Determination of concrete fracture toughness from modal dynamic response of notched beams

L. E. T. Ferreira, J. B. Hanai, L. V. Vareda, S. F. Almeida, A. I. Silva.
School of Engineering of São Carlos, University of São Paulo, Brazil

J. L. A. O. Sousa
School of Civil Engineering, Architecture and Urban Planning, State University of Campinas, Brazil

ABSTRACT: Modal dynamic analysis is proposed to evaluate concrete fracture toughness from notched-through beam tests. The dimensionless function of dependence of frequencies on geometry, which leads to identifying the variation of a natural frequency as a function of crack depth, is first determined for the (150 x 150 x 500) mm notched-through specimen. A modal/fracture mechanics approach is used to model the frequency reduction arising from the cohesive interface of the propagating fictitious crack. Therefore, an effective crack length is determined. The computationally obtained length is used to calculate the concrete fracture toughness. The method is applied to test results of high strength concrete. Within the experimental program, the beams are evaluated twice with regards to their frequency responses, before and after an imposed crack. The results proved to be compatible with fracture toughness values obtained from other existing methods. This fact demonstrates that toughness and damage evolution can be conveniently predicted from peak load and dynamic modal responses, within a simple procedure.

1 INTRODUCTION

Applying vibration analysis to the study of structural integrity is nowadays of increasing interest in most areas of engineering activities. This is particularly true regarding the structural monitoring of damage (Salawu 1997). Frequency changes and damage evolution in structures are phenomena which are mutually associated, which indicate that free vibration concepts can be coupled to fracture mechanics to study the crack process, as well as crack properties of different materials.

Moreover, other natural parameters, such as modes shapes, are of interest in globally evaluating the progressive damage process until failure, or in determining the location of damaged zones, such as discrete cracks occurring in structures (Dimarogonas 1996, Chinchalkar 2001, Law & Lu 2005).

Taking this into account, studies of different natures related to vibration responses of cracked elements have been published, most of then concerned with the linear-elastic crack problem (e.g. Ram & Lee 1992, Khiem & Lien 2004).

In the case of quasi-brittle materials, the determination of crack depth as a fundamental fracture parameter in measuring damage severity or crack resistance, can not be directly calculated from linear-elastic fracture mechanic concepts, due to the extent of the inelastic process zone, l_p, occurring ahead of the visible crack tip.

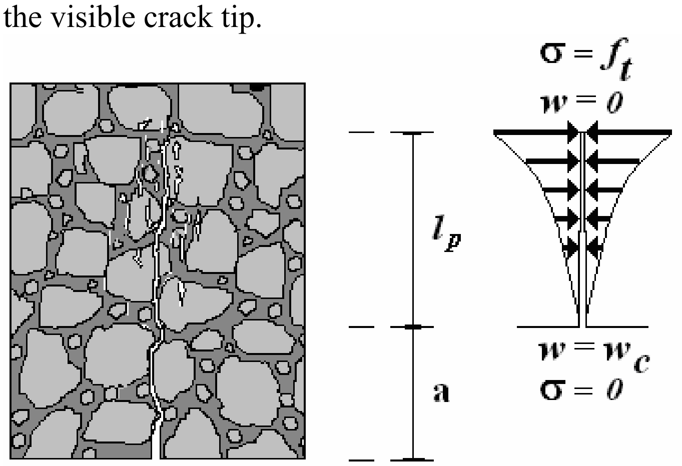

Figure 1. Quasi-brittle crack: inelastic process zone, l_p, ahead of the visible crack tip.

With origins in complex mechanisms, the inelastic processes are sometimes modeled via a simple interface connecting the newly formed crack faces, on which a cohesive stress acts representing the interface's ability to bridge stresses across the crack faces (Hillerborg 1985, Shah et *al*. 1995). The distribution of the cohesive stresses is schematically shown in Figure. 1.

