Testing and Modelling of Steel Fibre Reinforced Concrete Beams Designed for Moment Failure

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ABSTRACT: As part of a Norwegian industrial research project, several experimental investigations of the effect of steel fibre reinforcement (SFR) in concrete beams have been carried out in the period 2000-2003. Four test series, consisting of respectively 4, 3, 9 and 36 beams designed for moment failure, have been carried out. The following parameters were investigated: Ordinary vs. self compacting concrete, fibre volume, fibre length, cross section height, and influence of ordinary reinforcement.
In 10 of the beams, SFR was used in combination with a relatively small amount of ordinary reinforcement, which should ensure sufficiently ductile structural behaviour. While the two first series were parallel with ordinary or self compacting concrete, the third series consisted of beams with different cross section height (150, 250 and 500 mm). A significant size effect on the structural behaviour was expected, especially for beams without ordinary reinforcement. The fourth series consisted of smaller beams (150/150/600) sawn out from structural elements, ie walls and slabs in realistic sizes, produced on a construction site. A new standard test method, which also includes determination of residual strength and evaluation of the real fibre content, is proposed.
The post-cracking tension-model used in the finite element analysis, is based on a bilinear σ-w relationship. Both the discrete cracking and the smeared cracking approach have been used. A theoretical expression for the tensile force resultant in the fibres crossing a crack is deduced, and afterwards used to determine an expression for the residual tensile strength. Special attention is paid to the fibre fraction, which is defined as the relative amount of fibres crossing a crack plane. Its theoretical value is related to results from fibre counting, which for some of the series showed results over a wide variation range.

Keywords: concrete, steel fibre, beams, testing, modelling

1 INTRODUCTION

High strength steel fibre reinforcement (SFR) might replace ordinary reinforcement completely in concrete structures with relatively low reliability levels for structural safety, as slabs on grade, foundations and walls. Furthermore, in load carrying structures in general, SFR might be used in combination with a relatively small amount of ordinary or prestressed reinforcement. Although the economy has been a main limiting factor for practical use of SFR, it is presently a more interesting alternative due to lack of skilled workers and need for industrialisation of the construction industry. The combination of SFR and self-compacting concrete (SCC) is furthermore a very promising concept.

The experimental investigations are financially supported by a Norwegian industrial research project, chaired by the contractor Veidekke ASA. Four different series consisting of respectively 4, 3 9 and 36 beams, as described below, have been tested. The following parameters were investigated: Ordinary vs. self compacting concrete, fibre volume, fibre length, cross section size, influence of ordinary reinforcement, influence of the casting process and the type of structural element (ie wall or slab). In addition shear test have also been conducted, these are however not considered in this paper (Døssland & Kanstad 2003).

While the major objective for the Norwegian research project is to establish rules for design, execution and control (Fjeld 2003), the objectives by the beam testing are devoted to each of the previously mentioned variables. However considering design methods, the following objective was formulated:

> Evaluate a design method, based on residual tensile strength determined from the theoretical force resultant in the fibres crossing a crack.

2 EXPERIMENTAL PROGRAMME

Three different series of beams have been tested:

Series I (4 beams) (Kanstad & Døssland 2003): Beam dimensions: b/h/L=150/250/3000, Ordinary concrete, compressive cube strength: $f_{ck} \cong 50 N/mm^2$, minimum ordinary reinforcement (2Ø6 mm, ρ_L=0. 17%), Dramix steel fibres with hooked ends (Length/Diameter ratio=65), fibre length 35 or 60mm, fibre volume: v_f=0.5 or 1.0%.
Series II (3 beams)) (Kanstad & Døssland 2003): As series I, but made with self compacting concrete, $f_{ck} \cong 60 N/mm^2$,
Series III (9 beams)) (Kanstad & Døssland 2003): Three beam dimensions: b/h/L= (150/150/900), (150/250/3000) or (150/500/4000), Ordinary vibrated concrete. $f_{ck} \cong 50 N/mm^2$ and fibre length 60 mm. Ordinary reinforcement and fibre content: (v_f=1.0% and $\rho_L \cong$0. 5%), (v_f=1.0% and ρ_L=0) and (v_f=0.3% and ρ_L=0). The largest beam with ordinary reinforcement is shown in Figure 1
Series IV (36 beams) (Grimheden et al. 2003): Small beams: (150/150/600) Ordinary vibrated and SCC, (v_f=0.7% and $\rho_L \cong$0), beams sawn out from slabs or walls in realistic dimensions.

