Fracture Mechanics of Concrete and Concrete Structures -
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Fracture mechanics of early-age concrete

V.T.N. Dao, P.H. Morris & P.F. Dux
School of Civil Engineering, The University of Queensland, Australia

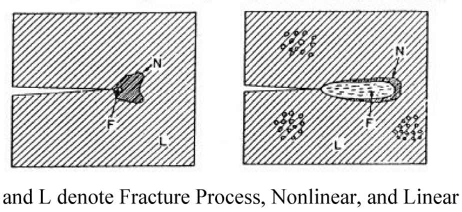
ABSTRACT: In this paper, the fracture mechanics of concrete structures is first reviewed. This clearly demonstrates that, despite the successful application of fracture mechanics to study the fracture behavior of mature concrete, its application to the cracking of very early-age concrete (i.e. within several hours after mixing) is still in its infancy, with very limited literature available. This is believed principally to be the result of difficulties that arise in the experimental determination of the properties of extremely fragile concrete at very early ages. In a recent research at the University of Queensland, a test apparatus and experimental procedures have been developed that enable the complete tensile stress-displacement behavior of concrete specimens at very early ages to be captured reliably. Based on the data obtained, the paper shows that current models for the stress-separation relationship of mature concrete may not apply for concrete at very early ages. Revised mathematical models for the stress-separation relationship of early-age concrete are proposed.

1 REVIEW OF FRACTURE MECHANICS OF CONCRETE STRUCTURES

The risk of failure due to the growth of cracks can be treated using the science known as fracture mechanics, which arose initially from the work of Griffith (1920) on the fracture of brittle materials such as glass. Its most significant applications, however, have been in controlling the brittle failure and fatigue failure of metallic structures such as pressure vessels, airplanes, and ships. Considerable development has taken place in the last several decades to account for the ductility typical of metals.

Portland cement concrete is a relatively brittle material and as a result, its mechanical behavior is critically influenced by crack propagation. Many attempts have been made to apply fracture mechanics concepts to cement-based composites, such as mortar and mature-age concrete. The first application of fracture mechanics to concrete appears to have been made by (Neville 1959a, b, c), while Kaplan (1961) appears to have published the first experimental study of the application of fracture mechanics to concrete. Excellent reviews of the application of fracture mechanics to mature cement and concrete are available in the literature, and it is thus not covered in detail here.

There are fundamental differences between the fracture behavior of concrete, a quasi-brittle material, and that of brittle and ductile-brittle materials. While the nonlinear zone is practically absent in brittle materials, most of the nonlinear zone in front of a crack tip in a ductile-brittle material (such as a metal) involves hardening plasticity or perfect yielding (Fig. 1). In ductile-brittle materials, the fracture process zone (FPZ), which is the zone in which the material undergoes softening damage, is quite small. In quasi-brittle materials (such as mature concrete, rock, and ceramics), however, plastic flow is almost nonexistent and the FPZ fills almost entirely the nonlinear zone (Fig. 1). It has been suggested that the width and length of the FPZ in mature concrete are of the order of three and twelve times the maximum aggregate size, respectively (Bazant & Oh 1983, Otsuka & Date 2000). The development of a sizable FPZ necessitates the application of nonlinear fracture mechanics (Bazant 2002), which differs significantly from the fracture mechanics traditionally applied to ductile-brittle materials. Linear elastic fracture mechanics is applicable only to relatively large-scale structures in which the effect of the nonlinear FPZ can be neglected.



(Note: F, N, and L denote Fracture Process, Nonlinear, and Linear Zones.)
Figure 1. Fracture process zone in metal and concrete (Bazant 2002).

Various models have been developed to describe the FPZ in front of a crack in concrete, including notably the fictitious crack model (FCM) proposed by Hillerborg et al. (1976) and the crack band model proposed by Bazant & Oh (1983) (Fig. 2). The former models the process zone as a geometrically discontinuous crack with characteristics after cracking that are described by a stress-crack opening relationship. The latter imagines the process zone to exist within a certain finite bandwidth in which the microcracks are uniformly distributed and describes the behavior after cracking by a stress-strain relationship. Figure 2 shows the stress distribution and softening stress-separation (or equivalent strain) curve for each model. The two models are essentially equivalent (Karihaloo 1995, Bazant 2003), and are sometimes referred to collectively as cohesive models or fracture process models. In this paper, the FCM is applied to study the fracture parameters of early-age concrete.

