Propagation analysis of fluid-driven fracture using the discrete crack approach

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ABSTRACT: The hydro-mechanical (HM) coupled problem in jointed or susceptible of cracking concrete or rock is formulated by means of the Finite Element Method (FEM) with interface elements of "zero-thickness" and double nodes. Both types of elements (continuum and interface) are formulated in terms of the displacements and the fluid pressure at nodes. After assembly, a highly non-linear system of equations is reached due to the strong dependence between the permeability and the aperture of joints. Poro-elasticity is assumed in the continuum. The cubic law is considered to govern the longitudinal flow along the discontinuity, and the influence of a transversal potential drop is introduced by means of a transversal transmissivity that accounts for the possible existence of an interface skin. Application examples consisting of hydraulic fracture and pressure driven wedge-splitting tests are solved following two numerical strategies: staggered and fully coupled.

1 INTRODUCTION

In the analysis of porous geomaterials such as concrete or rock, the pressure of the fluid filling the pores and the deformation of the porous medium are reciprocally affected, which is known as hydro-mechanical (HM) coupling. An additional degree of coupling appears in fractured porous media, since besides opening or closing, the discontinuity may propagate due to the effect of fluid pressure. These discontinuities may naturally exist in the medium or may develop through hydraulic fracturing phenomena. A hydraulic fracture is a discontinuity that is propagated by a highly pressurized fluid. These fluid driven fractures can naturally appear in structures such as dams or can be human-induced in the injection of slurried wastes, in grouting operations, to increase the productivity of petroleum reservoirs or to measure in-situ stresses in rock masses. HM processes in fractured porous medium are of main concern in aquifers, petroleum reservoirs, waste disposal sites, tunnels or slope stability among others.

From the numerical viewpoint, the solution schemes of the HM problem can be approached either with a staggered or a fully coupled strategy. In the first case, two different codes for the mechanical and flow behavior can be used for each problem in the context of an iterative procedure. In the second (fully coupled) an entirely new code which solves simultaneously both systems of equations is used.

Although extensive literature is available on the HM coupled formulation of porous media, HM models for fractured geomaterials are not so common (e.g. Guiducci et al. 2002), and even less common are the formulations that simulate hydraulic fractures in porous materials. In this case few totally coupled formulations exist (e.g. Simoni & Secchi 2003), whereas in the case of staggered formulations it is common to combine a finite differences (FD) code for the flow problem with a finite element (FE) simulator for the mechanics analysis (e.g. Boone & Ingraffea 1990).

This article describes a FEM formulation for the HM coupled problem in cracks and discontinuities provided the use of zero-thickness interface elements with double nodes, the use of which for mechanical analysis has been well established since some time already (Alonso & Carol, 1985; Gens et al. 1990; Carol et al. 2001), and for diffusion problems it has been recently proved to give reasonable results in standard problems (Segura & Carol 2004). The use of the same joint element for both flow and mechanical problems is numerically convenient. The article finishes with the numerical simulation of two problems: a hydraulic fracture case and a series of wedge splitting tests performed on concrete specimens under the influence of pressurized water that penetrates into the crack (Brühwiler & Saouma, 1995).

2 HM COUPLED EQUATIONS

The problem of fluid flow in a deforming porous medium is extensively treated in literature (e.g. Lewis and Schrefler 1998) and it is briefly mentioned here. The FEM equations that describe the coupled behavior of discontinuities are schematically introduced in this section. For a full and detailed description of the HM interface formulation the reader is referred to Segura and Carol (2007). Small-strain theory, isothermal equilibrium and negligible inertial forces are considered. Positive tension is assumed as in conventional continuum mechanics analyses.

2.1 *Mechanical equations*

The mechanical behavior of a saturated porous material is described through combination of the linear momentum balance equation, the effective stress principle, the constitutive relationship for the solid phase, and the compatibility equation.

The mechanical formulation of a saturated discontinuity is developed in its mid-plane considering the effective stress principle and the constitutive relationship between the stresses and the relative displacements of the discontinuity walls (normal and tangential), which give:

$$\mathbf{D_J} d\mathbf{a} - \mathbf{m_{mp}} dp_{mp} = d\mathbf{f_{mp}} \tag{1}$$

where $\mathbf{D_J}$ is the constitutive matrix, $\mathbf{a}$ is the relative displacement vector, $\mathbf{m_{mp}}$ is a vector that introduces the influence of fluid pressure in the normal direction, p_{mp} is the fluid pressure at the mid-plane and $\mathbf{f_{mp}}$ are force exchanges with the surrounding continuum medium per unit area of discontinuity.

