# Influence of steel fibers on full-scale RC beams under shear loading

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ABSTRACT: The use of Fiber Reinforced Concrete (FRC) has gained considerable attention in the last few years, particularly when crack propagation control is of primary importance, such as in slab-on-grade applications or in beams when shear reinforcement is partly or totally absent. Many experiments available in the literature showed that fibers, if provided in sufficient amount, are significantly effective as shear reinforcement. Fibers limit the growth of shear inclined crack, give visible warning prior the structure collapse and also provide a stable and diffused crack pattern within the shear critical area. However, the issue of size effect in members containing steel fibers, has not been deeply investigated and evaluated yet. This paper studies the beneficial influence of fibers on the size effect in members without conventional shear reinforcement. Moreover an extensive parametrical numerical study, performed by means of a FE program based on the Modified Compression Field Theory (MCFT), suitably adapted to FRC materials, is herein presented.

## 1 INTRODUCTION

During the last few decades many tests investigating shear behavior have been carried out on relatively small beams. It was found that the results of these tests can not be directly extended to full size beams. It was shown by Kani (1967) that there is a very significant size effect on the shear strength of members without transverse reinforcement. The shear strength of these members appears to decrease as the effective depth increases. Shioya et al. (1989) reaffirmed this fact and extended the available data to beams with a depth up to 3000 mm. Figure 1 shows that the average shear stress to cause failure of the largest beam was about one-third the average shear stress to cause failure of the smallest beam.

There is a general agreement that the main reason for this size effect is the larger diagonal crack width in larger beams. On the other hand, there is a disagreement concerning the modeling of this phenomenon, which has prompted a strong debate within the research community (Bentz, 2005).

A different approach was proposed by Bažant & Kim (1984), who stated that the most important consequence of wider cracks is the reduced residual tensile stress and aggregate interlock. Most of current formulas for predicting shear capacity of a member are in fact based on the concept of tensile strength.

However, this concept is theoretically justified only in the case of ductile failures governed by the theory of plasticity. For failures in which the stress decreases after reaching the strength limit, as in the case of tensile cracking, the strength concept is inconsistent when applied in a continuum analysis.

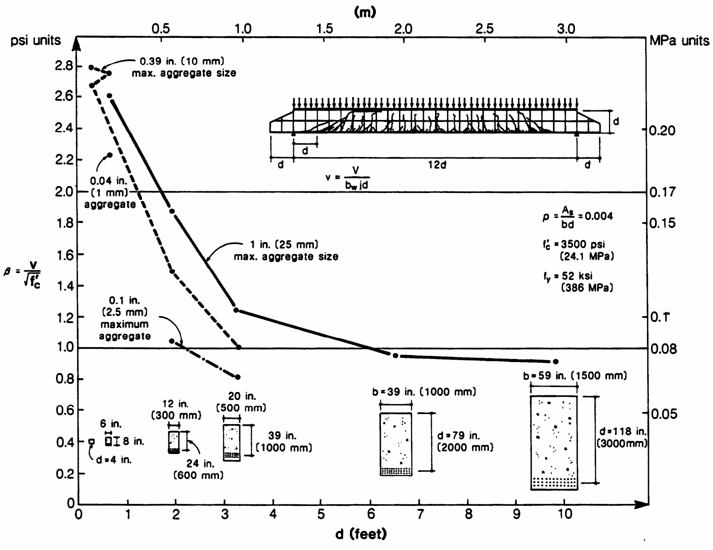


Figure 1: Shear stress at failure; results from Shioya (1989).

New approaches do not treat fracture like a point phenomenon, but recognizes that in brittle heterogeneous materials such as concrete, the fracture propagates with a relatively large fracture process zone in which progressive microcracking gradually reduces the tensile stress to zero.

Bažant & Kim (1984) investigated also the consequences of this nonlinear fracture mechanics approach for diagonal shear failure and proposed a size reduction factor for shear members (Bažant & Kim, 1984). The model represents a gradual transition from the strength criterion for small structures to the linear elastic fracture mechanics for very large structures.

Bažant & Cao (1986) and Bažant & Kazemi (1991) confirmed the significant agreement between nonlinear fracture mechanics and experimental evidence for prestressed concrete beams and a wide series of tests on reinforced concrete beams, with a size range being of 1:16. They also suggested important adjustments to the current design codes accounting for size effect coefficients.