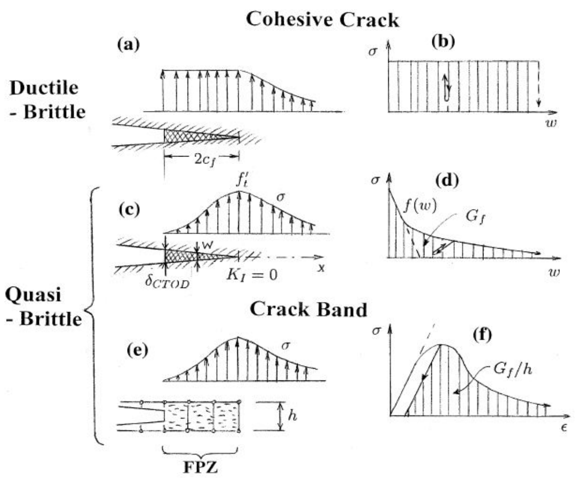


Figure 2. Stress distribution and softening curve: (a, b) cohesive crack model for ductile-brittle materials; (c, d) cohesive crack model for quasi-brittle materials; (e, f) crack band model for quasi-brittle materials (Bazant 2002).

The area under the entire softening stress- crack opening curve (Fig. 2d) represents the total energy dissipated by fracture per unit area of crack, and is defined as the fracture energy, G_F. That is,

$$G_F = \int_0^{w_c} \sigma(w)dw \tag{1}$$

where w_F is the crack opening at zero stress. This is obviously also the external energy required to create and fully break a unit surface area of crack. The G_F of mature concrete, determined by either direct or indirect tensile tests, normally ranges between 40 J/m^2 and 150 J/m^2 (Elfgren 1989, Karihaloo 1995). It is influenced to varying degrees by both microstructural and environmental/testing factors, including the aggregate size and quality, water-cement ratio, age of concrete, and loading rate (Petersson 1980a, b, Wittmann et al. 1987, Karihaloo 1995). Generally, the higher the strength of the concrete, the higher the fracture energy. This is in stark contrast to elastic-plastic metals whose fracture energy decreases with increasing tensile strength.

However, the increase in the fracture energy of concrete with its strength should not be confused with an increase in its ductility. The fracture energy alone cannot distinguish ductility from brittleness, nor does the reverse apply. Additional parameters must be evaluated to make this distinction. It has been suggested that the characteristic length, l_{ch}, which is defined by

$$l_{ch} = \frac{EG_F}{f_t^2} \tag{2}$$

can be used as an inverse measure of the brittleness of the concrete. The smaller the characteristic length, the more brittle the material, and vice versa (Hordijk et al. 1989, Karihaloo 1995, De Schutter & Taerwe 1997). The characteristic length l_{ch} is also the length of the FPZ. For most concretes, l_{ch} appears to range between 200 mm and 400 mm (Hordijk et al. 1989), but can sometimes have values greater than 1000 mm (Hillerborg 1985). Other typical values for the characteristic length of mature-age concrete and other materials are listed in Table 1.

Table 1. Typical characteristic lengths of mature concrete and other materials (Karihaloo 1995).

Material	l_{ch} (mm)
Glass	10^{-6}
Cement paste densified by silica fume	1
Hardened cement paste	5 - 15
Mortar	100 - 200
High strength concrete (50-100 MPa)	150 - 300
Normal concrete (agg. size smaller than 20 mm)	200 - 500
Dam concrete (agg. size of 38 mm)	700

It is noticeable from Table 1 that normal concrete is less brittle than high strength concrete but more brittle than dam concrete, which comprises larger aggregates. That is, the brittleness decreases with increasing aggregate size but increases with increasing concrete strength. Also, a comparison of the data for hardened cement paste, paste densified by silica fume, and high strength concrete with that for normal strength concrete indicates that the more compact the microstructure of the mix, the more brittle the material.

Despite the successful application of fracture mechanics to study the fracture behavior of mature concrete, its application to the early-age cracking of concrete is still in its infancy, with very limited literature available. A summary of relevant literature on the fracture behavior of early-age concrete is presented in Table 2 (Dao et al. 2009). In the often-mentioned state-of-the-art report on the properties of concrete at early ages (RILEM committee 42-CEA

Table 2. Summary of literature on fracture behavior of early-age concrete (Dao et al. 2009).

