Modeling of the influence of the damage on the behavior of concrete during tensile-compressive loading

S. Mertens & J. Vantomme
Dept. of Civil & Materials engineering, Royal Military Academy, Brussels, Belgium

J. Carmeliet
Laboratory of building physics, Dept. of civil engineering, Faculty of applied science, Katholieke Universiteit Leuven, Leuven, Belgium

Building physics group, Faculty of building & architecture, Technical University Eindhoven, Eindhoven, the Netherlands

ABSTRACT: Experimental tensile-compressive cyclic tests on DEN-concrete specimens reveal several phenomena: nonlinear elasticity, stiffness reduction, stiffness recovery, permanent deformation and hysteretic behavior, which changes shape during tensile unloading to compressive loading. In this paper, a constitutive model is proposed that captures all these effects, and which is based on strain decomposition into a classical linear elastic strain and a non-classical strain described by the Preisach-Mayergoyz model (PM-model).

1 INTRODUCTION

Cracking is one of the critical considerations when designing concretes structures. Continuum and discrete simulation models (e.g. nonlocal damage, gradient damage, PU-FEM, X-FEM) have been used to account for these complex cracking processes. Most of these codes use constitutive models (e.g. the damage-plasticity model) simplifying some of the observed phenomena. However, cyclic tensile-compressive tests on concrete specimens (Reinhardt, 1984) show a rather complex cyclic behavior (figure 1). In pre-peak tensile loading, a nonlinear load deformation curve is observed. In the post-peak region, strain-softening occurs during damage evolution. When unloading, the load deformation curve is characterized by a stiffness reduction in tensile loading and a stiffness recovery in compressive loading. In addition, permanent deformations and hysteretic loops are formed which change of shape when going from tensile to compressive loading and vice versa. Moreover, the hysteretic loops change form with damage. All these phenomena can be related to the presence and the growth of the damage in the form of micro-, meso- and macrocracks. Nonlinear elastic behavior is explained by the opening/closing of initial defects, while strain softening and stiffness reduction is attributed to the growth of the micro- and meso-cracks. Permanent deformations can be explained by the misfit of closing cracks and stiffness recovery upon complete closure of the cracks. Hysteretic behavior is attributed to the opening and the closing of cracks at different stresses.

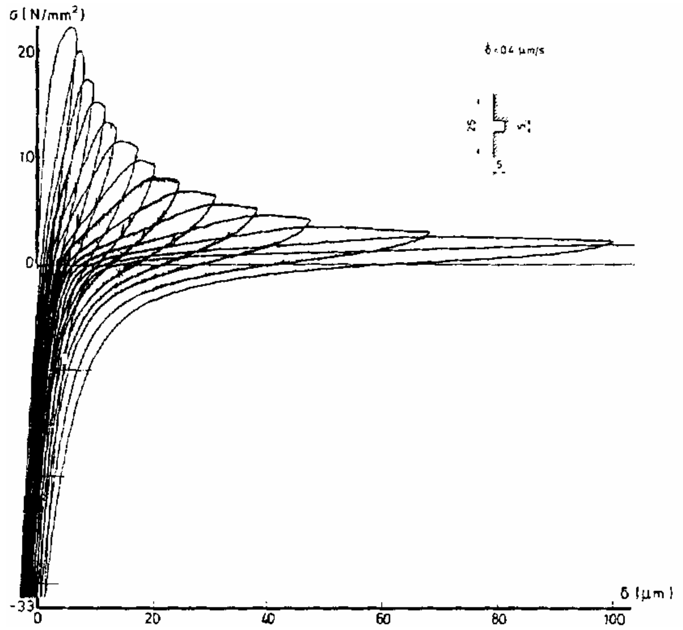

Figure 1: Experimental results according to Reinhardt(1984)

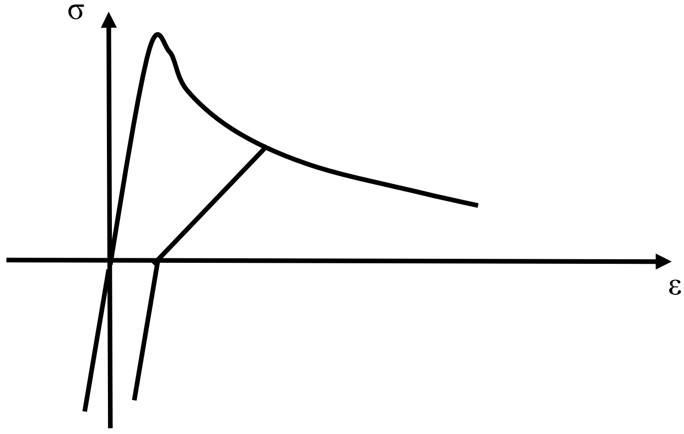

Figure 2: Combined damage-plasticity model.