The constitutive relationship is an important aspect of the discontinuity formulation and its choice depends on whether it is a pre-existing discontinuity (rock mechanics, e.g. Gens et al. 1990) or a developing crack (fracture mechanics, e.g. Carol et al. 1997, Caballero et al. 2006).

2.2 *Hydraulic equations*

The mass balance equation for water is combined with the general form of Darcy's law to describe the hydraulic behavior of the porous medium under the influence of the solid skeleton deformation.

In the case of flow in a saturated discontinuity, the same equations are posed along its mid-plane, leading to:

$$\operatorname{div}\left[\frac{\mathbf{T_l}}{\gamma^f}\left(\operatorname{grad} p_{mp} + \gamma^f \operatorname{grad} z_{mp}\right)\right] + q^+ + q^- = \frac{\partial a_n}{\partial t} \tag{2}$$

which is an analogous formulation to the one developed in Segura & Carol (2004), and where p_{mp} is the fluid pressure at the mid-plane, a_n is the joint aperture, q^+ and q^- are leakage fluxes from the surrounding porous medium, and $\mathbf{T_l}$ is the longitudinal transmissivity tensor of the discontinuity, which is highly dependent on the aperture and can be approximated through the cubic law (Snow, 1965). The cubic law basically assumes that flow along a discontinuity is laminar and occurs between a couple of smooth parallel plates, which makes the transmissivity proportional to the cube of the discontinuity aperture:

$$T_l = \frac{g}{12\nu} a_n^{\,3} \tag{3}$$

where g is the gravity acceleration and ν is the fluid kinematic viscosity.

A fluid pressure drop between the two discontinuity walls (i.e. transversal to the discontinuity) is also considered. This pressure drop is related to a transversal flow through a transversal conductivity coefficient K_t (Segura & Carol, 2004):

$$q_t = K_t \Delta p \tag{4}$$

where Δp is the pressure drop between both sides of the discontinuity.

2.3 *FEM formulation*

The HM coupled equations are discretized in space using the FEM and considering the nodal displacements ($\mathbf{u}$) and nodal fluid pressure ($\mathbf{p}$) as the main global unknowns. Standard finite elements are used for the continuum porous medium, and zero-thickness interface elements with double nodes are used to properly discretize and represent the HM behavior of each pre-existing discontinuity or developing crack (i.e. discrete crack approach).

In order to extend the formulation of the interface element from the mid-plane to the real element nodes located at the walls, it is assumed that the fluid pressure in the mid-plane of the joint is the average of the fluid pressure at the boundaries:

$$p_{mp} = \frac{p_u + p_l}{2} \tag{5}$$

where the subscripts u and l stand for upper and lower discontinuity walls. This assumption has proved to be reasonable in a standard diffusion problem in Segura & Carol (2004) in comparison with other interface elements for flow problems, although in some special occasions a triple-nodded interface element (Guiducci et al. 2002) would be necessary.

Following standard FEM procedures one can reach similar expressions for the continuum and the interface elements. After assembly, the following equation is obtained (Segura & Carol, 2007):

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{p} \end{bmatrix} + \begin{bmatrix} -\mathbf{K} & \mathbf{Q} \\ \mathbf{Q}^\mathrm{T} & \mathbf{S} \end{bmatrix} \begin{bmatrix} d\mathbf{u}/dt \\ d\mathbf{p}/dt \end{bmatrix} = \begin{bmatrix} -d\mathbf{f}^\mathbf{u}/dt \\ \mathbf{f}^\mathbf{p} \end{bmatrix} \tag{6}$$

where **u** and **p** are the displacements and fluid pressure at nodes, **E** is the permeability matrix, **Q** is the coupling matrix, **S** is the compressibility matrix, **K** is the stiffness matrix, and $\mathbf{f}^{\mathbf{u}}$ and $\mathbf{f}^{\mathbf{p}}$ are the right hand side force and flow vectors that include the gravity action as well as the influence of distributed loads and flows at the domain boundaries.