Other researchers believe that wider cracks reduce the ability to transmit crack interface shear stresses. The crack width is scaled like any other dimension of the member. The crack opening in the middle of the web is controlled by the strain of the longitudinal reinforcement, but this reinforcement is too far away to control the crack spacing. With the larger crack width, the friction of the shear forces is reduced and, consequently, the ultimate capacity is reduced as well.

As the crack spacing used to determine the shear stress at the limiting crack interface is a function of the specimen depth, no special factor is required to account for the size effect (Collins et al., 1996) Perhaps the strongest argument for this latter approach is that it leads to a consistent treatment of members with different arrangements of longitudinal reinforcement.

Tests of Kuchma et al. (1997) and of Frosch (2000) have demonstrated that the size effect is not significant in beams with web reinforcement and in beams without stirrups containing well distributed longitudinal reinforcement (all along the depth).

Other size effect laws were calibrated and published by Okamura & Higai (1981) and Reineck (1991), who considered the tests of Shioya et al. (1989) and, therefore, represent reasonable lower bound.

The analysis of Zararis (2001) shows that the diagonal shear failure in slender beams is due to a splitting of concrete that takes place in a certain region of the beam. It was realized that in this section the problem of the size effect on diagonal shear failure can be reduced to the problem of size effect on split-tensile failure. Tests on cylindrical disks of constant thickness done by many researchers (i.e. Sabnis & Mirza, 1979) confirmed the existence of size effect on split-tensile failure, and showed that up to a certain critical diameter the split-cylinder strength decreases as the diameter increases (ASCE-ACI Committee 445, 1998).

Recently, an interesting campaign was carried out at the University of Toronto with the purpose of investigating the safety of the shear design of large, wide beams (Lubell et al., 2004). The beam tested exhibited a brittle shear failure, typical for high-strength concrete beams, with a loud noise as the mid-span load reached 2440 kN. This failure load was only 52% of the failure load predicted by the ACI shear provisions, meaning that the beam would fail under the actual service loads. The maximum crack width measured was only 0.25 mm while the midspan deflection at failure was less than 1/500 of the span.

The Authors concluded that the ACI design procedure is currently unsafe, as it neglects the size effects in shear. On the contrary, the MCFT (Bentz et al., 2006) design procedures predicts that, if the depth of a member is doubled, the shear capacity will be less than double and that this strength ratio will become smaller as the size of the beam increases and as the aggregate size decreases.


Figure 2: Shear failure of a deep beam tested at the University of Toronto.

The ACI 318-02 shear design expression for members without stirrups can be very unconservative not only because it was based on test results from beams that had rather small depths, but also because those test beams typically had very large amounts of longitudinal reinforcement ($\rho>2\%$) to avoid any possibility of flexural failures.

Recent advancements in material technologies and productions, as well in standardization, provide new materials to introduce in the design process. A very promising material, easily available in the market, is Fiber Reinforced Concrete (FRC): it is characterized by a significant toughness enhancement, especially under tension. Fiber reinforced concrete was shown to give rather good performance in shear-critical beams (Minelli, 2005) and in other structural elements, as a valuable and effective substitution of secondary reinforcement (Di Prisco & Ferrara, 2001; Meda et al., 2005; Minelli et al., 2005).

In the work of Minelli & Plizzari (2006), a modification of the current EC2 (2003) equation for members not containing stirrups under shear was proposed for FRC, with a good agreement against more than 30 experimental results taken from the database of the University of Brescia. Among the principal assumption of such modeling, fibers were treated as reinforcement spread over the entire depth of a member, giving a shear contribution which is similar to that offered by small diameter bars placed all along the beam, as previously shown (Kuchma et al., 1997).

In other words, fibers can effectively reduce the size effect issue in reinforced concrete members not containing stirrups, if provided in sufficient amount and giving significant toughness and energy dissipation ability to the matrix.

The aim of this paper is to discuss this critical issue which has not been properly treated yet in the scientific literature.

Firstly, a number of experimental results on full scale shear-critical beams will be presented, focusing with special emphasis on recent tests conducted on 1 m deep members not containing stirrups. The effect of fibers and of the minimum amount of transverse reinforcement, as stated by EC2, will be compared against the response of identical plain concrete members. Secondly, a set of numerical analyses, carried out using a finite element program based on the compression field model procedures, will be presented, showing the promising effect of steel fibers in reducing the scale effects.

The aim of the ongoing research at the Universities of Brescia and Toronto is to develop a simple, although rational and reliable, procedure allowing engineers to incorporate SFRC in the design process.