Classical damage-plasticity models (figure 2) include a softening branch, the stiffness reduction in the tensile region and the stiffness recovery in the compressive region. Hysteretic behavior is however not included.

(Gylltoft, 1984) proposes a constitutive model that includes hysteretic behavior of the material in the tensile region ($\sigma \geq 0$), but not in the compressive region ($\sigma < 0$) (figure 3). In Gylltoft's model, stiffness reduction in tensile loading, compressive hysteretic phenomena and nonlinear elasticity are not taken into account.

(Reinhardt et al., 1986) propose a model that includes a tensile as well as a compressive hysteretic behavior (figure 4). The hysteretic behavior in tensile loading is similar as in Gylltoft's model, i.e. the unloading branch is parallel with the linear pre-peak elasticity behavior, and the tensile hysteretic behavior has the shape of a parallelogram. The compressive hysteretic behavior has the shape of a triangular. A drawback of the model is that the minimum stress in the loop has to be known in advance.

(Yankelevsky et al, 1989) propose a focal-point-model, where the hysteretic behavior is described by 7 so-called focal points (figure 5). The model is based on the experimental observation that the stiffness of the (tensile) loading and unloading hysteretic loop changes.

(Hordijk, 1991) proposes the continuous-function-model (CFM), which consists of 4 independent analytical equations, describing respectively the softening branch, the unloading behavior, the reloading behavior and the stress drop, when the reloading hysteretic loop reaches the softening branch. The model consists of 14 parameters, which need to be determined experimentally.

(Duda, 1991) proposes a rheological model for the simulation of cyclic tensile-compressive loading. The model consists of three parallel units: two units are composed of two springs and sliders in series connection (each able of forming a trapezium shaped hysteretic loop), and one unit is composed of a slider. The behavior of the units is influenced by the damage level (i.e. maximum attained displacement). This model can simulate all of the observed phenomena, except for the non-linear elasticity. The model consists of 14 parameters, which need to be determined experimentally.

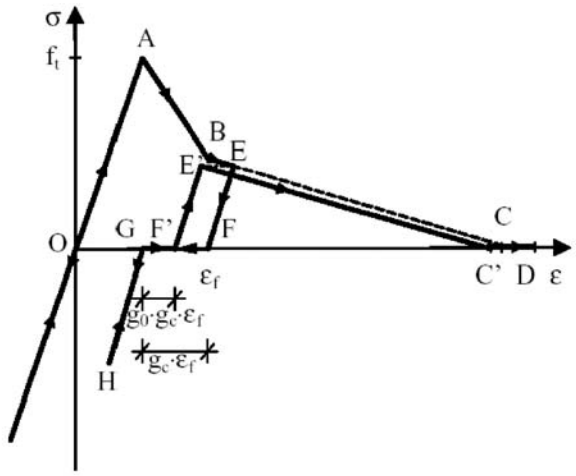

Figure 3: model proposed by Gylltoft(1984), where g_o and g_c are be considered material parameters.

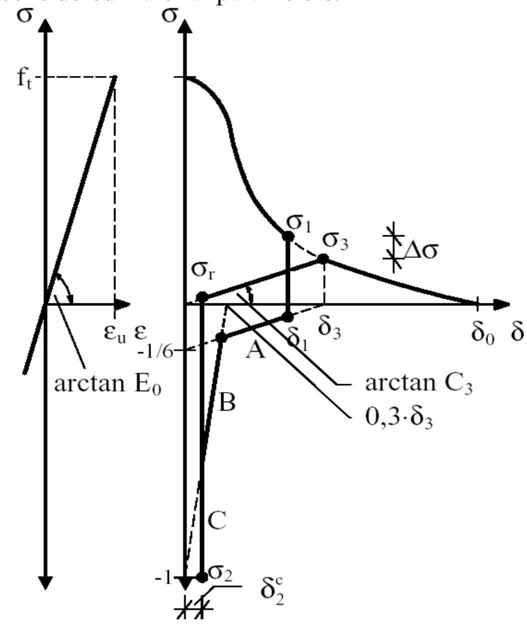

Figure 4: model proposed by Reinhardt et al,(1986).