Study	Testing	Conclusions
(Ostergaard et al. 2004)	Wedge-splitting test to determine stress-crack opening relationships of two high-performance concrete mixes with w/c of 0.31 and 0.48 at ages between 8 h and 28 days.	Tensile strength, Young's modulus, and fracture energy were found to increase with age. However, ductility expressed in terms of characteristic length was found to decrease with age.
(Zollinger et al. 1993)	Three-point bending test of concrete aged between 12 h and 28 days.	Increases in both critical stress intensity factor and critical effective crack length or process zone size with age were observed, indicating that early-age concrete is more brittle than mature concrete.
(Kim et al. 2004)	Wedge-splitting test for concrete of ages from 1 day to 28 days.	Both critical stress intensity factor and fracture energy increase with age, especially at early ages, and converge to a limit at 28 days. Similar trends were noted by (Zollinger et al. 1993).
(De Schutter & Taerwe 1997)	Three-point bending test of unnotched prisms of 150 mm x 150 mm x 600 mm; span 500 mm; concrete aged between 1 day and 28 days	Results similar to those of most other researchers were observed.
(Morris & Dux 2005)	Using LEFM to estimate fracture energy of cement mortars aged up to 8 h from mixing, by measuring the total suction and assuming crack depth.	Fracture energy was estimated about 73 N/m, which suggested that plastic cracking of cement mortar involves a significant zone of plastic straining or microcracking adjacent to the crack tip.

1981), nothing can be found concerning fracture energy. Even at a relatively recent major international conference concerning early-age concrete (Kovler et al. 2004), little attention was paid to the experimental study of the evolution of the softening behavior. The dearth of available literature on the fracture behavior of early-age concrete is principally the result of difficulties in the experimental determination of the tensile stress-separation curves of early-age concrete (Dao et al. 2009). Consequently, a new test apparatus and experimental procedures have been developed at the University of Queensland (Dao 2007), which have enabled the collection of reliable data on the complete tensile stress-displacement behavior of concrete specimens at ages of 1.5 h or more after mixing. The variation of the fracture energy with the age and the tensile strength of early-age concrete are plotted in Figures 3-4, respectively. Other properties that are crucial for the study of concrete at these very early ages are described in Dao et al. (2009).

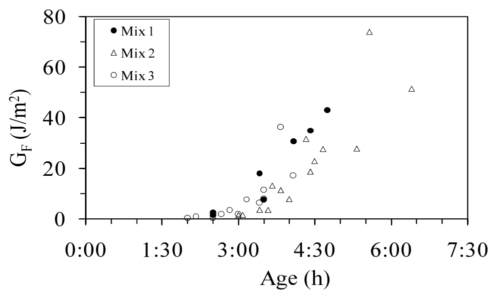


Figure 3. Fracture energy versus age of concrete.

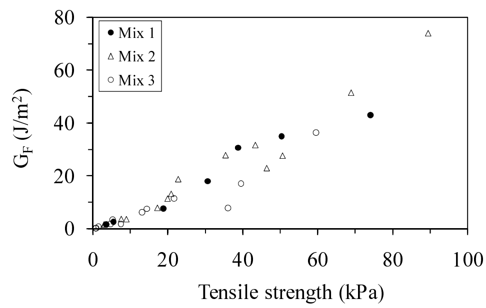


Figure 4. Fracture energy versus tensile strength of concrete.

2 STRESS-SEPARATION RELATIONSHIPS

Various models, including linear, bi-linear and non-linear functions (Hillerborg et al. 1976, Gopalaratnam & Shah 1985, Reinhardt 1985, Foote et al. 1986, Liaw et al. 1990, Karihaloo 1995, Elices et al. 2002), have been proposed for the stress-separation relationships of mature concrete. Based on these models for mature concrete and on the early-age tensile strengths and fracture energies in (Dao et al. 2009) (Fig. 4), the following models for the softening behavior of concrete at very early ages have been developed. In all cases, best fits were obtained by ordinary least-squares methods.

