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Theoretical development of CP method in predicting expansive cement concrete cracking

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ABSTRACT: The CP method is extended in order to give more precise prediction of the expansive cement effects on thermal cracking prevention for early age concrete. The feature of the extended CP method is based on the conservation law of chemical energies, which is quite different concept than the initial strain calculation that has been used extensively in the past. Then, the formulation for thermal stress analysis based on the law is proposed. Finally, the effect of expansive cement on reducing thermal stress is numerically clarified using the extended CP method.

1 INTRODUCTION

In Japan, technologies to control thermal cracking of massive concrete structures has been remarkably developed for the last few decades. Most major contribution to predict thermal cracking may be development of compensation plane method (CP method) initially proposed by the second author and later reinforced in the parts of restraining factors by the Technical Committee on Thermal Stress of Massive Concrete of JCI (1985). The JSCE Standard (2002) makes comments that the calculation method should be incrementally linear FEM or the equivalent and refers to the CP method for design and construction of concrete structures.

The CP method consists with an assumption that a section plane remains plane or a normal vector to the middle plane remains normal after deformation in blocks where concrete ages are still very young and varies with ages. It is widely recognized that the method enables to calculate hygro-thermal stress in pseudo three dimensionally and quite accurately.

Now, in preventing initial cracking or in concrete retrofitting, the use of expansive cement has been increasing rapidly and sometimes used as key engineering to comply with these needs. However, the problem of analytical prediction of its effect still remains as an unsolved matter and a new development of an analytical tool for the prediction is keenly requested.

In this study, the extended CP method is presented in order to give more precise prediction of the expansive cement effects on of thermal cracking prevention for early age concrete. The feature of the proposed method is based on the conservation law of chemical energies. It has become evident in the past research that chemically induced mechanical energy is constant for different amount of restraints for a unit volume of expansive cement. This law has been originally recognized by Tsuji (1988), through his experiments. The authors have examined the law and recognized some modification is necessary for Tsuji's proposal. However, the authors confirmed the general validity of the law experimentally under biaxially restrained conditions as well as uniaxially restrained condition which we think as quite comprehensive with our modification. The details are written in the paper with an example of the use of expansive cement on thermal cracking prevention with the theory of the extended CP method.

2 THE CONSERVATION LAW OF CHEMICAL ENERGY AND ITS VERIFICATION FOR EXPANSIVE CEMENT COCRETE

2.1 *Concept of the conservation law of chemical energy*

According to the first law of thermodynamics, energy conservation in a general structural system can be expressed as

$$\Delta Q + \Delta H + \Delta M = 0 \tag{1}$$

where ΔQ is internal energy acquired in the system, ΔH is heat transfer to outer environments and ΔM is mechanical energy transferred to outer environments. Assuming $\Delta H = 0$, i.e. the system is isothermal, the energy conservation yields

$$-\Delta Q = \Delta M \tag{2}$$

Now, consider that chemical expansion exerts the work for the outside region of the chemically expanding material as shown in Figure 1. The region A expands while the region B restrains its expansion. In this case, ΔM will correspond to mechanical works exerted by all materials composing the structural system, i.e. region A and region B. Therefore, Equation (1) indicates the chemical energy developed by the expansive cement is equal to the strain energy induced in the material.

More precisely, work $d\xi_B$ done by the region A to the region B, i.e, the area outside of the region A can be given by the next equation.

$$d\xi_B = \oint R ds = \int_{V_B} \sigma : d\varepsilon dV_B + \sum_i P_i du_i \tag{3}$$

where R and s are forces and displacement vectors at the boundary of the expanding portion A, respectively. V_A and V_B are volumes of region A and B, respectively. P_i and u_i are external force and corresponding displacement vectors, respectively. σ and ε are stress and strain tensor, respectively. Inversely, the work done to the region A by the outside area of A can be given as

$$-\xi_B = -\oint R ds = \int_{V_A} \sigma : d\varepsilon dV_A \tag{4}$$

Decomposing Equation (4) and considering creep and shrinkage components, the right-hand terms of Equation (4) may yield as follows