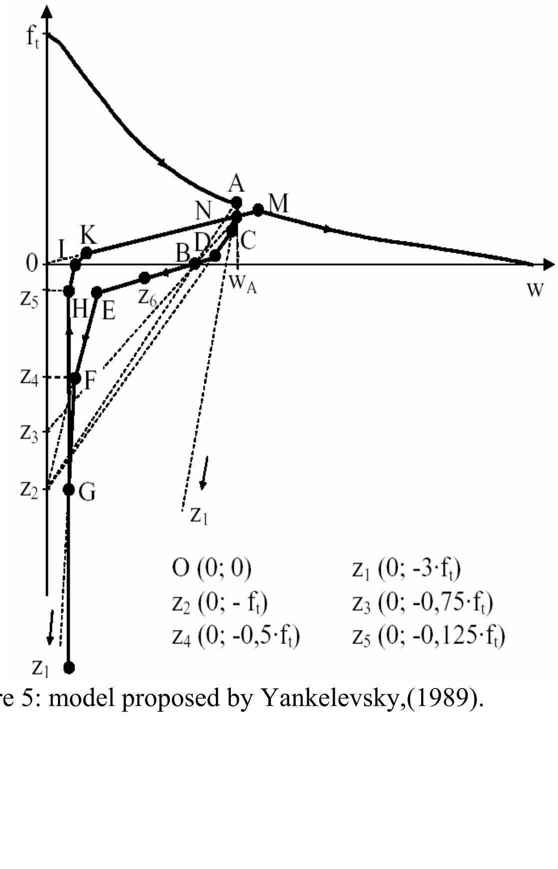

Figure 5: model proposed by Yankelevsky,(1989).

(Oliveira, 2002) proposes an interface cyclic model which incorporates all described phenomena except the nonlinear elastic behavior for the meso-modelling of masonry. The model is based on the plasticity theory and uses the yield envelope proposed by (Lourenço, 1996). The hysteretic unloading/reloading behavior is incorporated using two unloading surfaces. The unloading surfaces are controlled by a mixed hardening law and appropriate experimentally measured hardening evolution rules.

The above-mentioned constitutive models are or unable to describe all the observed phenomena, or the determination of all parameters is hard to perform. Moreover, these models are generally based on parametric description of the loops and lack physical understanding of the underlying microscopic and mesoscopic phenomena.

In this paper, we present a two-scale model that is based on the modeling of the physical behavior of the material at the micro- and mesoscale (i.e. as mentioned above, e.g. opening/closing of the micro- and mesocracks…). The proposed model is able to incorporate nonlinear elasticity, softening, stiffness reduction in tensile loading, stiffness recovery in compressive loading, the tensile-compressive hysteretic loops and permanent strains. The constitutive model is based on the Preisach-Mayergoyz phenomenological model (PM-model). In section two, the PM-model is briefly explained, and the PM-space distribution is identified. In section 3, the constitutive model is presented. Section 4 shows the obtained results and in section 5 conclusions are drawn.

PM-MODEL

1.1 *Motivation to use the PM-model.*

Guyer et al. (1999) show that materials can be subdivided in 2 classes: microscopic elastic materials (metals without dislocations, undamaged individual crystals, many fluids, intact plastics…) and nonlinear mesoscopic elastic materials (sand, soil, cement, concrete, ceramics and damaged microscopic elastic materials). In a nonlinear, hysteretic mesoscopic material, the solid 'grains' behave as rigid units, while the contact between the grains (the bond system) controls the nonlinear, hysteretic behavior of the material. More specifically, according to Van Den Abeele et al. (2002) the nonlinear, hysteretic response in this class of materials is caused by the low-aspect ratio features (e.g. contacts in microcracks, macro-cracks, asperities …). According to Guyer (1999), this class of materials cannot be described by the traditional elasticity theory as applied for microscopic elastic materials. Guyer et al. (1995) propose the Preisach-Mayergoyz space model, where the macroscopic hysteretic nonlinear response is described by outcome of an assemblage of micro- and mesoscale elements. Since the nonlinear, hysteretic response of concrete in cyclic loading results from micro- and mesoscopic features related to damage (such as microcracks, mesocracks, contacts in macrocracks, etc.), it is logical that the nonlinear, hysteretic elastic behavior depends upon damage.