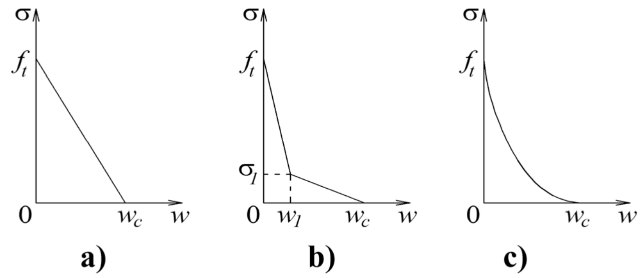


Figure 5. Idealized stress-separation relationships.

2.1 *Linear model*

The softening function for linear stress-separation behavior (Fig. 5a) is

$$\sigma = f_t\left(1-\frac{w}{w_c}\right) \tag{3}$$

The fracture energy defined in Equation 1 thus becomes

$$G_F = \int_0^{w_c} \sigma(w)dw = \frac{f_t w_c}{2} \quad (4)$$

The best-fit value for w_c obtained for the data plotted in Figure 4 is 1.31 mm, about two orders of magnitude higher than typical values of between 0.01 mm and 0.02 mm for mature concrete (Hillerborg et al. 1976). It is, moreover, significantly smaller than the value of about 5 mm obtained from the experimental study of early-age concrete by Dao et al. (Dao et al. 2009). Consequently, the linear model, which is also inconsistent with the corresponding experimental stress-separation relationships, is inappropriate for concrete at very early ages.

2.2 *Bi-linear model*

The softening function for the bi-linear stress-separation behavior (Fig. 5b) is

$$\sigma = f_t - (f_t - \sigma_1)\frac{w}{w_1} \quad \text{for } 0 \le w \le w_1 \quad (5)$$

and

$$\sigma = \sigma_1 - \frac{\sigma_1(w - w_1)}{(w_c - w_1)} \quad \text{for } w_1 < w \le w_c \quad (6)$$

The fracture energy (Equation 1) is thus

$$G_F = \frac{1}{2}(f_t - \sigma_1)w_1 + \frac{1}{2}\sigma_1(w_c - w_1) + \sigma_1 w_1 \quad (7)$$

Adopting the experimentally determined value of 5 mm (Dao et al. 2009) for w_c, the best-fit values for w_1 are 0.808 mm, 0.558 mm, and 0.308 mm for values of σ_1 of $0.1f_t$, $0.15f_t$, and $0.2f_t$ respectively. This range of values for σ_1 is consistent with both the theoretical model (Fig. 5b) and the experimental stress-separation relationships for early-age concrete obtained by (Dao et al. 2009). Obviously, when σ_1 equals zero, the model becomes linear (Equations 3 and 4). For values of σ_1 greater than $0.25f_t$, w_c becomes very small or even negative.

These results indicate that the bi-linear model, which has the virtue of simplicity, can model the stress-separation curve sufficiently accurately for practical purposes.

2.3 *Non-linear model*

A number of non-linear functions for the stress-separation behavior (Fig. 5c) have been suggested (Gopalaratnam & Shah 1985, Reinhardt 1985, Foote et al. 1986). Here, the following two non-linear functions are investigated:

$$\sigma = f_t\left(1 - \frac{w}{w_c}\right)^m \quad (8)$$

$$\sigma = f_t\left[1 - \left(\frac{w}{w_c}\right)^n\right] \quad \text{where } 0 < n < 1 \quad (9)$$

Combining Equations 5 and 9 with Equation 1 while adopting the experimentally determined value of 5 mm (Dao et al. 2009) for w_c, gives best-fit values for m and n of 6.647 and 0.151, respectively. The latter is close to the corresponding value of n for mature concrete, which ranges between 0.2 and 0.4 (Reinhardt 1985).

3 SUMMARY AND CONCLUSIONS

In this paper, the fracture mechanics of concrete structures has been briefly reviewed. This has highlighted clearly the limitations of the application of fracture mechanics to the early-age cracking of concrete, despite its successful application in the study of the fracture behavior of mature concrete.

Mathematical models for the stress-separation relationship of early-age concrete have been derived based on recent experimental data for concrete at ages of 1.5 h or more after mixing. It has been demonstrated that not all existing stress-separation relationships for mature concrete are applicable to concrete at very early ages, and the revised models proposed in this paper should be preferred. The bi-linear model appears to be the best of the three models investigated. However, further work is being carried out by the authors to collect more data and refine the revised stress-separation relationships.

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