## 2 EXPERIMENTS

### 2.1 *Materials and Specimen Geometry*

Among the 47 shear tests carried out at the University of Brescia since 2001 (most of them can be found in Minelli, 2005), the following discussion will focus on those experiments conducted to assess the influence of the minimum shear reinforcement, represented either by classical transverse reinforcement or steel fibers, and the size effect in shear.

Two series of specimens are presented herein: the first refers to sample beams having a total depth of 500 mm (“Small Size Specimens”), while the second consists of elements 1000 mm deep (“Large Size Specimens”).

Concerning the first set of experiments, five shear-critical beams loaded with a three point loading system having a shear span-to-depth ratio of 2.5 (which is recognized to be the most critical in terms of shear strength (Kani, 1967) were tested. All beams had the same geometry aimed at analyzing the effect of the addition of a randomly distributed fibrous reinforcement to concrete.

A beam depth of 500 mm was chosen, with a gross cover of 45 mm. The beam spanned 2280 mm, while the overall length of the specimen was 2400 mm. Two deformed longitudinal bars, having a diameter of 24 mm, were present in each specimen, corresponding to a reinforcement ratio of 1.04%.

Deeper beams (Large Size Specimens) were cast with a total depth of 1000 mm, an effective depth of 910 mm, a width of 200 mm and a span of 4550 mm (Figure 3). A three point loading scheme was chosen resulting in a span-to-depth ratio again of 2.5. The steel reinforcement (6ϕ20 mm rebars) was located in two identical bottom layers giving a reinforcing ratio of 1.03%.

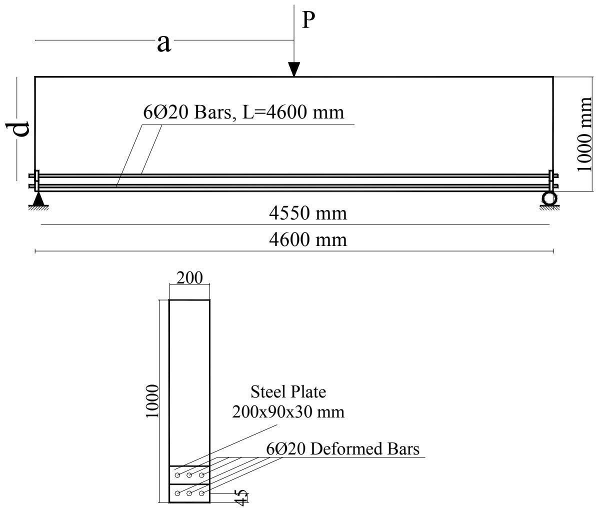


Figure 3: Geometry of Large Size Specimen.

Table 1 summarizes the geometrical characteristics of the beams. All smaller specimens were cast by using a normal strength concrete (NSC), while two different series of larger beams were tested, the first one cast in the same batch as the smaller beams, and the second using a high strength matrix (HSC). Table 2 reports the composition of the two different concrete batches.

Table 1: Geometry characteristics of specimens.

| | H=500 mm | H=1000 mm |
|---|---|---|
| Total Length | 2400 mm | 4600 mm |
| Span | 2280 mm | 4550 mm |
| Shear Span a | 1140 mm | 2275 mm |
| Width | 200 mm | 200 mm |
| Total Depth | 500 mm | 1000 mm |
| Gross Cover | 45 mm | 90 mm |
| Effective Depth d | 455 mm | 910 mm |
| Reinforcement Area | 905 $mm^2$ | 1884 $mm^2$ |
| Reinforcement Ratio | 1.04% | 1.03% |

One of the Small Specimens was cast without any transverse reinforcement (PC-50), two with the minimum amount of transverse reinforcement (MSR-50, with stirrups 2ϕ8@300mm) as required by EC2 (2003), and two with 20 $kg/m^3$ of steel fibers (FRC-50) having a length of 50 mm and a diameter of 1 mm (aspect ratio l/ϕ=50).

The fracture properties of FRC were determined according to the Italian Standard (UNI 11039, 2003), which requires bending tests (4PBT) be performed on small beam specimens (150x150x600 mm).

The equivalent post-cracking strengths related to the SLS and ULS were equal to $f_{eq(0-0.6)}$=2.53 MPa and $f_{eq(0.6-3)}$=2.50 MPa, respectively.

Three Large Size Specimens were cast for each concrete strength (NSC and HSC): the reference element (PC-100), the sample containing the minimum amount of shear reinforcement (MSR-100, with stirrups 2ϕ8@650mm) and the latter containing 20 kg/m$^3$ of hooked steel fibers (FRC-100). Note that the notation PC refers to a beam cast with plain concrete, whereas FRC always indicates a beam with fibers as the only shear reinforcement. Finally, MSR refers to minimum shear reinforcement specimen.