This set of equations is discretized in time using the finite differences method; the values of **u** and **p** are linearized within a time step:

$$\begin{aligned}\mathbf{u}_{n+\theta} &= \mathbf{u}_n + \theta\,\Delta\mathbf{u}_{n+1}\\ \mathbf{p}_{n+\theta} &= \mathbf{p}_n + \theta\,\Delta\mathbf{p}_{n+1}\end{aligned} \tag{7}$$

where the value of θ determines the time integration scheme. The following equation is reached, which determines the nodal displacements and fluid pressure at a given time step $n+1$ relative to their values at previous time-step n:

$$\begin{bmatrix} -\mathbf{K} & \mathbf{Q} \\ \mathbf{Q}^{\mathbf{T}} & \mathbf{S}+\Delta t\theta\mathbf{E} \end{bmatrix}_{n+\theta} \begin{bmatrix} \Delta\mathbf{u} \\ \Delta\mathbf{p} \end{bmatrix}_{n+1} = \begin{bmatrix} -\mathbf{F}^{\mathbf{u}} \\ \mathbf{F}^{\mathbf{p}} \end{bmatrix} \tag{8}$$

3 NUMERICAL STRATEGIES

As already mentioned in the introduction, several numerical techniques can be employed to solve the system of equations (8). One option is to solve it simultaneously, what is known in the literature as a monolithic or fully coupled strategy. Due to the highly non-linearity introduced by the dependence of the permeability matrix on the cube of the displacements, advanced iterative techniques may be needed.

The staggered procedure splits the system of equations (8) into two equations to be solved in a staggered manner:

$$\mathbf{K}\,\Delta\mathbf{u}_{n+1} = \mathbf{F}^{\mathbf{u}} + \mathbf{Q}\left(\Delta\mathbf{p}_{n+1}\right)^{k} \tag{9}$$

$$\left(\mathbf{S}+\Delta t\theta\mathbf{E}\right)\Delta\mathbf{p}_{n+1} = \mathbf{F}^{\mathbf{p}} - \mathbf{Q}^{\mathbf{T}}\left(\Delta\mathbf{u}_{n+1}\right)^{k} \tag{10}$$

where $(\Delta\mathbf{u}_{n+1})^k$ and $(\Delta\mathbf{p}_{n+1})^k$ are the predictors of the solution at iteration k. Of course $(\Delta\mathbf{u}_{n+1})^k$ is the last solution available of (9) , and $(\Delta\mathbf{p}_{n+1})^k$ is the last available solution of (10). Provided the introduction of the appropriate coupling loops, the solution of the coupled system is obtained by iteratively solving equations (9) and (10) with a geomechanical code and a fluid flow simulator respectively until a certain tolerance on the solution is satisfied for each time step.

4 APPLICATIONS

The features and capabilities of the formulation are illustrated with two examples: a hydraulic fracture problem and a series of wedge splitting tests performed on concrete specimens and that introduce the effect of fluid pressure on the propagating crack (Brühwiller & Saouma, 1995).

The staggered strategy makes use of the codes DRAC and DRACFLOW developed at the UPC Geotechnical Engineering Department and which follow the discrete crack approach to respectively solve the mechanical and the hydraulic problems in fractured medium. A new code, based on the two previously named, has been built up to solve the system of equations (8) simulataneously (i.e. fully coupled approach).

The following predictor (Saetta et al. 1991) is used for the pressure field in the staggered procedure:

$$\mathbf{p}^{k+1} = \mathbf{p}^{k} + \gamma\left(\mathbf{p}^{k+1} - \mathbf{p}^{k}\right) \quad \text{where} \quad 0 \le \gamma \le 1 \tag{11}$$

where the subscript k stands for iteration.

Interface mechanical behavior is reproduced by means of a work-softening elasto-plastic constitutive law that incorporates two fracture energies G_f^{I} and G_f^{IIa}. This constitutive model is described and analyzed in detail in Carol et al. (1997), and has been used in many other analyses (e.g. Carol et al. 2001, Caballero et al. 2006) involving 2D and 3D crack opening and propagation. Fluid flow behavior along the opening discontinuity is assumed to follow the cubic law.

4.1 *Hydraulic fracture phenomenon*

The first problem analyzed (Boone & Ingraffea 1990) consists of injecting fluid along an incipient crack located at the lower left corner of the domain (Fig. 1). This produces the propagation of a fracture along the lower boundary, where the zero-thickness interface elements have been inserted. As a preliminary analysis to the transient case, the problem is studied in steady-state conditions, i.e. analyzing which would be the ultimate length of a fracture for a given flow injection that is progressively increased.