1.2 *Composition of the PM-model.*

The PM-model is build up of a set of micro- and meso-scale elements, which are called non-classical elements or hysterons. In the PM-model, it is assumed that a hysteron can only be in two states, open or closed. The element opens when a certain stress σ_0 is exceeded and remains open as the loading continues to increase (figure 6a). The element closes at σ_c (different from σ_0) and remains closed when the loading further deceases. The stresses σ_0 and σ_c for each hysteron can be used as the element's coordinates in a "PM space", thus creating a density of elements in the σ_0 - σ_c space (with $\sigma_0 \geq \sigma_c$). Elements that show no hysteresis reside on the diagonal ($\sigma_0 = \sigma_c$), while hysterons showing increasing hysteretic behavior are situated further from the diagonal. By keeping track of which elements are closed or open, the resulting nonlinear elastic strain, also called the non-classical strain can be calculated by the following integration in the P-M space:

$$\varepsilon_{nc}(t) = \int_{-\infty}^{\infty} \int_{\sigma_c}^{\infty} \gamma_{\sigma_o \sigma_c} \sigma(t) \cdot \mu(\sigma_o, \sigma_c)\, d\sigma_o d\sigma_c \tag{1}$$

where ε_{nc} = the non-classical strain; $\gamma_{\sigma_o \sigma_c} \sigma(t)$ = the state of the hysteron with opening stress σ_o and closing stress σ_c at time t; and $\mu(\sigma_o, \sigma_c)$ = the PM-distribution function.

Figure 6b shows the PM space in tension-compression. The PM space in tensile direction is bounded by the tensile strength f_t. In the post-peak softening range the tensile strength gradually reduces due to damaging of the material, which results in a shrinkage of the tensile PM space. We assume that in compression, the stress is lower than the compressive strength, so no compressive damaging of the material is considered. The compressive PM space is bounded by a constant limiting value $\sigma_{\lim}$. By choosing an adequate PM-distribution function, different non-linear phenomena observed at the macro-level can be modeled: hysteretic behavior, stiffness reduction, stiffness recovery and permanent deformations.

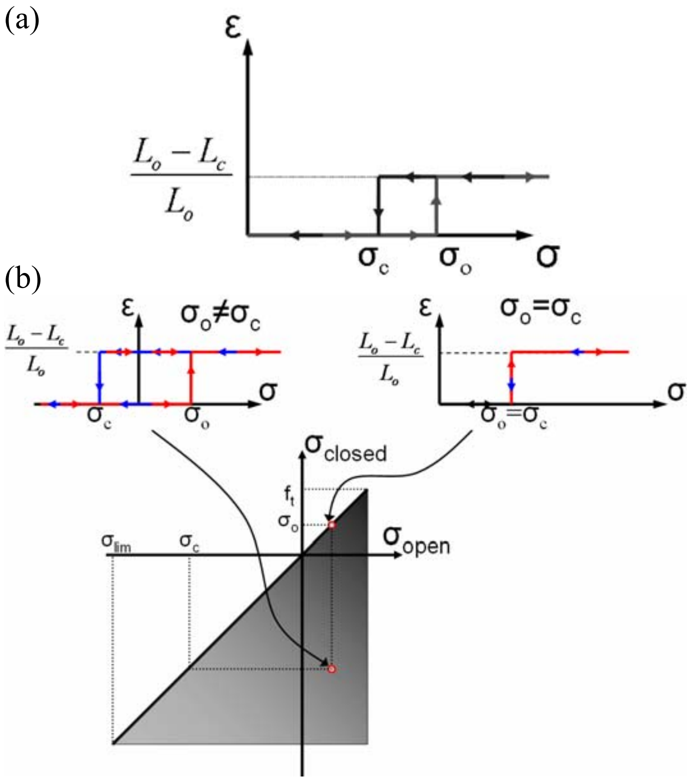

Figure 6 (a) Representation of the behavior of an hysteron. (b) The mapping of the individual hysteron in the PM-space.