One should note that the fiber amount of 20 kg/m$^3$ is quite low; some structural (e.g., CNR DT 204, 2006) do not allow for the usage of a such a low fiber content. However, aim of the present comparison is the evaluation of steel fibers as a minimum transverse shear reinforcement and how one can achieve a good shear response with a relatively low fiber content. The Authors, in other words, decided to use a low fiber content to enforce their study toward an extensive and, at the same time, economical utilization of FRC.

Table 2: Mechanical properties of concrete.

| | NSC | HSC |
|---|---|---|
| Cement Content [kg/m$^3$] | 345 | 380 |
| Maximum Aggregate Size [mm] | 20 | 20 |
| Plasticizer [l/m$^3$] | 5.2 | 3.8 |
| Compressive Cubic Strength [MPa] | 25.7 | 55 |
| Elastic Modulus [GPa] | 31 | 37 |

## 2.2 *Experimental Results*

In the following, the main experimental results (from Figure 4 to Figure 8) are presented and discussed. With regard to the small size specimens (H=500), Figure 4 plots the load-displacement curve of the five experiments and Figure 5 shows the corresponding shear-crack width vs. load curve.

The specimen made of plain concrete (PC-50) showed the well known brittle shear collapse, characterized by early diagonal cracking and unstable propagation for a low load level. No ductility after cracking nor visible warning were detected.

On the contrary, the two elements made of FRC as well as the two cast with the minimum amount of stirrups showed visible cracking with a stable propagation, accompanied also by a not negligible increase in bearing capacity; in fact, specimen FRC-20-2 doubled the ultimate load and showed a displacement at failure four time bigger than the reference sample (PC-50). Also the plot of the main shear crack (Figure 5) confirms the role played by a minimum amount of transverse reinforcement, constituted either by fibers or stirrups.

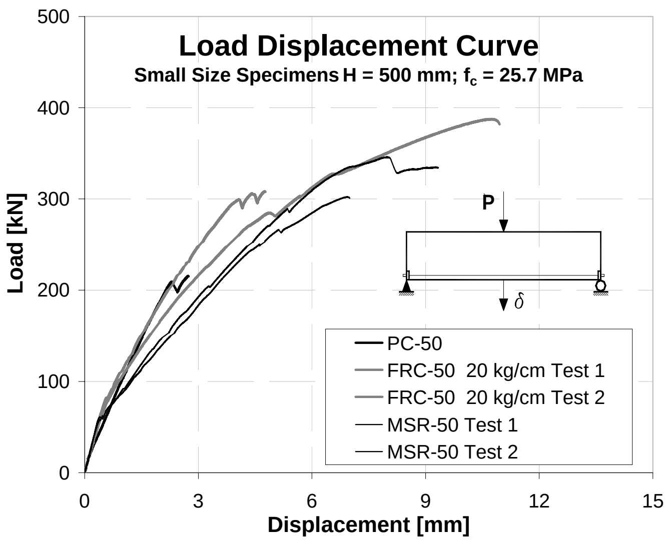


Figure 4: Load-Displacement Curve; series H=500 mm.

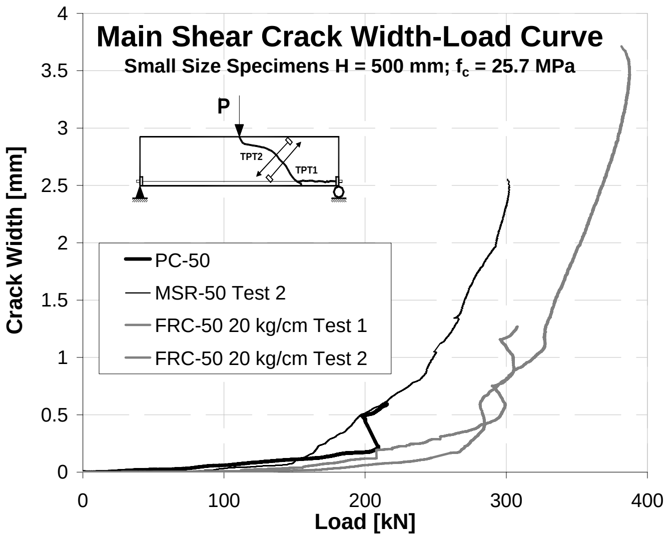


Figure 5: Shear Crack Width-Load Curve; series H=500mm.