$$\begin{aligned} &\int_{V_A} \sigma : d\varepsilon dV_A \\ &= \int_{V_A} \sigma : \left(d\varepsilon^{e} + d\varepsilon^{cr} + d\varepsilon^{sh} + d\varepsilon^{che}\right) dV_A \\ &= \int_{V_A} \sigma : d\varepsilon^{e} dV_A + \int_{V_A} \sigma : d\varepsilon^{cr} dV_A \\ &+ \int_{V_A} \sigma : d\varepsilon^{sh} dV_A + \int_{V_A} \sigma : d\varepsilon^{che} dV_A \\ &= d\xi_A^{e} + d\xi_A^{cr} + d\xi_A^{sh} + d\xi_A^{che} \end{aligned} \tag{5}$$

in which superscripts e, cr, sh and che denote elastic, creep, shrinkage and chemical expansion, respectively. Substituting Equations (4) and (5) into Equation (3), the following equation can be obtained

$$-d\xi_A^{che} = d\xi_B + d\xi_A^{e} + d\xi_A^{cr} + d\xi_A^{sh} \tag{6}$$

where

$$-d\xi_A^{che} = \int_{V_A} \sigma : d\varepsilon^{che} dV_A \tag{7}$$

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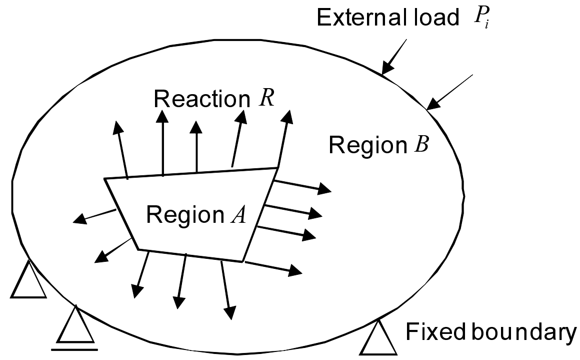
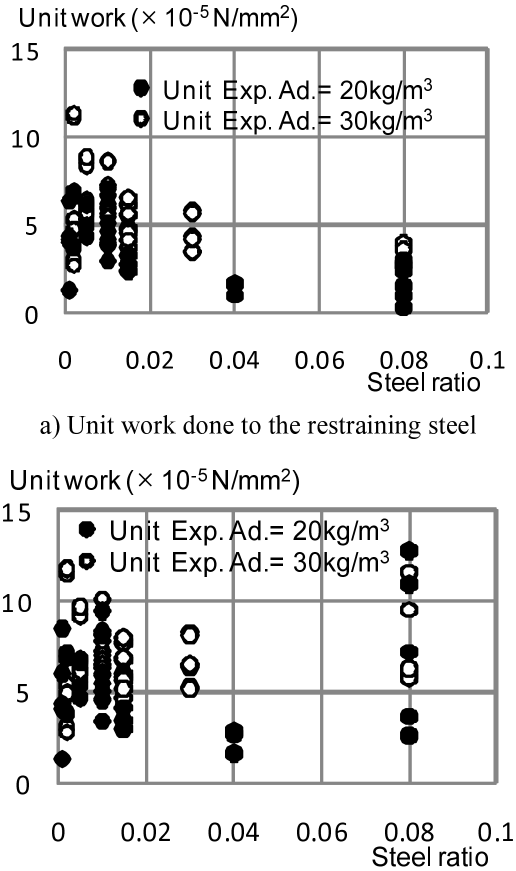


Figure 1. A total structure including expanding portion.



a) Unit work done to the restraining steel

b) Summation of unit work done to the restraining steel and the expansive cement concrete itself

Figure 2. Relationship between steel ratio and unit work.

The first term of the right-hand side of Equation (6) is the work done for outside or for a restraint. However, the second to the last terms of the right-hand side are the works done for the expanding material itself. According to Equation (2), chemical energy $d\xi_A^{che}$, which is developed as the results of chemical reaction, are equal to the sum of the work done for the outside and the work done for itself mechanically.

The law requires $d\xi_A^{che} = const$ and $d\varepsilon^{che}$ must be positive for chemical expansion and negative for chemical contraction.