Focusing on the case of a cracked beam from the dynamic viewpoint, different situations need to be examined. Within the linear-elastic case, as crack propagates, i.e., as crack length increases, the natural frequency responses gradually decrease, as a consequence of the successive changes in the specimen's compliance. In this context, both, the modal and the crack problems will be completely determined from the knowledge of the extents of the crack length, at each step of crack propagation.

In the case of quasi-brittle materials, it is expected that the same mechanisms causing softening are also responsible for part of the total reduction in natural frequency responses.

However, the crack length of the propagating crack is not known *a priori*, because the extent of the process zone is also unknown.

This fraction of the overall reduction in frequencies adds the one concerning the increase in the specimen's compliance, which, in turn, comes from the extension of the (free) crack faces, in the linear-elastic sense. These different mechanisms acting on frequency responses of quasi-brittle materials make the coupled modal-fracture mechanics problem non-linear.

However, in fracture mechanics studies of quasi-brittle materials, the computation of crack extensions, in general, follow specific non-linear guidelines in order to calculate effective crack lengths, a_{eff}, (Jenq & Shah 1985, Karihaloo & Nalathambi 1989).

This will enable one to combine both, the modal and the crack problems, into a single non-linear situation within which the main unknown factor will be the effective crack depth, a_{eff}.

This approach is based on the assumption widely accepted within the framework of the quasi-brittle crack theory that the energy dissipation necessary for crack propagation will take place within a preferential plane, or a crack band (Bazant & Oh 1983), especially in the case of the pre-notched beam discussed herein.

In this paper, the modal parameters of a linear-elastic cracked beam, at different crack depths, are firstly determined for a given beam geometry. Subsequently, these parameters are used together with the frequency responses experimentally obtained, before and after imposing cracks to the specimens, to compute effective crack lengths and the fracture toughness of the material.

Finally, the values of fracture toughness obtained with the proposed method are compared with the fracture toughness values obtained from Two Parameter Crack Model methodology (Jenq & Shah 1985).

2 DINAMIC RESPONSE OF A NOTCHED BEAM ELEMENT

The beam element considered in the present discussion is treated as a continuous, linear and conservative system, so that dumping is disregarded. Furthermore, the structural element is thought to vibrate freely.

In the case of a structural element fulfilling the hypothesis of Euler-Bernoulli's beam theory, the frequencies, f_i, associated with the different modes of (free) vibration can be computed with (Blevins 1984):

$$f_i = \frac{\lambda^2{}_i}{2\,\pi\,L^2}\sqrt{\frac{EI}{m}} \qquad (1)$$

where λ_i are dimensionless parameters associated with the different mode shapes which depends on boundary conditions, E is the Young's modulus, m is the mass per unit length of the beam and L is the beam's length.

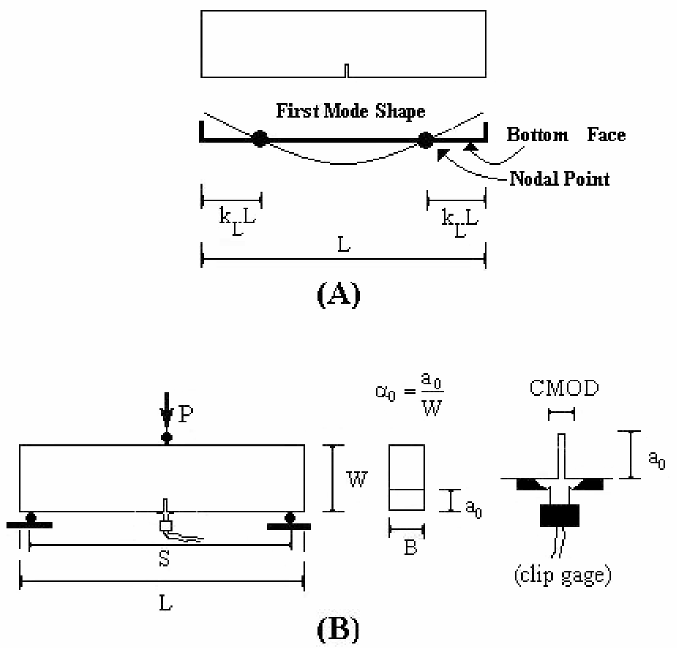

Figure 2. Notched-through beam element: First mode shape, location of the nodal points and loading arrangement.