In the two first series, SFR was used in combination with a relatively small amount of ordinary reinforcement. The major objective was to investigate the validity of superimposing the effect of minimum reinforcement and SFR with respect to crack distribution, moment capacity and ductility. Series III had main focus on the effect of cross section height for different reinforcement conditions.

For the two first series the load was applied load controlled, and it was not possible to measure the descending branch of the load-deflection curve. Before the third series the test rig was reconstructed to obtain full deflection control. The following types of instrumentation was used: Strain gauges on the ordinary reinforcement, LVDT's for deflections, strains and crackwidths in addition to hand operated extensiometer for strains and crackwidths.

The fourth series is basis for a proposed standard test method, which also includes rules for determination of the residual strength and evaluation of the fibre distribution and orientation in actual cases (Thorenfeldt 2003) , (Fjeld 2003).

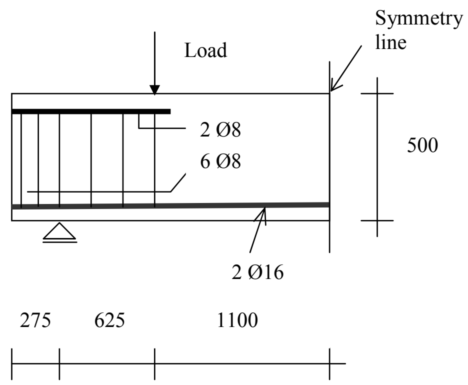

Figure 1. Geometry of the largest beam

3 FINITE ELEMENT MODELLING

In general modelling of tensile failure in reinforced concrete it is important to include the descending branch or the strain softening behaviour in a sensible way. This is especially important for fibre reinforced concrete because the effect of fibres is significant only after cracking occurs. The post cracking behaviour may be described either by smeared or discrete cracking. Steel fibres bridging cracks are not as ductile as ordinary reinforcement and the behaviour is strongly dependent on the crack width. Consequently it is important to use the discrete crack approach as basis, and to model the crack spacing quite accurately. This is obviously a problem in practical design, but not in this case where the crack pattern is known from the experiments.

As a basis for modelling, the influence of fibres after cracking is considered according to (Thorenfeldt 2003). Compared to the rather comprehensive work by (Li et al. 1991), several simplifications due to the modelling of single fibres are made.

Obviously it is the force resultant in the fibres crossing a crack which is the most important factor in tension modelling. However, the section ratio, defined as the relative amount of fibres crossing a section, is also interesting because the theoretical value can be related to results from fibre counting and used to describe for instance systematic anisotropic fibre orientation. It is therefore interesting to deduce this theoretically. Considering Figure 2, the horizontal plane may represent an area of the crack plane while the planterium represents the directions which the fibres will be evenly distributed over if isotropic conditions are assumed. The fibre fraction with angle ($\phi \pm \Delta\phi/2$) can then be expressed as:

$$\rho_{f\,\Delta\phi} = (2\pi r \sin\phi \; r\Delta\phi)/2\pi r^2 = \sin\phi \,\Delta\phi \qquad (1)$$

The corresponding volume ratio for this fraction is:

$$v_{f\,\Delta\phi} = v_f \sin\phi \,\Delta\phi \qquad (2)$$

In which v_f is the fibre volume. The section ratio ρ_x is defined as the area of the fibres per unit concrete area. Integration of Equation (2) multiplied by $\cos\phi$ for all fibres with direction angle ϕ between 0 and $\pi/2$ gives the section ratio for all fibres crossing the crack plane:

$$\rho_x = v_f \int \sin\phi \cos\phi \, d\phi = v_f \,[(1/2) \sin^2\phi] = v_f /2 \qquad (3)$$

The result means that 50% of all the fibres present in a unit volume, will cross a plane in any direction. If it is assumed that all the fibres are evenly distributed in a plane, the section ratio normal to this plane will be $0.637v_f$. Furthermore if the orientation is unidirectional the section ratio will be equal to v_f.