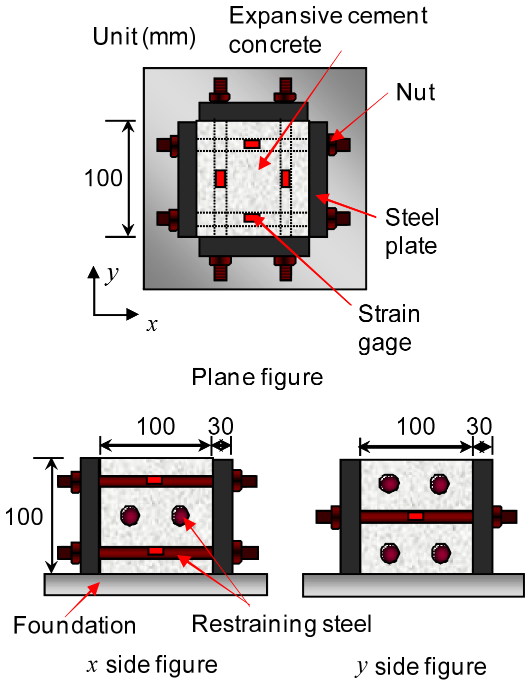


Figure 3. Outline of the apparatus

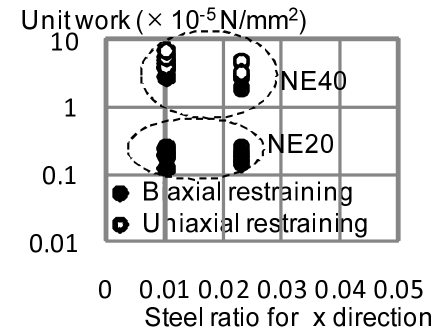
2.2 *Verification of the conservation law of chemical energy for expansive cement concrete*

The conservation law of chemical energy has been firstly recognized by Tsuji et al. through his experiments as has been mentioned. Since then, much effort has been poured into experimental investigations of the behavior of expansive cement concrete with emphasis on the verification of the law under uniaxial restraint condition by many researchers. The authors have carried out a survey of experimental results of uniaxial restraint tests of expansive cement concrete which have been already published in the literature, and verified unit work done to restraining steel bar and expansive concrete itself with varying steel ratio shown in Figure 2 (Ishikawa et al. 2008). In the figure, the unit work at 3 days is plotted. It is found that summation of these works i.e. chemical energy may be almost invariant despite of steel ratio. It should be emphasized here that this finding has corrected the mislead opinion that the chemical energy reduces with the increase of steel ratio.

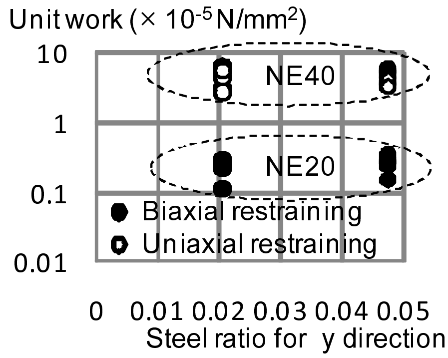
In addition to it, the authors have examined whether or not the chemical conservation law of expansive cement concrete will consists under biaxial restraint conditions (Hayashi et al. 2009). The experiments have been carried out using original apparatus shown in Figure 3. Then, evaluation of summation of unit work done to the restraining steel and the expansive cement concrete itself, referred to as "unit work", are measured. Mix proportion used in the experiments is such that unit water content is 175 kg/m^3, water-binder ratio is 57%, maximum aggre-

Table 1. Unit work at 3 days with various restraining conditions.

Unit Expansive Admixture	Steel ratio for x direction	Steel ratio for y direction	Unit work for x direction (10^{-4}N/mm^2)	Unit work for y direction (10^{-4}N/mm^2)
	0.01	0.02	2.80	3.05
			5.00	6.52
			4.21	5.67
		0.05	3.63	5.09
			3.98	5.73
	0.02	0.02	4.55	5.81
			2.75	5.77
		0.05	1.90	3.90
			3.26	5.60
			3.22	5.04
NE40	0.01	0.00	3.88	-
			5.40	
			6.83	
	0.02	0.00	2.85	-
			4.83	
			3.16	
	0.00	0.02	-	2.70
				4.42
				5.62
	0.00	0.05	-	3.45
				4.04
				3.34
	0.01	0.02	0.12	0.11
			0.23	0.25
			0.25	0.29
		0.05	0.19	0.24
			0.18	0.25
NE20			0.20	0.28
	0.02	0.02	0.20	0.26
			0.15	0.22
			0.20	0.27
		0.05	0.15	0.15
			0.16	0.28
			0.24	0.34



a) For x direction



b) For y direction

Figure 4. Relationship between unit work and steel ratio.