As a consequence of the approach, transverse shear and rotational inertia effects are not considered. Instead, in this study the assumptions of Timoskenko's beam theory are used.

Denoting the crack length as a, the relative crack length, which will be used in most of the following discussions, is defined as $\alpha = a/W$, where W is the depth of the beam.

To consider the frequency responses of a notched beam element, a dimensionless function of dependence on geometry, $\nu\,(\alpha_0)$, that also takes into account the constant λ_i, is applied to Equation 1. In this case, the transversal frequency, f, corresponding to the lowest mode of vibration will be given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{EI}{ML^3}}\ \nu(\alpha_0) \qquad (2)$$

where M is the mass of the element and $\alpha_0 = a_0/W$ is the normalized notch depth. Young's modulus can be determined from the notched geometry by rearranging Equation 2:

$$E = \frac{4\pi^2 ML^3}{I}\left(\frac{f}{\nu(\alpha_0)}\right)^2 \qquad (3)$$

Considering the cross section shown in Figure 1b, the nominal moment of inertia, I_0, as a function of the initial notch depth, a_0, can be calculated as follows:

$$I_0 = \frac{B(W - a_0)^3}{12} \qquad (4)$$

The substitution of Equation 4 into Equation 3 leads to a general formula that can be used to compute Young's modulus, before crack propagation:

$$E = \frac{48\pi^2 ML^3}{B\,[W(1-\alpha_0)]^3}\left(\frac{f}{\nu(\alpha_0)}\right)^2 \qquad (5)$$

To consider the frequency responses of a cracked beam element at different crack depths, a_i, the dimensionless function of dependence on geometry, $\nu(\alpha)$, concerning the full or almost all the crack path will be needed *a priori*.

Here, the computation of this function is carried out using the finite element method.

3 FINITE ELEMENT ANALYSIS

To study the crack process, a two-dimensional (linear-elastic) free-vibration analysis was carried out using the ANSYS program in plane stress assumption using the lumped mass approximation. The geometry (Fig. 2b) and elastic parameters adopted through the computations are given in Tab. 1.

The beam was modeled with a mesh composed of triangular quadratic isoparametric finite elements. The crack was propagated along the central plane of the beam from a 3 mm wide initial notch.

Table 1 - Geometrical and material parameters used through the simulations (1 daN = 10N ≈ 1kgf).

Lenght L (cm)	Depth W(cm)	Width B(cm)	Notch Depth a_0 (cm)	E (daN/cm^2)	Mass (kg)
50.00	15.00	15.00	2.25	300000.00	28.125

Throughout the computations, different variables affecting the frequency and displacement responses have been studied, namely Poisson's ratio, ν, and the initial notch depth, a_0. In the former case, the values of 0.1, 0.2 and 0.3 have been used.

In the latter, the profundities a_0 of 20.0, 22.5 and 25 mm, which correspond to α_0 values of 0.133, 0.150 and 0.167, respectively, have been investigated.

In each step of the analysis, the crack tip was moved in small steps by consecutive increments Δa of crack advance. Within the analysis, an increment of 5mm was used.

The crack-tip singularity affecting the mode shapes was handled with a rosette of twelve triangular quarter-point elements (radius of 0.05cm) around the "current" crack tip.

3.1 *Dimensionless function of dependence on geometry*

The numerical computations were performed over a relative crack length range of $\alpha_0 < \alpha \leq 0.80$. The equations for the dimensionless functions $\nu(\alpha)$ were determined by fitting 4th degree polynomials to the results numerically obtained and are given by Equation 6.

