack in tension-shear model

Figure 6 reveals the different final crack patterns predicted by the two crack models. The crack pattern of the rotating crack model swings according to the spin of the principal strain direction while the crack pattern of the multiple fixed crack model is fixed after crack initiation. Such differences in crack pattern may affect the results significantly when unloading or crack closure occurs. This is investigated in the next element test.

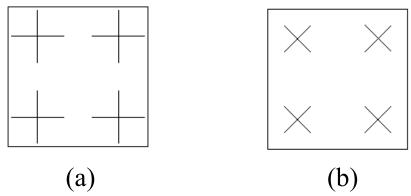

Figure 6. Final crack patterns by (a) multiple fixed crack (b) rotating crack in tension-shear model (plot intended for 2×2 integration scheme)

3.2 *Reversed pure shear-tension model problem*

For the second test, the same bilinear plane-stress element with unit dimension is now subjected to a different non-proportional loading path. The comparative study is still limited to the multiple fixed crack and rotating crack models.

Firstly, the element is loaded by pure shear straining until the induced principal tensile stress reaches the tensile strength (see Figure 7a). After such cracking is initiated, the pure shear loading is continued until the value of shear strain reaches 0.0006 (primary principal tensile strain is equal to 0.0003) after which the reversed pure shear is applied to the element ending up with zero strain stored in the element (see Figure 7b & 8). Finally, continuous biaxial tensile straining with the ratio of $\Delta\varepsilon_{xx}$: $\Delta\varepsilon_{yy}$ equal to 1: 20 is performed (see Figure 7c). The same plain concrete properties of Table 1 are adopted. However, in the multiple crack model, a low shear retention factor β=0.001 is used. The test problem has been designed such that the role of the crack plane being either fixed or non-fixed is significant.

As expected, the crack patterns during the first pure shear loading predicted by both multiple fixed and rotating crack models are the same (see Figure 9a1 and 9a2). The continuous biaxial tension after applying reversed pure shear, leads to marked differences in crack patterns between the two models (see Figure 9b1 and 9b2). For the rotating crack model, the crack direction will immediately shift to the direction normal to the primary principal tensile strain, i.e. normal to the y-direction. The crack pattern then rotates from the one in Figure 9a2 to that in Figure 9b2. For the fixed crack model, the inclined crack is still present and this memory of the crack is maintained. The normal stress-strain response in both x and y directions is depicted in Figure 10 and 11.

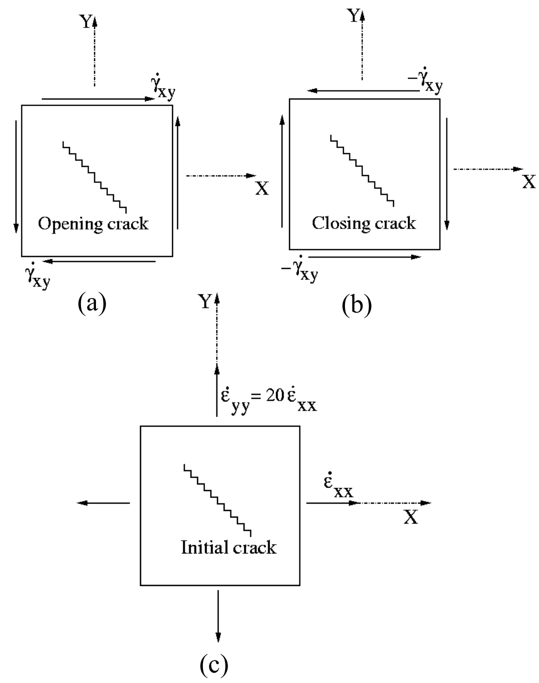

Figure 7. Reversed pure shear-tension model problem (a) pure shear up to first cracking and further until ε_1 equal to 0.0003, (b) reversed pure shear until zero straining, (c) continuous biaxial tension

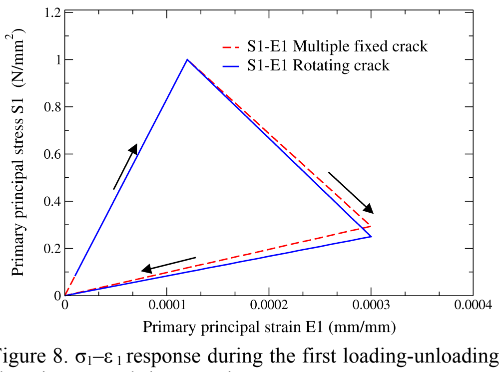

Figure 8. σ_1–ε_1 response during the first loading-unloading pure shear in reversed shear-tension test

Indeed, for the rotating crack model after we introduced the initial crack, the primary principal strain and the primary principal stress direction do not comply anymore. The primary principal tensile stress is aligned in the x direction whereas the primary principal tensile strain is always in the y direction during biaxial tension. This is because isotropy of material is violated and anisotropy from the initial crack dominates the behavior.