2 MODELLING

2.1 *Basics of constitutive model.*

The constitutive model is based on the decomposition of the total strain into a classical elastic strain and a non-classical strain

$$\varepsilon = \varepsilon_e + \varepsilon_{nc} = \frac{\sigma}{E_o} + \varepsilon_{nc} \tag{2}$$

The classical elastic strain ε_e is described using the classical theory of linear elasticity. The corresponding Young's modulus E_o is the modulus of the undamaged material (i.e. without initial damage). The non-classical strain ε_{nc} is determined by the PM-model. In the next section, a method is proposed to determine the PM-distribution.

2.2 *Identification of the PM-distribution.*

Figures 7a, 7b and 7c give respectively the first, an intermediate and last unloading-loading loop of stress versus non-classical strain. The non-classical strain is obtained by subtracting the linear elastic strain from the total strain. Comparing the different hysteretic loops, it can be concluded that the PM space related to the loops, highly depends on the damage level d of the material or $\mu(\sigma_o, \sigma_c; d)$. Damage is defined according to continuum damage theory, where the damage variable d ranges from 0 for the intact material to 1 at complete macro-cracking. During the unloading-loading loop ($\sigma \leq f_t$), no further damage evolution is assumed, which means that a single loop is determined by one PM distribution with damage d. The damage level attributed to a loop can be determined based on the residual strength of the material f_t, which equals the maximum stress $\sigma_{\max}$ of the unloading-loading loop (see Figure 7). The residual strength in the post-peak loading (softening curve) is given by:

$$f_t = (1-d)\cdot f_{t0} \tag{3}$$

With f_t equals σ_{max}, which is the maximum stress that can be transferred in the material with damage d, f_{t0} = the initial tensile strength of the undamaged material. At the start of the unloading curve ($\sigma=\sigma_{max}$) all hysterons are in the open state. The corresponding non-classical strain $\varepsilon_{nc,\max}$ is determined by the integration of the PM space until $\sigma_0 = \sigma_{max}$. The non-classical strain during unloading is then given by:

$$\varepsilon_{nc}(\sigma(t)) = \varepsilon_{nc,\max} - \int_{\sigma(t)}^{\sigma_{\max}} \int_{\sigma_c}^{\sigma_{\max}} \mu(\sigma_o, \sigma_c)\, d\sigma_o d\sigma_c \tag{4}$$

The non-classical strain ε_{nc} during reloading from the minimal reversal stress σ_{rev} of the loop is given by:

$$\varepsilon_{nc}(\sigma(t)) = \varepsilon_{nc,rev} + \int_{\sigma(t)}^{\sigma(t)} \int_{\sigma_c}^{\sigma(t)} \mu(\sigma_o, \sigma_c)\, d\sigma_o d\sigma_c \tag{5}$$

In equation (5), $\varepsilon_{nc,rev}$ is the non-classical strain obtained with equation (4) during unloading from the maximum stress σ_{max} until the reversal stress σ_{rev}.

The identification of the PM distribution is generally an underdetermined identification problem. Therefore, following (Carmeliet et al. 2002), the PM space is described by a sum of analytical functions. We introduce the following functional form, based on an exponential decay in density of hysterons away from the diagonal, i.e. we assume:

$$\mu(\sigma_0, \sigma_c; d) = \sum_{i=1}^{n} \rho_i(\sigma_m; d)\exp(-\kappa_i(\sigma_d; d)) \tag{6}$$

with the mean stress:

$$\sigma_m = \frac{\sigma_o + \sigma_c}{2} \tag{7}$$

and the stress amplitude:

$$\sigma_d = \frac{\sigma_o - \sigma_c}{2} \tag{8}$$

In equation (6), κ_i is a decay parameter, which is dependent on the damage d, ρ_i a diagonal density function given by:

$$\rho_i(\sigma;d) = a_i(d)\exp\big(b_i(d)\sigma\big) \tag{9}$$

The damage dependent parameters a_i, b_i and κ_i are determined by indirect identification minimizing the least square difference between measured and predicted loop. Figure 7a, 7b and 7c compare the measured and fitted data for respectively the first, an intermediate and the last loop. The identification analysis showed that the loops can be adequately modeled by the sum of two functions (n=2, or two PM-populations). Figures 8a-e shows the dependence of the parameters on damage.