The coefficients from a through e originated in the nonlinear fittings for different values of ν are given in Tab. 2.

$$\nu(\alpha) = a + b\alpha + c\alpha^2 + d\alpha^3 + e\alpha^4 \qquad (6)$$

For the notched specimen (before crack propagation), the $\nu(\alpha_0)$ values are affected also by the initial notch depth, a_0. The values of these constants are given in Tab. 3, for Poisson's ratios which were studied, so that intermediate α_0 and ν values can be linearly interpolated.

It has been observed in the range investigated, that the variation of the notch depth, a_0, is of importance only for the intact geometry, therefore affecting Equation 5.

After crack propagation, its interference in frequency responses is negligible, due to the small variation of mass concerned with different notch depths, so that for all practical purposes, Equation 6 can be safely used.

Table 2 – Coefficients for the function of dependence on geometry – ν (α)

ν	a	b	c	d	e
0.10	593.1081	416.0307	906.4000	-3163.4058	2992.3754
0.20	593.8430	414.1586	980.7819	-3295.5156	3068.5907
0.30	594.6467	411.1085	1061.9722	-3436.8336	3149.1527

Table 3 – Constant $v(\alpha_0)$ values for different notch depth.

ao	αo	v(αo)		
(cm)		ν = 0.1	ν = 0.2	ν = 0.3
2.00	0.13333	655.599	657.122	658.657
2.25	0.15000	664.835	666.559	668.300
2.50	0.16667	673.701	675.637	677.598

If Equation 2 is used in conjunction with Equation 6, Young's modulus enters in daN/cm^2 (1 daN = 10N ≈ 1kgf) and the linear dimension in cm (centimeters), respectively, to achieve frequency responses of the same order of magnitude of $v(\alpha)$ values.

To use GPa (gigapascal) and m (meter) instead, $v(\alpha)$ values computed with Equation 6 as well as the constant for the notched specimen before crack propagation, will have to be multiplied by 10^3.

3.2 *Position of nodal points*

As expected, the frequency and displacement responses of a beam with a propagating crack will be different for each crack depth considered.

Taking this into account, the whole coupled crack-modal analysis represents a set of studies concerned with bodies of different geometries. Due to this fact, as the crack advances, the nodal points for the vibration mode investigated will move towards the center of the beam.

This moving behavior has been previously discussed and exploited in inverse techniques for sizing and locating cracks in vibrating beams (e.g. Dilena & Morassi 2002].

The location of the nodal points at the bottom face of the beam is also a function of the Poisson's ratio of the material.

Regarding beam ends (Fig. 2a), for each step of crack propagation the nodal point positions are now given as a fraction of the beam's length, L, and Poisson's ratio, ν, and can be computed with Equation 8.

$$L_{NP} = k_L \ L \tag{8}$$

The k_L curves can be expressed with the nonlinear model given by Equation 9, for Poisson's ratios equal to 0.1 and 0.3. For intermediate values of ν, a k_L factor as a function of the normalized crack length, α, can be obtained by linear interpolation.

The coefficients to be used with Equation 9 are presented in Tab. 4.

$$k_L(\alpha) = a + b\alpha + c\alpha^2 + d\alpha^3 + e\alpha^4 + f\alpha^5 \tag{9}$$

Table 4 – Coefficients for the computation of k_L, using Equation 9.

ν	a	b	c	d	e	f
0.1	0.229907	-0.020039	0.358762	-0.859155	0.847116	-0.309700
0.3	0.233444	-0.030502	0.391712	-0.929505	0.912846	-0.330675

The implications of the moving behavior of the nodal points on experimental determination of frequency responses will be discussed in section 5.

4 THE EFFECTIVE APROACH

Young's modulus, E, of the material is first obtained using the frequency response of the notched beam in the "intact state". The specimen is then subjected to the center-point loading configuration shown in Figure 2b.

In most cases, the test can be performed under load control if a very small loading rate is specified, so that the specimen can be safely unloaded at the peak-load, P_{max}. This allows the implementation of the methodology without requiring complex, displacement-controlled load equipment.

Subsequently, a new modal evaluation is performed. Now, the frequency response, f, will be lower, though reflecting the change on beam's compliance associated with crack advance.