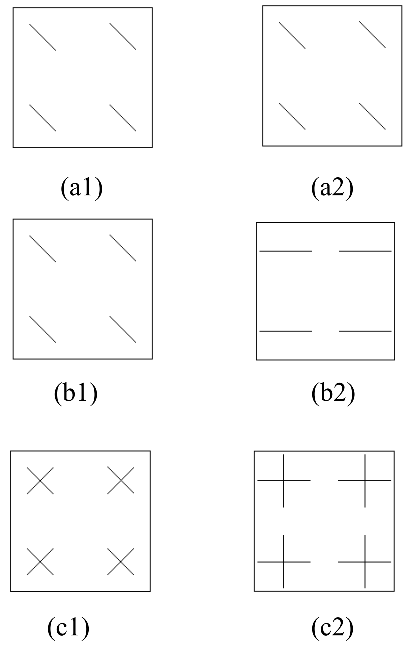

Figure 9. Evolved crack patterns by multiple fixed crack (a1, b1, c1) and rotating crack (a2, b2, c2) in reversed pure shear-tension model (Note: a is the pattern during the first pure shear and reversed pure shear straining, b is the pattern when the first small increment of the continuous biaxial tensile straining is applied, c is the final crack pattern after continuous biaxial tensile straining is completed)

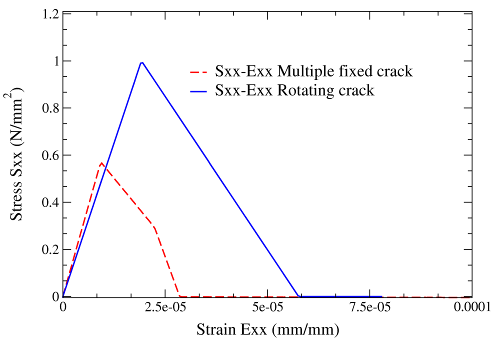

Figure 10. Compared σ_{xx}-ε_{xx} response, multiple fixed vs rotating crack in reversed shear-tension test

Although the rotating crack model can control the maximum principal tensile stress in Figure 12, it does underestimate the normal stress-strain response in y direction and on the other hand, it does overestimate the response in x direction, compared with the fixed crack model. The final crack patterns predicted by two crack models are different as shown in Figure 9c1 and 9c2. In fact, for the rotating crack model, the stress at the onset of unloading in Figure 8 is memorized for reloading in the y-direction, which physically speaking is not correct.

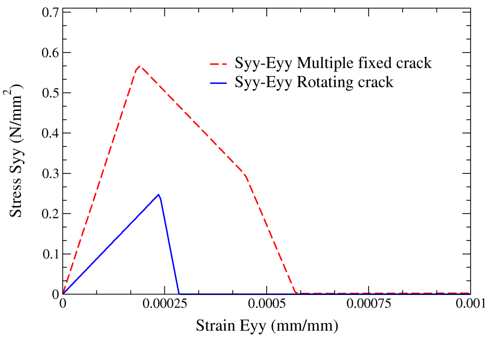

Figure 11. Compared σ_{yy}-ε_{yy} response, multiple fixed vs rotating crack in reversed shear-tension test

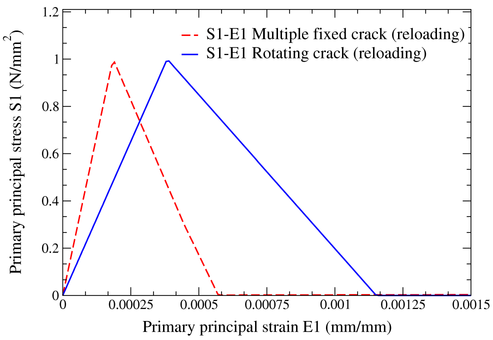

Figure 12. Compared σ_1-ε_1 response, multiple fixed vs rotating crack in reversed shear-tension test

With a higher value of β = 0.1 for the multiple fixed crack model, the shear stress along the fixed crack planes increases significantly and consequently σ_{xx} and σ_{yy} are built up as shown in Figure 13 and 14. A third horizontal crack plane is activated as shown in Figure 15. It is evident that the predicted fixed crack results can be sensitive to the Mode II shear term. This aspect is still open and future investigation into improved crack shear relations is required. In summary, this elementary test demonstrates that proper inclusion of the physical crack plane has advantages in case of non-proportional loading. By ignoring the existing crack pattern and crack interaction during non-proportional loadings, the strength and residual behavior may be underestimated or overestimated.

The drawback of the non-fixed crack plane concept mentioned above is not limited to the elasticity-based rotating crack model. Similar problems with a spinning plane of degradation may be encountered when using a plasticity-based crack model (e.g. Feenstra 1993), or a damage based crack model (e.g. Mazars & Pijaudier-Cabot 1989). For non-proportional loading, in addition the unloading option is relevant. The elastic unloading option in plasticity-based models may give a too stiff response; see also Feenstra & Rots (2001) for cyclic loading problems.