As diffusely reported in the literature, the primary role of the minimum shear reinforcement is to limit the growth of inclined cracks, to improve ductility, and to ensure that the concrete contribution to shear resistance is maintained at least until yield of the shear reinforcement. In other words, it is commonly agreed that, before failure, the R/C structure must give a warning by cracking and visible deflection; this represents the requirements for the minimum reinforcement ratio. Beams that do not contain web reinforcement may fail in a relatively brittle manner immediately after the formation of the first diagonal crack (Figure 5).

Moreover, one should keep in mind that the shear capacity of such members can be substantially reduced by factors such as (1) repeated loading which propagates existing cracks and lowers the apparent tensile strength of concrete; (2) tensile stresses caused by restrained shrinkage strains; (3) thermal strains or creep strains; stress concentrations due to discontinuities such as web openings, (4) termination of flexural reinforcement, or (5) local deviation of tendon profiles (Collins & Mitchell, 1993).

Such reinforcement can be omitted if there is no significant chance of diagonal cracking. It can also be omitted if the member is of minor importance, or if the member is part of a redundant structural system that allows substantial redistribution and, hence, will show adequate ductility.

A low amount of fibers provides all the aforementioned benefits, being a valuable and economical alternative to the traditional stirrups, whose handling and placing can be expensive especially when dealing with precast beams or, in general, with structural elements characterized by non-rectangular or square cross-sections. FRC gives the same structural response of members containing the minimum transverse reinforcement.

Figure 6 and Figure 7 show the load-displacement curves for the large size specimens, respectively for the NSC series and HSC series.

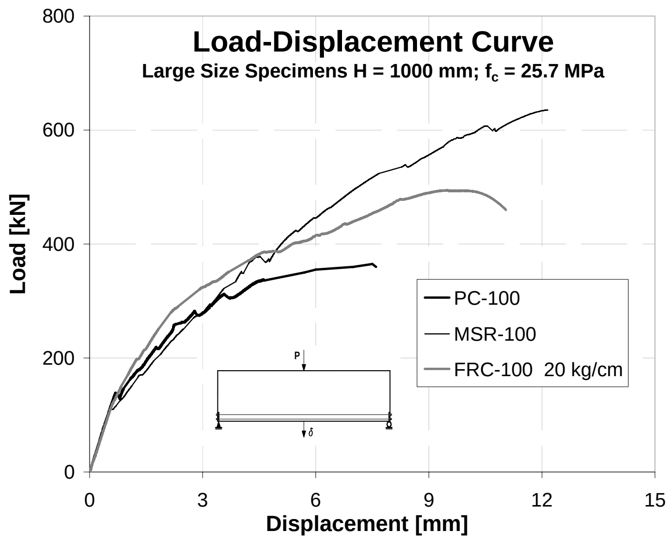


Figure 6: Load-Displacement Curve; series 1 (H=1000 mm).

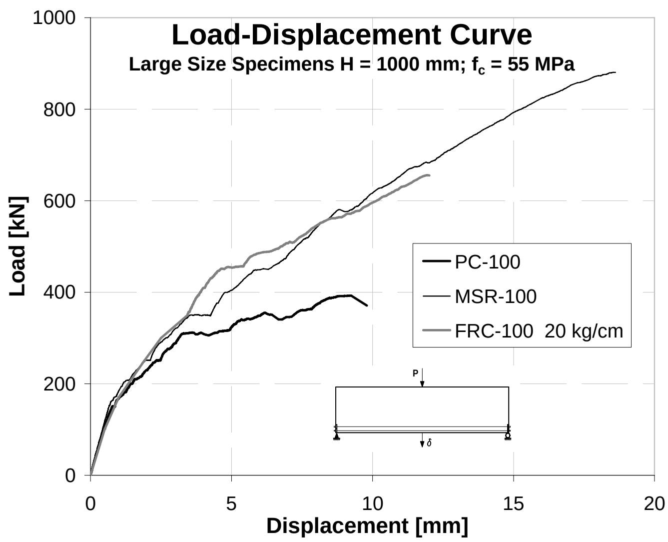


Figure 7: Load-Displacement Curve; series 2 (H=1000 mm).

Figure 8 exhibits the width of the main shear crack vs. the load for the HSC series. Note that the main shear crack is the average of 6 measurement performed in both shear spans, as recalled in the plot of Figure 8.