Returning to the isotropic conditions, and assuming that the fibres behave in accordance with the theory of plasticity, and that the average stress is σ_F, the corresponding normal force resultant per unit concrete area can be determined as:

$$F_{xp} = v_f \sigma_F \int \sin\phi \cos^2\phi \, d\phi = - v_f \sigma_F \,[\cos^3\phi /(2+1)] = v_f \sigma_F/3 \qquad (4)$$

As basis for this equation it is also assumed that the fibres keep their original direction after cracking. For the previously considered two dimensional and one-dimensional fibre orientation the corresponding forces will be $0.5\, v_f \sigma_F$ and $v_f \sigma_F$, respectively.

Furthermore it is necessary to know a value for the average fibre stress, σ_F, which should be a function of the crack width as shown in Figure 3. In this work it is assumed that the average fibre stress just after cracking (ie at a crack width of 0.1 mm) is approximately 50% of the fibre strength which corresponds to 500 N/mm^2. This is theoretically considered by (Thorenfeldt 2003).

Some advantages by the present model, compared to other approaches, is that it is linearly related to the fibre content, and that anisotropic fibre orientation relatively easily can be accounted for. As our test results later will show this is very important.

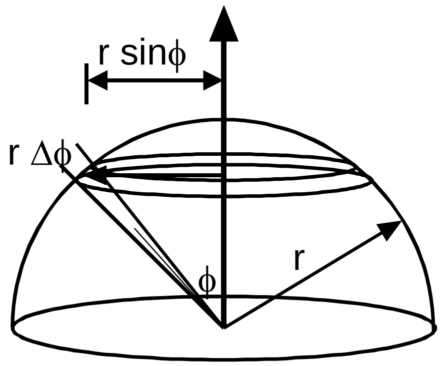

Figure 2. Determination of force resultant in the fibres crossing a crack. Thorenfeldt (2003).

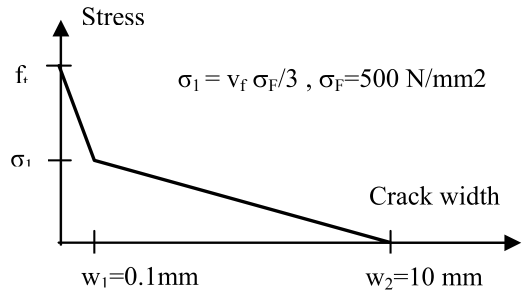

Figure 3. Stress-crackwidth relation used in the present study

Typical examples of crack patterns observed in beams with fibres and ordinary reinforcement are shown in Figure 11a-c. The cracks are relatively evenly distributed along the beam, and consequently the smeared crack approach may be used. To obtain a relevant stress-strain relationship, the crack strain is achieved by dividing the crack width on the average crack spacing observed in the test. Crack patterns of three beams with 1% fibres only are presented in Figure 11d-f. In the analysis of these beams, the stress-crack-width relation is used directly. To model the cracks approximately where they occurred in reality, interface elements have been used.

4 RESULTS

4.1 *Fibre distribution and orientation*

The moment capacity for similar beams made of ordinary and self compacting concrete (series I and II) are compared in Figure 4. It is very interesting to note that while the moment capacity increase caused by addition of 0.5% and 1.0% fibres were 24.5% and 43.9% for the beams made of ordinary concrete, the corresponding capacity increases were as large as 55% and 98% for the beams made of self compacting concrete. A relatively small part of the difference might be explained by the higher compressive strength (20%) and the improved bond properties of the self-compacting concrete. However is the main reason due to different fibre distribution and orientation. This is verified by counting fibres on different orthogonal planes of sawn prisms from the beams. Defining the fibres into three categories; Isotropic oriented, plane oriented and uni-directionally oriented, it was shown that for the beams of self compacting concrete only approximately 40% of the fibres were isotropic oriented. Consequently, the beams had relatively large parts of the two latter categories. It was also shown that the fibre concentration was considerably larger in the lower parts than in the upper parts of the cross section. (approximately 60% in the lower half and 30% in the lower quarter) (Døssland and Kanstad 2003).