We observe that the parameters a_i and b_i for both populations increase with damage. These parameters describe the diagonal function. This means that during damage evolution the mechanical effect of the total number of hysterons on the nonlinear behavior of the material, or the non-classical strain, becomes more pronounced. The decay parameter of the first population κ_1 increases first with damage, whereafter it decreases to zero for high damage values. This means that in the beginning of the damage process the hysterons of the first population become more situated near the diagonal and thus become less hysteretic. At the end of the damage process, these hysterons will be situated uniformly over the PM-space, and thus generate very hysteretic behavior. These hysterons could probably be attributed to nonlinear effects in the crack process zone, which grows during damage evolution.

The second population of hysterons becomes only important at high damage values (d>0.8). This population is characterized by a decay coefficient κ_2 approximately equal to zero, meaning there is no decay in the PM distribution away from the diagonal. This means that these hysterons, which only originate at high damage level, are very hysteretic, and are caused by the presence of nonlinear mechanisms – probably contacts in macrocracks - when macrocracks are fully developed.

In conclusion, the hysteretic behavior of the material can be divided in two stages. In the first stages during damage evolution, the effect of the hysterons increases, while moving to the diagonal (indicated by an increase of κ_1). At high damage level, a second population of very hysteretic hysterons appears (indicated by a low value of κ_2), while the hysterons of the first population become more and more hysteretic (indicated by a decrease of κ_1).

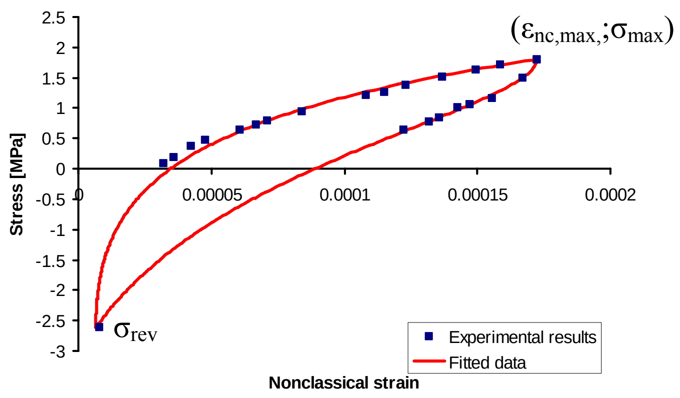

Figure 7a: Comparison of the results for the first hysteretic loop.

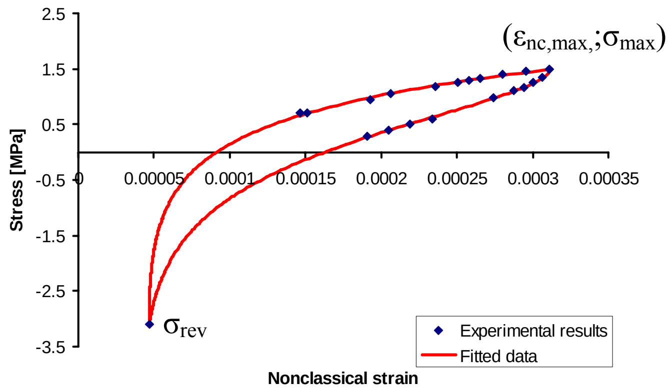

Figure 7b: Comparison of the results for an intermediate loop.

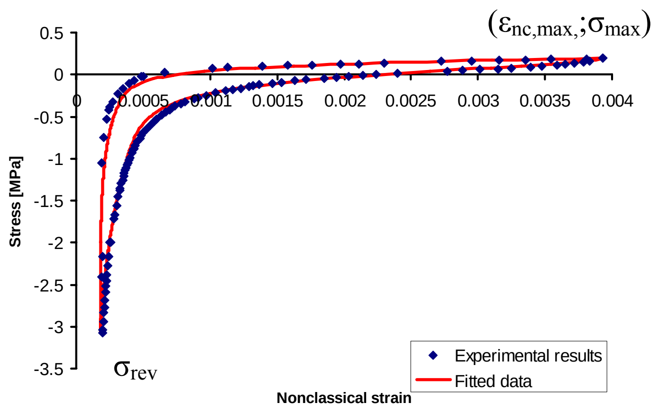

Figure 7c: Comparison of the results for the last hysteretic loop.