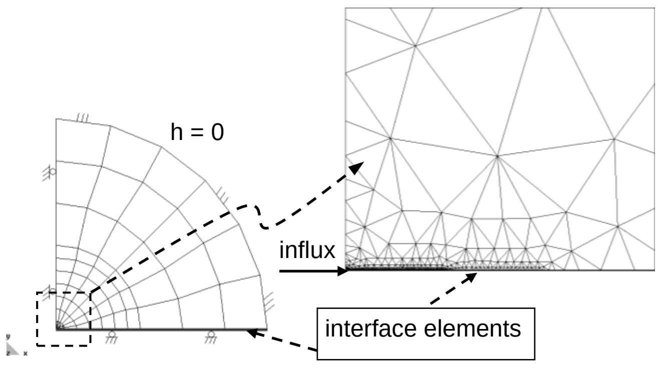

Figure 1. Scheme of the hydraulic fracture problem.

The two numerical strategies (staggered and fully coupled) are used, reaching very similar results. However, it has to be noted that the convergence of the staggered procedure needs considerable iterations in advanced stages of the analysis. Therefore, a fully coupled formulation of the problem would seem more desirable.

Figure 2 shows the development of the crack as the injected fluid flow is increased. Figure 3 shows how once the fracture has reached a sufficient development, the pressure profiles at the injection zone change and the fluid enters the porous medium along all the open fracture length. At the same time, and in connection to that, the fluid potential drop along the open fracture is very low.

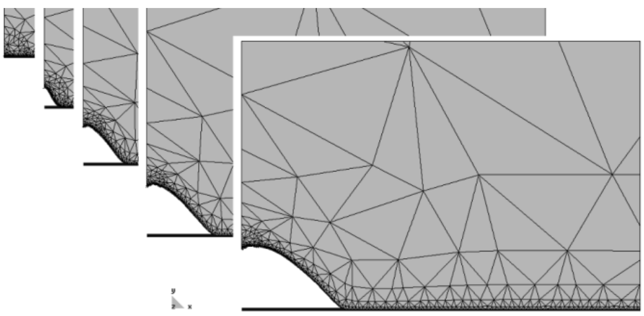

Figure 2. Fracture propagation (deformed mesh).

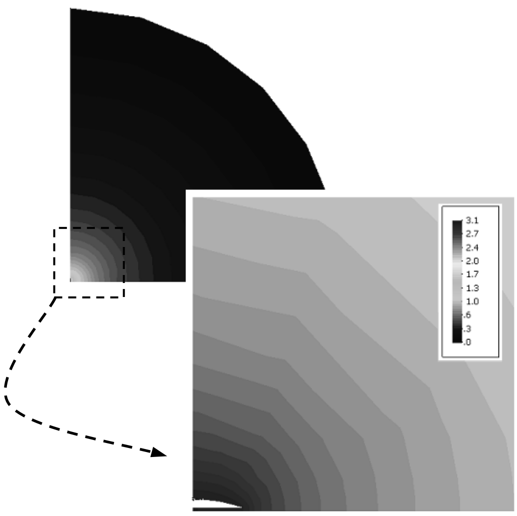

Figure 3. Hydraulic head (m) profiles.

4.2 *Fluid pressurized wedge splitting tests*

The fully coupled approach is used to reproduce a series of wedge splitting tests under the effect of pressurized fluid within a developing crack shown in Brühwiller & Saouma (1995). This series of tests was continued and further studied by Slowik & Saouma (2000), who take into account the influence of other factors such as the loading rate.

This work concentrates on the former experiments by Brühwiller & Saouma (1995), but a more extensive discussion including also comparison with the experimental results by Slowik & Saouma (2000) can be found in Segura & Carol (2007).

The experiment consists in simultaneously applying two independent loads in the concrete specimen: a mechanical splitting force (F), and an internal water pressure (σ_w) applied through the injection of fluid at the mouth of the notch. This experiment has its application on dam engineering, and it reproduces the effect of the external hydrostatic water pressure acting on the upstream face of a dam (F) and the effect of the fluid pressure that penetrates into the crack (σ_w). The fluid pressure at the notch is kept constant during the experiment at σ_{w0}. The concrete specimens geometry is shown in Figure 4.