The frequency response obtained and the normalized initial notch depth, $\alpha_0 = a_0/W$, are used with Equation 5 to compute a fresh value of E, say, E_i, which will result much lower than the original one.

The relative effective crack length is found by iterating for α in Equation 5 until the condition $E_i \approx E$ is satisfied, within a pre-defined tolerance.

When this condition is fulfilled, the converged value of α will stand for the effective normalized crack length, α_{eff}, which is used to compute the effective crack length:

$$a_{eff} = \alpha_{eff} \ W \tag{10}$$

The fracture toughness, K^m_{IC}, is finally evaluated using the *LEFM* equation for the three-point bend configuration:

$$K^m_{IC} = \frac{1.5\,(P_{\max} + Mg/2)\ S\sqrt{\pi\, a_{eff}}}{B W^2} f(\alpha_{eff}) \tag{11}$$

where P_{max} is the peak-load, M is the mass of the specimen, g is the gravity acceleration and the superscript m stands for "modal".

The dimensionless function of dependence on geometry and boundary conditions, $f(\alpha)$, computed by the authors with FEM for a beam with S/W = 3 loaded in three-point bend, can be calculated by substituting α_{eff} in Equation12:

$$f(\alpha)=60.398928\,\alpha^5-86.787007\,\alpha^4+47.418483\,\alpha^3-8.2347737\,\alpha^2+0.092058453\,\alpha+0.9983671 \quad (12)$$

Many loading-unloading cycles, followed by natural frequency estimations can be performed to evaluate the changes on the beam's compliance. However, a single cycle is sufficient to calculate the fracture toughness.

On the other hand, the test can be performed under crack mouth opening displacement (*CMOD*) control in order to introduce a considerable amount of crack into the beam in a stable fashion. This is archived by monotonically loading the specimen and unloading it at 95% of P_{max}, beyond the peak-load (Shah et *al.* 1995).

In this case, only two load-unloading are strictly necessary for a complete check on fracture resistance.

Since two successive ascendant branches will result from this procedure, the consequent variation on flexural compliance between cycles enables one to use the Two-Parameter Crack Model (Jenq & Shah 1985) to check the K^m_{IC} value obtained from the modal analysis.

5 PRELIMINARY EXPERIMENTAL ANALYSIS

The analytical approach described above has been preliminary applied to the results of tests performed on three concrete beams tested in three-point bend. The span, depth and width of the beams were 450mm, 150mm and 150mm respectively.

The depths and widths of all notches were 3mm and 25mm, respectively.

The material investigated was a high strength concrete with a mix proportion, in mass, equal to 1: 2.13: 1.83 (cement, sand and gravel) with a maximum aggregate size of 19 mm and a water-to-cement ratio of 0.31. Silica fume has been added to this mix in the proportion of 7% concerning the cement's weight.

The mechanical properties obtained at an age of 28 days in axial compression and Brazilian splitting-tension tests of (10x20) cm cylinders were f_c=70.57 MPa and f_t=5.25 MPa, respectively. Young's modulus, *E*, obtained in compression tests was equal to 43.42 GPa.

5.1 *Modal Responses*

Young's modulus, *E*, of the material was first determined using the frequency responses of the notched beams in the intact state. To accomplish this, an accelerometer was placed at the bottom face of the beams, about 20 mm away from the central notch (Borsaikia et *al*. 2006).

The beams were sustained by two nylon wires positioned at the nodal points corresponding to the lowest natural frequency, computed with Equation 8. For this, a Poisson ratio equal to 0.2 was used. The arrangements used are shown in Figure 3.