Figure 5 presents the relation between the measured and the theoretical (isotropic) number of fibres in the cross section of the beams in test series III where ordinary vibrated concrete was used. Although a scatter was observed also here, the fibres are much more evenly distributed than for the SCC beams.

In series IV, the objective was primarily to study the influence of the casting process on the fibre distribution and orientation in real concrete structures on a construction site. Therefore small beams (b/h/L= 150/150/600) were sawn out from large real structural elements, i.e. wall and slab elements with thickness 150 mm made of ordinary or self compacting concrete. In the walls the beams were sawn in either vertical or horizontal direction. The location, related to the pouring position was also varied.

The distribution of the section ratio, which theoretically should be $v_f/2$=0.35% (Equation 3), is presented in Figure 6. The section ratio is determined by counting the fibres on a section 50 mm away from the failure plane. The results are so far not very encouraging, but the major experience is that the casting process must be better controlled in fibre reinforced structures than what is common practice for ordinary reinforced concrete. Obviously also more investigations are needed within this subject. A more promising experience is due to the relation between the section ratio and the residual tensile strength presented in Figure 7. The residual strength is determined as 0.37 times the average equivalent bending strength recorded at 0.5 and 2.5 mm deflection.

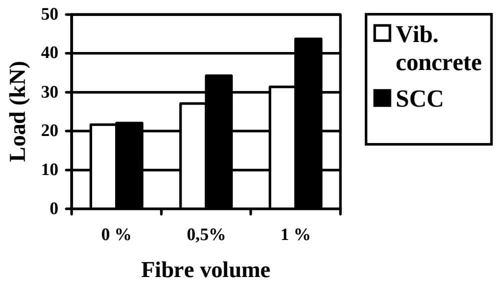

Figure 4. Moment capacity of beams made of ordinary and self compacting concrete with different steel fibre content. 60 mm long fibres with hooked ends

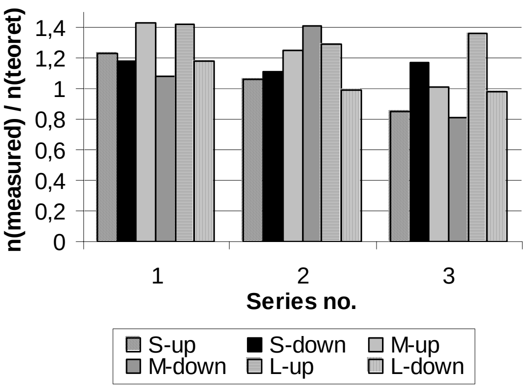

Figure 5. Measured versus theoretical (isotropic) number of fibres for series III: Beams made of vibrated concrete. S, M and L are the small, medium and large beams. The fibres are measured on the upper and lower half of the cross-section.

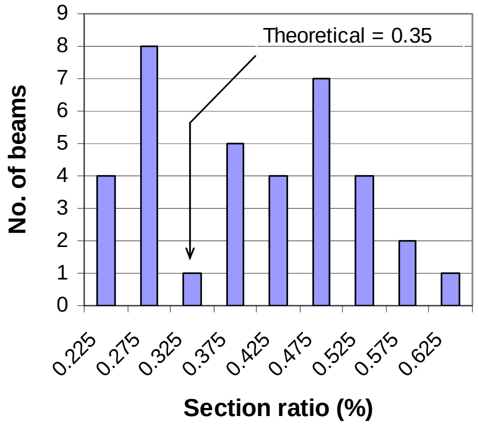

Figure 6. Section ratio for small beams (Series IV, b/h= 150/150), sawn out from real concrete structural elements.