Differently form the small size specimens, the traditional shear reinforcement turned out to be significantly more effective than steel fibers, both in terms of bearing capacity and ductility. Note that fibers gives better performance, compared to the MSR specimens at the service level, improving the tension stiffening effect and therefore reducing the displacement. Also cracks are fewer and narrower, at service level, then those of MSR specimens, for both NSC and HSC series. However, with increased loads and displacements, and after a shear crack of around 3 mm, fibers are no longer able to resist further load and displacement, bringing the member to the "block mechanism", which characterizes the shear collapse.

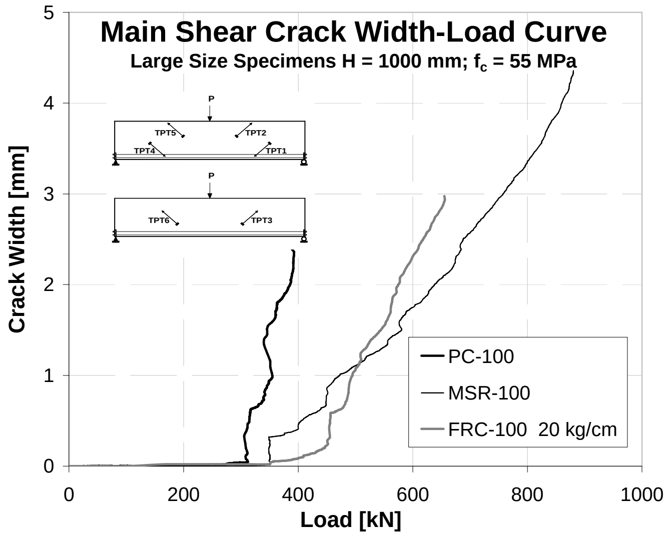


Figure 8: Shear Crack Width-Load Curve; series 2 (H=1000mm).

Table 3 reports a summary of the main experimental results of the experimental campaign herein discussed. Note that the ultimate shear stress ($v_u$) as well as the ultimate shear force ($V_U$), was determined by considering also the self-weight of the members. Moreover, in the calculation of the bearing capacity of the member, which corresponds to a flexure failure ($V_{U,FL}$), the effect of fibers was neglected. One should note that the design guidelines CNR DT 204 (2006) provide a simple method to calculate the contribution of fibers to bending strength; in our case there is an improvement of 10% only. The results for the plain concrete members agree reasonably well with the experimental results plotted in Figure 1, whereas very good results can be obtained for FRC members which, especially for the small beams, are able to approach the flexural bearing capacity without the addition of stirrups.

Definitely, the addition of FRC does not completely solve the size effect issue, especially if fibers are provided in such a low amount. The following chapter will investigate this aspect better through a series of numerical analyses on different FRC compositions.

Table 3: Summary of the main experimental values.

| SPECIMEN | | $P_u$ | $v_u$ | $v_u/(f_c)^{1/2}$ | $\delta_u$ | $V_u/V_{u,FL}$ |
|---|---|---|---|---|---|---|
| | | [kN] | [MPa] | [-] | [mm] | [-] |
| PC-50 | | 216 | 1.22 | 0.24 | 2.74 | 0.52 |
| MSR-50 1 | | 346 | 1.93 | 0.38 | 9.33 | 0.82 |
| MSR-50 2 | | 302 | 1.69 | 0.33 | 7.03 | 0.72 |
| FRC-50 1 | | 388 | 2.16 | 0.43 | 10.95 | 0.92 |
| FRC-50 2 | | 308 | 1.72 | 0.34 | 4.77 | 0.73 |
| PC-100 | NSC | 365 | 1.07 | 0.21 | 7.60 | 0.43 |
| MSR-100 | NSC | 635 | 1.81 | 0.36 | 12.60 | 0.73 |
| FRC-100 | NSC | 494 | 1.42 | 0.28 | 11.05 | 0.57 |
| PC-100 | HSC | 393 | 1.14 | 0.15 | 9.79 | 0.45 |
| MSR-100 | HSC | 880 | 2.48 | 0.33 | 18.62 | 0.99 |
| FRC-100 | HSC | 656 | 1.86 | 0.25 | 12.01 | 0.74 |

## 3 NUMERICAL ANALYSES

### 3.1 *Introduction*

Numerical analyses were initially performed to validate the FE program VecTor2 (Wong & Vecchio, 2002) which is based on the MCFT (Vecchio & Collins, 1986). The latter represents a well known model for representing the nonlinear behavior of reinforced concrete structures; it is essentially a smeared, rotating crack model for cracked reinforced concrete elements. On the basis of a number of panel tests, constitutive relationships were developed to describe the behavior of cracked reinforced concrete in compression and in tension.