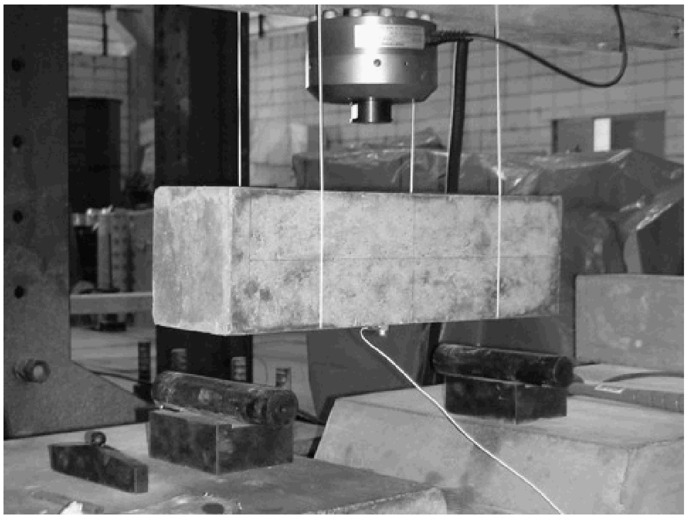

Figure 3. Evaluation of frequency responses of a notched beam in the "intact" state.

At this stage, each beam was excited several times on the opposite side of the accelerometer using a modal hammer and the average frequency responses, individually applied to Equation 5.

Within a second phase of tests, all specimens were subjected to fracture using an INSTRON system in a center-point loading configuration in order to introduce substantial amounts of crack into the specimens.

The specimens were loaded under *CMOD* (crack mouth opening displacement) control up to the peck-loads, P_{max}, and unloaded at about 95% past P_{max}.

New modal evaluations were conducted after complete unloading of the specimens.

Considering that all beams were cracked at this stage and that the correct positions of the nodal points could not be known *a priori* (because the crack depths were not known), all beams were sustained at nodal positions of $\alpha \approx 0.3$, which approximately corresponds to the cracked configuration of a concrete beam loaded in three-point bend.

Supplementary loading and unloading operations were conducted in two specimens (SP2 and SP3), in order to provide information regarding changes to the specimen's compliance, as well as to frequency responses due to the crack growth.

The changes occurring in the specimen's compliance were used to calculate the fracture toughness, K^S_{IC}, using the Two Parameter crack model (Jenq & Shah 1985) to check the fracture toughness values obtained with the proposed model.

6 RESULTS AND DISCUSSIONS

The values of Young's modulus, *E,* calculated with Equation 5 from responses of the intact specimens are presented in Tab. 5.

Table 5 – Young's modulus computed from responses of the intact specimens (1daN=10N≈10kgf).

Specimen	Frequency	α_0	E (intact)	E (intact)
	(Hz)	-	(daN/cm^2)	(GPa)
SP-1	1973.00	0.1667	474874.250	47.487
SP-2	1943.00	0.1667	455527.887	45.553
SP-3	1973.00	0.1667	469703.221	46.970
Mean:			466701.786	46.670
Stand. Dev.:			8178.301	0.818

It has been observed that, for the material tested, the mean value of *E* dynamically obtained (46.67 GPa), was about 7.5% greater than the mean value found with (10x20) cm cylinders in compression tests (43.42 GPa).

For each specimen tested, Equation 5 was used with an increment for α of $1.0\text{x}10^{-8}$ to compute successive values of Young's modulus, E_i, until convergence ($E_i \approx E$). For this, a tolerance of 0.00001 was used (with E given in kN/mm^2). The results obtained are given in Tab. 6.

Table 6 – Fracture toughness computed with the proposed model.

Spc	Frequency		E (converged)	aeff	KIC	KIC
	Initial (Hz)	Final (Hz)	(daN/cm^2)	(cm)	(daN.cm^ -1.5)	(kN.mm^ -1.5)
1	1973.00	1919.00	474874.246	2.920	130.07	0.04113
2	1943.00	1898.00	455527.879	2.862	108.05	0.03417
3	1973.00	1915.00	469703.214	2.948	113.52	0.03590
Mean					117.21	0.03707
Stand. Dev.					9.36	0.00296

The fracture toughness values computed with the methodology of the Two Parameter crack model are presented in Tab. 7 for specimens sp2 and sp3, in view of the fact that the data file containing the reloading path of specimen sp1, was lost during the tests.