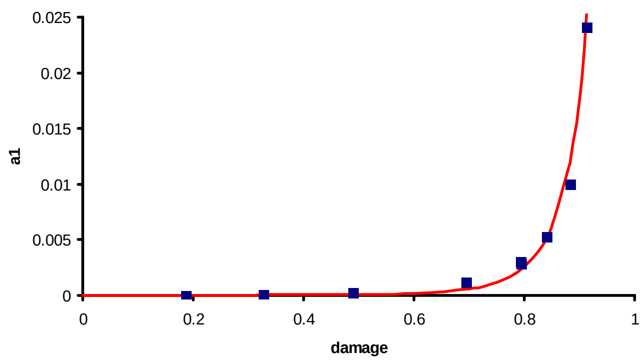

Figure 8a: Evolution of the parameter a_1 with the damage d ($a_1 = a_i$ of the first term).

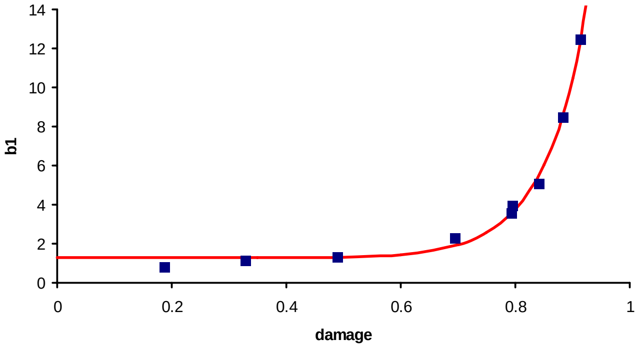

Figure 8b: Evolution of the parameter b_1 with the damage d.

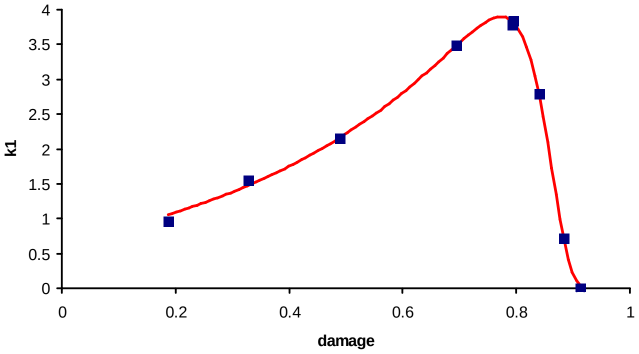

Figure 8c: Evolution of the parameter κ_1 with the damage d.

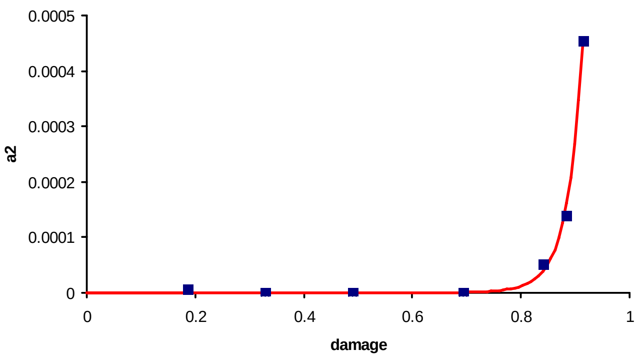

Figure 8d: Evolution of the parameter a2 with the damage d.

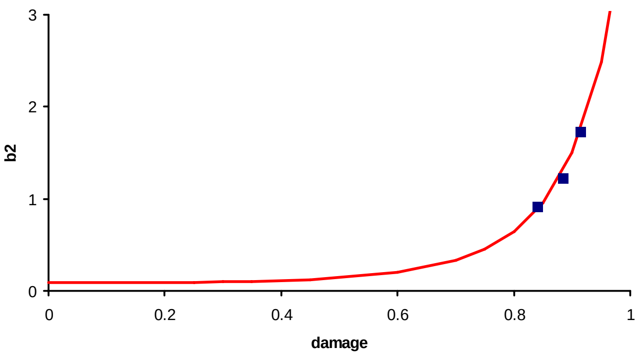

Figure 8e: Evolution of the parameter b2 with the damage (d).