Numerical analyses were performed by assuming perfect steel-to-concrete bond (no slip). The beam was discretised with four-node 2D plane stress elements. Transverse reinforcement was modeled as embedded in concrete elements.

While concrete in compression was easily modeled using constitutive laws available in the literature, it is worthy discussing the assumption regarding tensile behavior. Concrete in tension was assumed to be linear up to the tensile strength. The post-peak behavior, particularly important when fibers are present in the matrix, was calculated from FE back analyses of the experiments performed according to the Italian Standard (UNI 11039, 2003); further details are reported in Minelli (2005).

The cohesive laws were initially determined as stress-crack opening (σ-w) laws and, since VecTor2 is a smeared cracking model, were eventually transformed into stress-strain relationships (σ-ε) by dividing the fracture opening (from back analysis) by a "characteristic length".

These relationships were then incorporated in VecTor2, and used to perform the FE analyses of all small and large beams with steel fibers. The characteristic length was assumed as the effective flexural depth of members, which represents an approximation of the crack spacing of beams with little or no shear reinforcement. The maximum size of the aggregate, which is a quite important parameter for shear, was assumed equal to 20 mm, as in the experiments.

Special emphasis was devoted to the crack-width control, which is based on a maximum crack width that can be selected by the user among a series of proposal. This parameter turned out to be quite influential on the overall response of members subject to a shear mode of failure. The maximum crack width was here selected equal to a fifth of the aggregate size for plain concrete members (Vecchio, 2000) whereas it was chosen equal to 3 mm for the FRC members (Minelli & Vecchio, 2006), according to the experimental results. Note that such a limit does not significantly affect the structural response of members with transverse reinforcement (Vecchio, 2000).

Finally, rebars were simulated by using a polylinear stress-strain curve, including strain hardening, according to the experimental data.

Figure 9 reports the results of numerical analyses performed on the Large Size Elements made of NSC. Space restrictions do not allow the authors to show all results, even though the following discussion fully applies to all analyses referred to the experiments presented above.

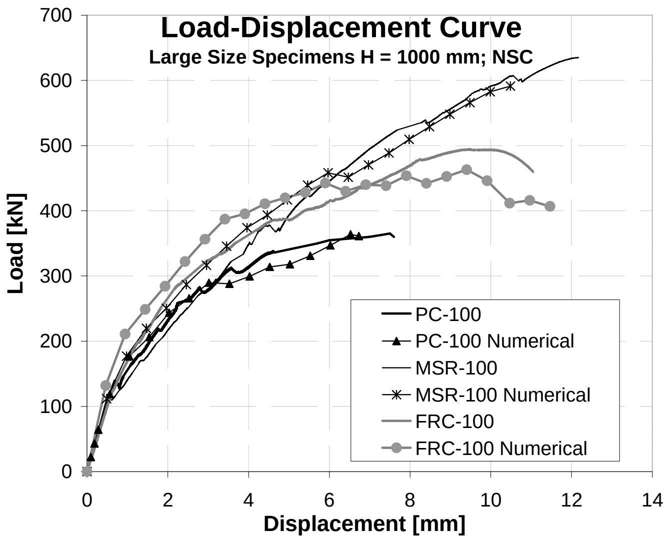


Figure 9: Numerical analyses vs. experiments, Large Size Specimens, NSC.

The MCFT-based procedures already gave a number of examples demonstrating their ability in accurately representing the behavior of R/C members with little or no shear reinforcement (Vecchio, 2000). Fewer cases are reported for FRC elements (Minelli, 2005). Adding the corresponding tension softening law (as determined form back analysis) and suitably improving the crack width control, the compression field model was able to produce reasonably well the experiments, in terms of bearing capacity, stiffness, crack patterns and ductility. Modeling of PC-100 specimen turned out to be very accurate, whereas a general overestimation of the stiffness of the cracked stage was found for FRC-100 specimen. However, good accuracy in the ultimate bearing capacity and in the crack pattern was seen. As far as the MSR-100 specimen is concerned, a 7% underestimation of the peak load was observed, even though the crack pattern and the overall behavior was well modeled. In conclusion, the numerical study confirmed the ability of fibers in improving the post-cracking stiffness and in reducing the shear cracking.