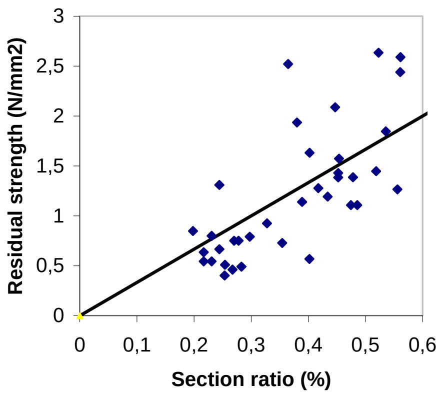

Figure 7. Residual strength versus measured section rato for small beams (Series IV, b/h= 150/150).

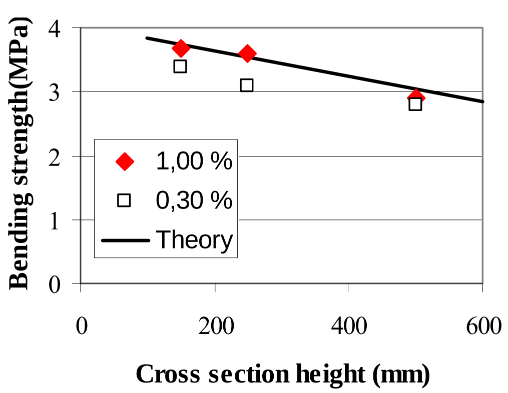

Figure 8. Bending strength vs cross section height

4.2 *Bending strength*

Regarding the third series , including the size effect study for the beams with SFR only, a significant size effect on the bending tensile strength was observed, Figure 8. This strength was determined from the first stiffness reduction on the load-curvature relationship, which should coincide with the first crack. It was also seen that the bending tensile strength is higher for the concrete with 1.0% fibres than for 0.3%. In the figure the results are compared with the commonly used size factor: k_v=1.5 - h (h in meter) and it is seen that the agreement is rather good.

4.3 *Experimental vs calculated results*

Figure 9 presents measured versus calculated curvature for the beams of series III with different cross section height and ordinary reinforcement in addition to 1% fibres. The load was increasing for a long period after the tensile reinforcement yielded, and therefore the ductility was satisfactory for all the beams. The experimental curvature has been calculated from the strains measured by LVDT's with measurement length 200 mm. The beams were analysed using the smeared crack approach, and the average measured crack spacing on 115 mm, 103 mm and 193 mm for the beams with height 150, 250 and 500 mm respectively, have been used to determine the stress-strain relationship in the post-cracking range as described in Chapter 3. The crack patterns are shown in Figure 11a-c. Considering size effects on the moment capacity, it is interesting to note that for the two smaller dimensions the capacity is underestimated by the analysis, while the situation is opposite for the largest beam. The size effect is therefore probably not fully described by the finite element modelling. Similar results were also reported by (Erdem, 2003). Because the amount of longitudinal reinforcement is approximately 0.5%, and because less than 1/6 (=17%) of the fibres have the same efficiency for bending moment, the fibres contribute with less than 1/3 of the moment capacity.

Furthermore Figure 10 presents calculated and measured results for the beams with 1% steel fibres only. The beams were analysed using the discrete crack approach as previously described in Chapter 3. For the two smallest beams, the agreement is satisfactory, while it is not so for the largest beam.

Although all the measurements are not yet fully evaluated, it is seen that both the maximum load and the ductility seem to decrease with increasing cross section height.

5 CONCLUSIONS

Four different experimental series with beams exposed to four point bending have been carried out, and the following parameters were investigated: Ordinary vs. self compacting concrete, fibre volume, fibre length, cross section height, influence of ordinary reinforcement and position of tested element sawn out from real structural elements.

The beams have been analysed by the finite element method using a stress crackwidth relationship as starting point. The tensile stress after cracking is determined as a function of the amount of fibres crossing the crack plane, and the average fibre stress. For beams with ordinary reinforcement the smeared crack approach was used, while for the beams with fibres only, the discrete crack approach has been used.

The influence of cross section height on the bending strength (1st crack) is similar to ordinary concrete

Considering beams with fibres and ordinary reinforcement, the effect of the steel fibre content on the deformational properties and the failure capacities was significant. In beams with 0.17% ordinary reinforcement, the moment capacity increase caused by addition of 0.5% and 1.0% 60 mm long fibres with end hooks, were 24.5% and 43.9% for beams made by ordinary vibrated concrete. For SCC-beams, the corresponding capacity increases were as large as 55% and 98%. The explanation is due to favourable fibre orientation and concentration in the latter beams.