2.3 *General constitutive model.*

The stress-strain relation for isotropic damage including non-classical strain effects (hysteresis, stiffness decrease and stiffness recovery) is described as:

$$\dot{\sigma} = E(d, \sigma_{history}) \cdot \dot{\varepsilon} + \dot{E}(d, \sigma_{history}) \cdot \varepsilon \qquad (10)$$

Where ε is the total strain and $E(d,\sigma_{history})$ is the tangent stiffness in the stress-strain space. The tangent stiffness is determined by:

$$E(d, \sigma_{history}) = \frac{\partial \sigma}{\partial \varepsilon} = \frac{1}{\frac{1}{E_0} + \frac{\partial \varepsilon_{nc}}{\partial \sigma}} \qquad (11)$$

Where $\partial \varepsilon_{nc} / \partial \sigma$ is determined from the PM space model. It should be noted that the permanent strains are taken into account in the PM space model. This corresponds in the PM-space model with hysterons that remain in open state and do not close, upon loading. Permanent deformations are coupled with the damage present in the material because the PM model evolves with damage.

Damage growth is defined by a damage growth criterion and a damage evolution law. Damage growth in tension is defined by a Rankine kind criterion.

$$f_d = \varepsilon - \phi \qquad (12)$$

Where f_d defines the damage surface in tension, ε is the total applied strain and ϕ is an internal parameter. At the beginning, this parameter equals the initial tensile fracture strain. If f_d equals 0, then according to the Kuhn-Tucker conditions damage evolves. The damage evolution function is described by:

$$d = 1 - \alpha_1 \cdot e^{-\alpha_2 \cdot (\phi - \phi_i)} - (1 - \alpha_1) \cdot e^{-\alpha_3 \cdot (\phi - \phi_i)} \qquad (13)$$

where α_1, α_2 and α_3 are parameters which are determined by the tensile softening branch of the material, ϕ is an internal parameter determined by equation (12) and ϕ_i the initial value of the internal parameter.

Finally, the cyclic tension-compression behavior is simulated by the model formulated by equations 1, 2, 3, 6, 7, 8, 11, 12 and 13. The proposed model describes all phenomena using damage evolution and un/reloading (figure 9).

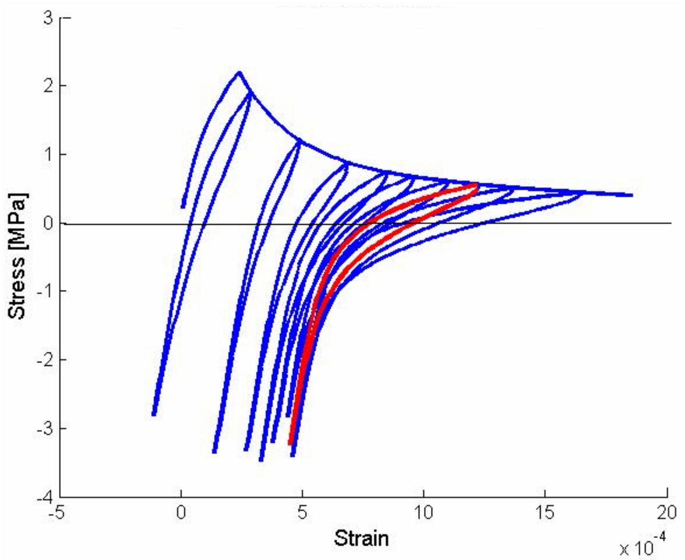

Figure 9: Results obtained with the proposed constitutive model.

3 CONCLUSIONS

In this paper, a model is presented to describe uniaxial cyclic tensile/compressive loading including phenomena like tensile softening damage, stiffness decrease during tensile damage, stiffness recovery during compressive loading, permanent deformations and hysteresis between loading and unloading branch. The model is based on strain decomposition in elastic and non-classical strain, covering all nonlinear effect. The non-classical strain is considered as the outcome of a population of hysteretic elements - called hysterons – which are described in the PM space by two distribution functions. The PM distribution is dependent on the damage level. The second population only originates at high damage levels or when macro-cracks are formed. At low damage levels, the populated PM-space is located near the diagonal. As the damage increases more hysterons originate, which are located near the diagonal, causing the hysterons to behave less hysteretic. Because the parameters that describe the PM-distribution increase with the damage, it can also be stated that the total effect (or number) of the hysterons increases. It can be observed that the combined model is able to take into account all of the observed phenomena that occur during tensile-tensile and tensile-compressive loading: i.e. in the pre-peak region: nonlinear elasticity, hysteresis and in the post-peak region: stiffness reduction, stiffness recovery, permanent deformations, tensile-tensile and tensile-compressive hysteretic loops.

4 ACKNOWLEDGEMENTS

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