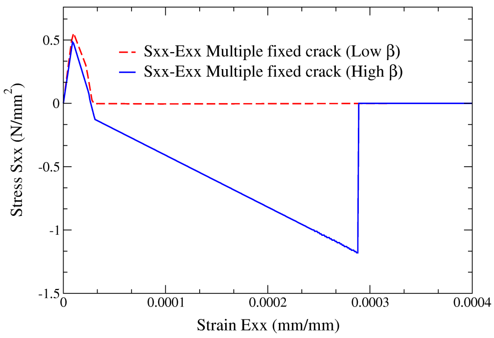

Figure 13. Compared σ_{xx}-ε_{xx} response, high vs low shear retention factor β in reversed shear-tension test

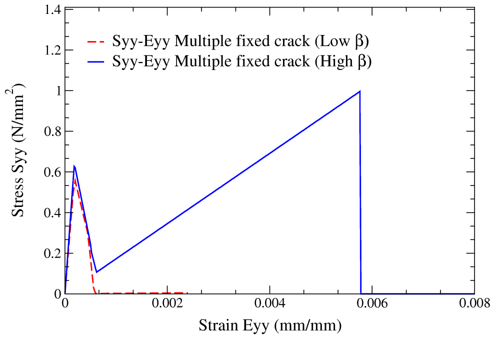

Figure 14. Compared σ_{yy}-ε_{yy} response, high vs low shear retention factor β in reversed shear-tension test

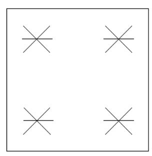

Figure 15. Altered final crack plane for shear retention factor β = 0.1 in multiple fixed crack model

4 CRACKED WALL SUBJECTED TO GROUND MOVEMENTS

In this section, a large-scale wall associated with non-proportional loadings is discussed with some practical viewpoints regarding the importance of including the physical plane of degradation when determining the effect of initial cracks on structural behaviour. The wall is taken from the recent study by Boonpichetvong & Rots (2002). That study aimed at predicting the settlement damage of the masonry wall induced by soft ground tunnelling. The non-proportional loading scheme involves the action of dead load of the masonry, live load from each floor and the subsequent expected soil settlements. An example of possible cracking in the masonry wall due to the tunnelling activity is given in Figure 16.

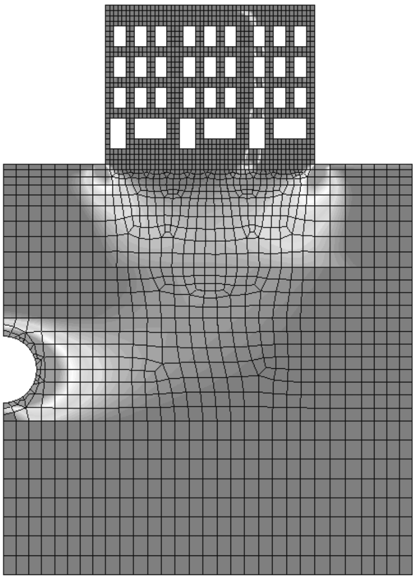

Figure 16. Possible settlement damage of wall subjected to ground movement

With the selection of a low tensile strength to reflect the case of historical structures, substantial initial cracks may be predicted under the action of dead and live loads (see Figure 17).

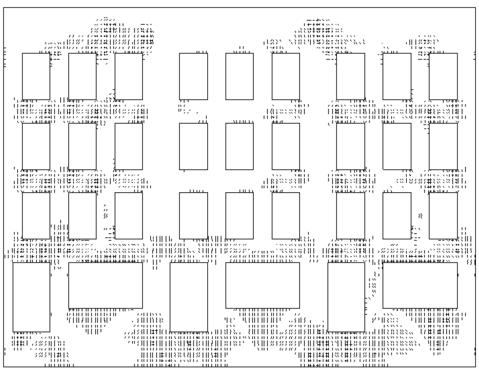

Figure 17. Predicted initial cracks under the action of vertical dead and live loads

In addition to these initial cracks, the properties of the concrete or masonry may be changed due to the effect of long-term sustained loading causing creep. These initial conditions must be taken into account in FE analysis to acquire a reliable settlement damage prediction due to the short-term tunnelling activity. For this situation, selecting the proper crack model becomes important. Choice must be made firstly between fixed and non-fixed crack concepts.

In practice, a procedure that directly transforms recorded damage patterns obtained by building condition surveys into initial material parameters for FE analysis is required. Crack maps of historical buildings are translated to orientation-dependent model parameters that serve as initial conditions for the settlement load case. The fixed crack plane concept is to be preferred from this point of view. The authors believe that the concept of decomposed strain formulation of fixed smeared cracking, which provides for coupling with creep, thermal and other effects is still very attractive. With the non-fixed crack concept, the inherent spinning plane of degradation often makes it difficult to simulate the interplay between existing initial cracks and newly emerging cracks. A return to the fixed crack philosophy has recently also been reported by Papa & Taliercio (2003).

5 CONCLUSIONS

The significance of incorporating the physical plane of degradation in fracture analyses for non-proportional loadings and existing initial cracks is investigated in this paper. The results reveal that the non-fixed crack plane concept may provide incorrect material response for such cases. The fixed crack plane concept has more potential to determine the effect of initial cracks on residual structural performance.

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