Table 7 - Fracture parameters computed with the methodology of the Two Parameter crack model.

Specimen	ac	KIC	KIC
	(mm)	(daN.cm^ -1.5)	(kN.mm^ -1.5)
sp2	25.426	101.511	0.03210
sp3	39.728	136.348	0.04312
Mean:		118.930	0.03761
Stand. Dev.:		24.63	0.00779

7 CONCLUSIONS

In this paper, a new formulation based on the modal dynamic analysis was proposed to evaluate the concrete fracture toughness from notched-through beam tests. A set of dimensionless functions of dependence of frequencies on geometry, was computed for the (150 x 150 x 500) mm notched-through specimen. These functions allowed for the computation of Young's modulus of the material.

A modal/fracture mechanics approach was presented to model the frequency reduction arising from the cohesive interface of the propagating fictitious crack in a quasi-brittle material, therefore allowing the computation of effective crack lengths.

The computationally obtained lengths were used to calculate the fracture toughness of high strength concrete.

Taking into account the quite limited experimental program presented to check the model proposed in this paper, particularly regarding the number of specimens tested, it was observed that the fracture toughness obtained via modal analysis differs in a reasonably way from the data calculated using the Two Parameter Crack Model.

Naturally, a more robust experimentation is needed. Furthermore, an experimental program involving specimens geometrically similar in two dimensions is necessary to have a complete check on the size effect phenomenon.

ACKNOWLEDGEMENTS

The authors would like to thank the continued support provided by FAPESP – Fundação de Amparo à Pesquisa do Estado de São Paulo and CNPQ – Conselho Nacional de Pesquisa e Desenvolvimento Tecnológico, Brazilian Founding Agencies.

REFERENCES

Bazant, Z. P., Oh, B. H. 1983. Crack band theory for fracture of concrete. *Materials and Structures*, 16: 155-177.

Blevins, R. D. 1984. *Formulas for natural frequencies and mode shapes,* Malabar, Robert E. Krieger Publishing Co.

Borsaikia, A., Talukdar, S., Dutta, A. 2006. Study of modal parameters and vibration signatures of notched concrete prisms, *Cement and Concrete Research*, 36: 592 – 598.

Chinchalkar, S. 2001. Determination of crack location in beams using natural frequencies, *Journal of Sound and Vibration,* 247(3): 417-429.

Dilena, M., Morassi, A. 2002. Identification of crack location in vibrating beams from changes in node positions, *Journal of Sound and Vibration*, 255(5): 915-930.

Dimarogonas, A. D. 1996. Vibration of cracked structures: a state of the art review, *Engineering Fracture Mechanics,* 55(5): 831-857.

Hillerborg, A. 1985. The theoretical basis of a method to determine the fracture energy G_F of concrete, *Materials and Structures*, 18, 291-296.

Jenq, Y., Shah, S. P. 1985. Two Parameter Fracture Model for Concrete, *Journal of Engineering Mechanics* ASCE, 111(10): 1227-1241.

Khiem, N.T., Lien, T.V. 2004. Multi-crack detection for beam by the natural frequencies, *Journal of Sound and Vibration,* 273: 175–184.

Karihaloo, B. L., Nalathambi, P. 1989. An Improved Effective Crack Model for the Determination of Fracture Toughness of Concrete, *Cement and Concrete Research*, 19: 603-610.

Law, S. S., Lu, Z. R. 2005. Crack identification in beam from dynamic responses, *Journal of Sound and Vibration*, 285: 967-987.

Ram, T. Y., Lee, T. Y. 1992. Detection of cracks in structures using modal test data, *Engineering Fracture Mechanics*, 42(2): 381-387.

Shah, S. P., Swartz, S. E., Ouyang, C. 1995. *Fracture Mechanics of Concrete – Applications of Fracture Mechanics to Concrete, Rock and Other Quasi-Brittle Materials*, John Wiley & Sons, Inc.

Salawu, O. S. 1997. Detection of structural damage through changes in frequency: a review, *Engineering Structures*, 19 (9): 718-723.