In a series of small beams from real structural elements, the section ratio determined by counting the fibres showed a large scatter. Consequently the casting process must be better controlled in fibre reinforced structures than what is common practice for ordinary reinforced concrete. There is a relatively clear relation between the measured section ratio and the experimentally determined residual tensile strength.

The moment capacity of the beams with 1% fibres and ordinary reinforcement is size dependent. A simplified approach, based on average steel fibre stress and smeared cracking, does not fully describe this effect.

Furthermore it seems that the relative ultimate capacity and the ductility of the beams with 1% SFR only both are decreasing with increasing cross section height.

6 ACKNOWLEDGEMENT

The experimental work was financially supported by two Norwegian research project: (1) Steel fibres in concrete structures chaired by the contractor Veidekke ASA, and (2) Self-compacting concrete chaired by the cement producer Norcem ASA.

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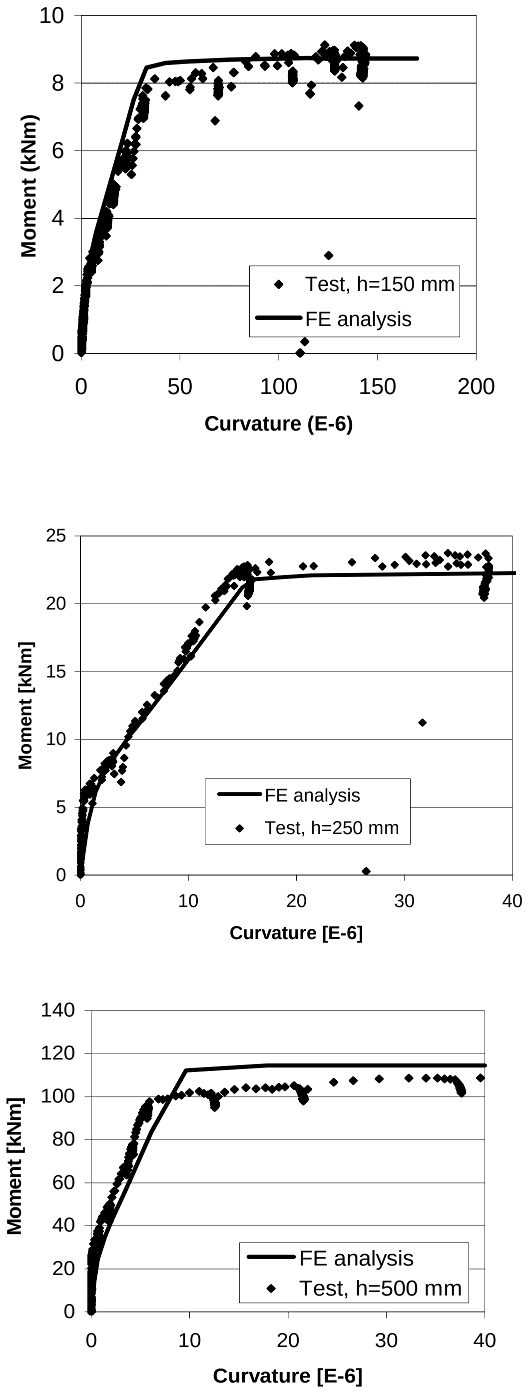


Figure 9. Experimental vs calculated curvature for the beams of different cross section height in series III. 1% steel fibres and approximately 0.5% ordinary reinforcement.