Once the FE program was validated against many experiments (Figure 9), an extensive numerical study was performed to better clarify the influence of fibers in reducing the scale effects. Figure 10 plots the ultimate shear (from numerical analyses) to maximum shear (from flexure failure) ratio vs. the effective depth. The ratio between the two shear stress values has been selected to show both scale effects and modes of failure. Different constitutive tension softening laws were chosen for a series of beams having a depth ranging form 250 to 2000 mm and characterized by the critical a/d ratio of 2.5. The longitudinal reinforcement was selected in all case as equal to 1%. NSC-PC and NSC-FRC1 refer to the concrete mixtures reported in the present paper, while NSC-FRC2 refers to a content of fibers of 40 kg/m$^3$. Finally, HSC-FRC3 concrete composition concerns a high strength concrete with 50 kg/m$^3$ of high-carbon steel fibers, having a considerably high performance and a strain-hardening behavior under flexure (Minelli & Vecchio, 2006). Numerical results are plotted against the experimental values related to Table 3, showing a discrete agreement.

From this plot one can notice that the decay in the ultimate shear strength with increasing dimensions diminishes as the fiber toughness increases, as one would have expected. When fibers are added in large amounts and sufficient toughness, they result, even for significantly deep members, in a collapse failure which moves from shear (brittle) to flexure (generally ductile) without the addition of any stirrup reinforcement. This is of great importance from a design point of view, giving the possibility to reduce or also totally substitute stirrups with fibers and, moreover, to significantly reduce the scale effects. Note that the plot shows a vertical line in correspondence of an effective depth of 1 m. In many cases, in fact, codes require, from that point on, the addition of a distributed skin reinforcement in the two directions, basically for crack control. This requirement, as well as the one of the minimum shear reinforcement, can be met by using a relatively low amount of fibers, according to Figure 10 and to the experimental results herein discussed.

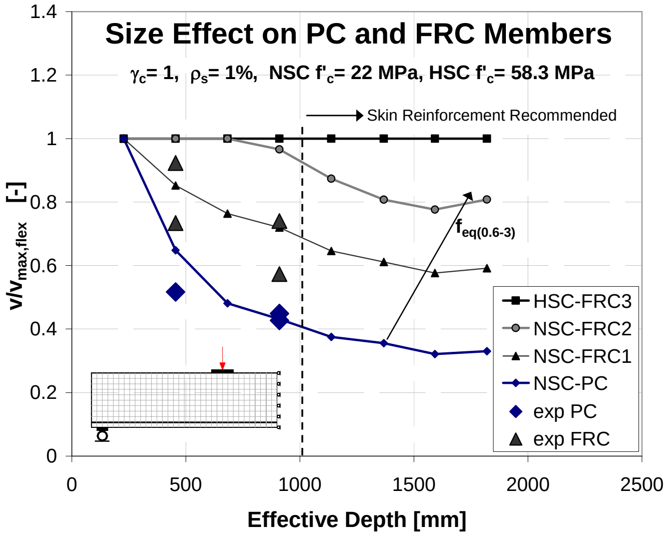


Figure 10: Size effect development for different fiber reinforced concretes; results from numerical analyses.

## 4 CONCLUSIONS

In the present paper, some critical issues related to the behavior of structural R/C members with little or no shear reinforcement or with the addition of fiber reinforced concrete were discussed.

The following main conclusions can be drawn:

- Fibers in reinforced concrete elements under shear significantly reduce the scale effects, even if provided in relatively low amount. To fully avoid such an experimental drawback, a greater fiber content (which means a bigger toughness) is required;
- Fibers can effectively replace the minimum amount of transverse reinforcement and the skin reinforcement required to control the crack propagation and give visible warning before the structural collapse; fibers, in fact, allow the shear cracking to gradually grow and the overall deflection to become visible before the collapse.
- The finite element program herein adopted, which implements several compression field model-based procedures, is able to represent in a consistent and effective way the behavior of member with little or no shear reinforcement as well as with fibers. The scale effect determined through numerical analyses were found to be similar, for all materials studied, to the experimental ones.

Further studies are intended to assess these preliminary experimental and numerical results toward a consistent design of members made of FRC. Suitable adaptations of current codes, as already presented for FRC beams (Minelli & Plizzari, 2006) should extensively evaluated with regard to the size effect issue.

## ACKNOWLEDGMENTS

The Authors would like to give their appreciation to steel factory Alfa Acciai (Brescia) for supplying the reinforcement for the experiments. Moreover, the assistance of engineers Bertozzi Alessandro, Federico Ginesi and Adriano Reggia is gratefully acknowledged.

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