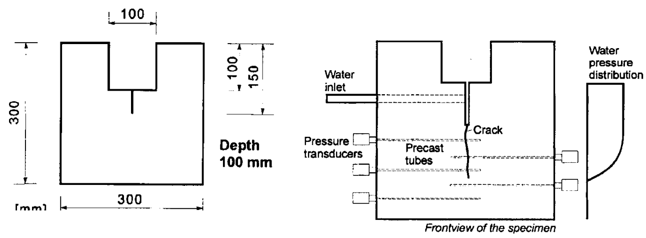

Figure 4. Experiment geometry for the wedge splitting tests (Slowik & Saouma, 2000).

The experimental results depicted in Figure 5 show that as the fluid pressure at the notch is increased, the maximum splitting force values ($F_{s,max}$) and the corresponding crack mouth opening displacement at $F_{s,max}$ both decrease as it would be expected, since the fluid pressure is helping to split the specimen. The descending branch is also steeper as the fluid pressure is increased. The real fracture properties are determined from the reference curve (i.e. with no fluid pressure). The remaining curves provide apparent fracture toughness and fracture energy, which decrease as the applied fluid pressure increases. These apparent properties could be used in a purely mechanical analysis to account in an indirect manner for the effect of fluid pressure. However, if a coupled analysis is performed, the reference properties of the material should be used, and the effect of water pressure should be naturally captured by the hydro-mechanical coupled model.

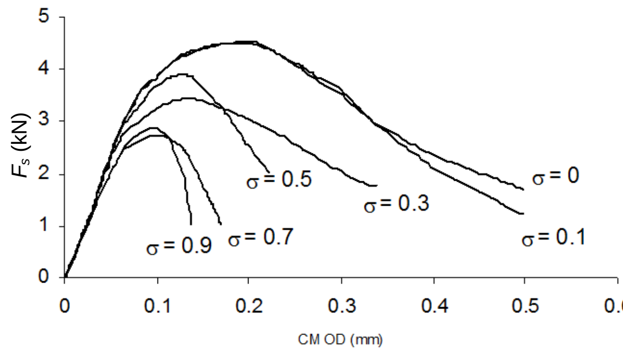

Figure 5. Splitting force against CMOD experimental curves as function of hydrostatic pressure (Slowik & Saouma, 2000)

Figure 6 shows the numerical results obtained with the HM coupled approach described in previous sections. Comparing these results with Figure 5, we can see how the model captures the effect of fluid pressure on the mechanical failure of the specimen.

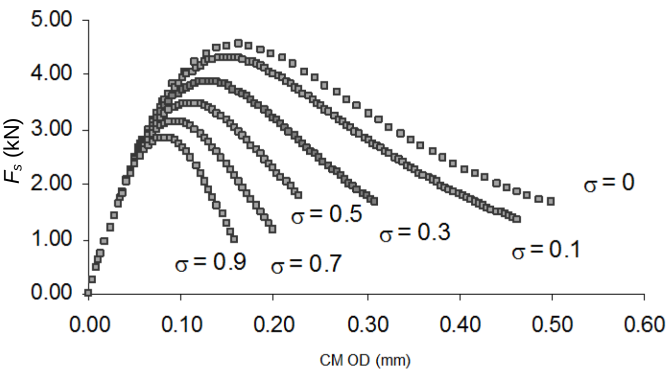

Figure 6. Splitting force against CMOD numerical curves as function of hydrostatic pressure.

5 CONCLUDING REMARKS

The HM coupled problem in jointed or susceptible of fracturing porous materials has been formulated by means of the Finite Element Method with interface elements of "zero-thickness" and double nodes to discretize and describe the HM behavior of discontinuities. Poro-elasticity is assumed in the porous medium, which is discretized with standard finite elements. The main unknowns for both types of elements (continuum and interface) are the fluid pressure and the displacements at the nodes. The double-node interface element also incorporates a transversal potential drop trough the discontinuity.

Two different numerical strategies are analyzed to solve the system of equations that govern the problem: the fully coupled and the staggered approaches.

Preliminary results on fluid-driven fracture have been obtained, which satisfactorily reflect the physics of the process. The fully coupled approach has been used to properly describe the failure of concrete specimens under the influence of fluid pressure.

More details on the work described herein, as well as additional examples may be found in Segura & Carol (2007).

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