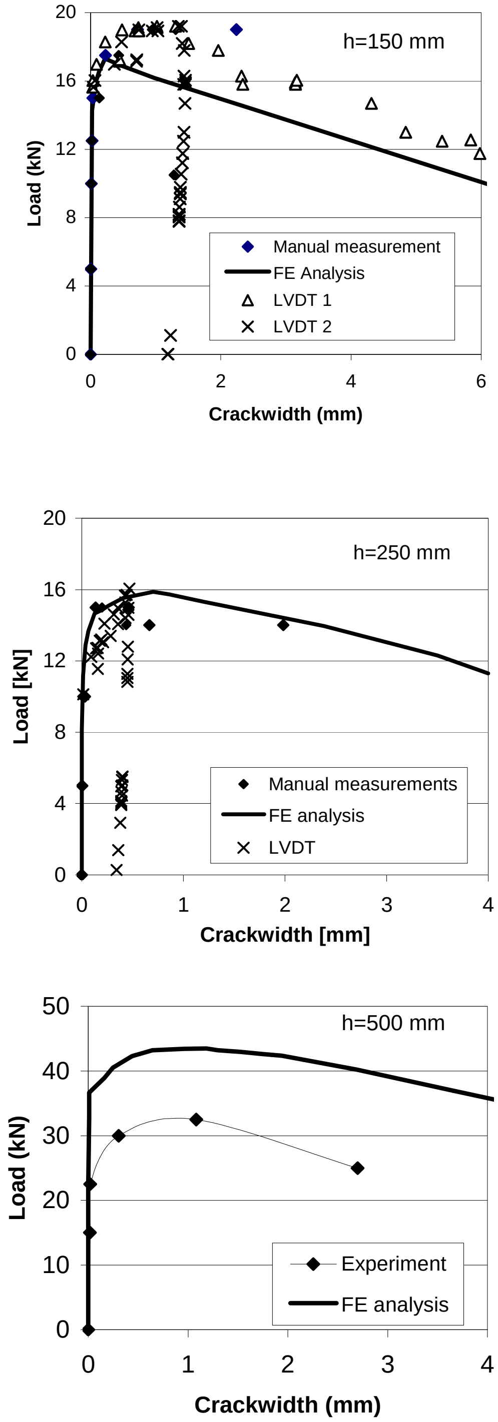
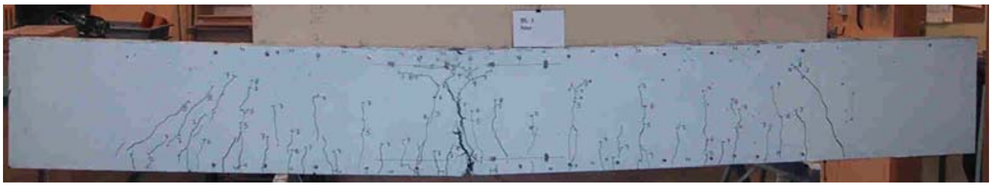


Figure 10. Experimental vs calculated crackwidth development for the beams with different cross section height in series III. 1% steel fibres only.

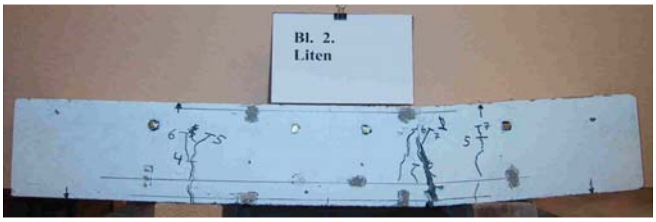
a)

b)

c)



d)



e)

f)

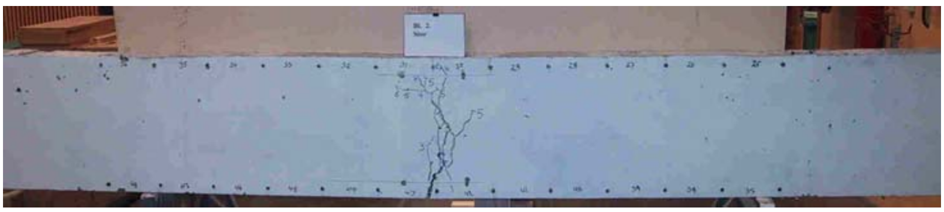


Figure 11. Crack patterns for the beams of series III. (a) h=150, 1% steel fibres and ordinary reinforcement, (b) h =250, 1% sf and ord reinf., (c) h =500, 1% sf and ord reinf, (d) h=150, 1% sf, (e) h=250, 1% sf, (f) h=500, 1% sf.