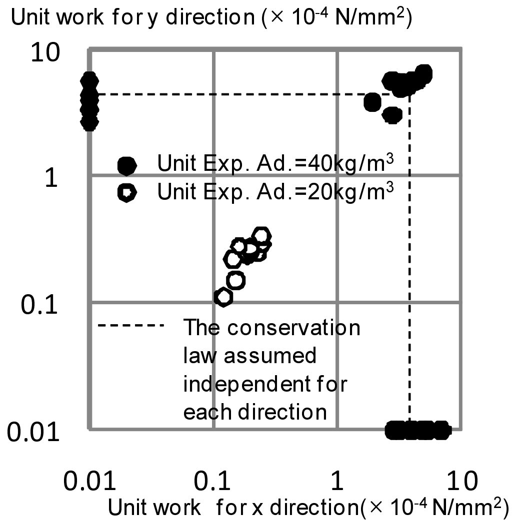


Figure 5. Relationship between unit work for two directions.

gate size is 20mm and sand-total aggregate ratio is 47%. Low-addition type expansive additive of lime system is used as expansive admixture. Then, the unit work is indirectly estimated by the steel strain histories measured under constant environmental temperature of 20 deg C. The unit work at 3 days with various steel ratio obtained from the experiments are listed in Table 1. "NE20" and "NE40" mean that unit expansive admixture contents are 20 and 40 kg/m^3, respectively.

The unit work and steel ratio for each direction relationships are shown in Figure 4. Figure 5 shows relationship between summation of the unit work at 3 days for x, y directions. The unit work under uniaxial restraining case is plotted on the axes, which has been carried for only the case of NE40.

In these figure, under biaxial stress conditions, it will be said that the chemical work seems to be independent on the direction and constant regardless to the direction or steel amount, which gives significant advantages to 3D FEM calculation. It is also shown that the chemical energy may logarithmically increase with increasing the unit admixture content proportionally.

3 EXTENSION OF CP METHOD IN PREDICTING EXPANSION CEMENT CONCRETE CRACKING

3.1 *Formulation of the extended CP method based on the chemical conservation energy*

In this section, the CP method is extend to consider chemical expansion problem based on the conservation law of chemical energy in order to enable this method to predict thermal cracking problem using expansive cement concrete. The basic concept of the extended CP method is quite the same as the conventional one, which has been world-widely known. Given a section composed of expansive cement concrete, shown in Figure 6, temperature rises in the section, probably very high in the inside and somewhat lower in the outer side. The compensation plane is defined for the situation such that the free initial strain deducted by the strain on the compensation plane multiplied with rigidity of material, induces zero axial force and zero flexural moment to the section. At expansive cement concrete region, the free initial strain is a summation of thermal and chemical expansive strain which is not known yet. Then, $\Delta\bar{\varepsilon}$ and $\Delta\bar{\phi}$, which determine axial location and inclination of the compensation plane, will be given as

$$\Delta\bar{\varepsilon} = \frac{E_f \int_{A_f} \Delta\varepsilon^t dA_f}{EA} + \frac{E_C \int_{A_C} \left(\Delta\varepsilon^t + \Delta\varepsilon^{che}\right) dA_C}{EA} + \frac{E_S \int_{A_S} \Delta\varepsilon^t dA_S}{EA} \tag{8}$$

$$\Delta\bar{\phi} = \frac{E_f \int_{A_f} \Delta\varepsilon^t (y - y_g) dA_f}{EI} + \frac{E_C \int_{A_C} \left(\Delta\varepsilon^t + \Delta\varepsilon^{che}\right)(y - y_g) dA_C}{EI} + \frac{E_S \int_{A_S} \Delta\varepsilon^t (y - y_g) dA_S}{EI} \tag{9}$$

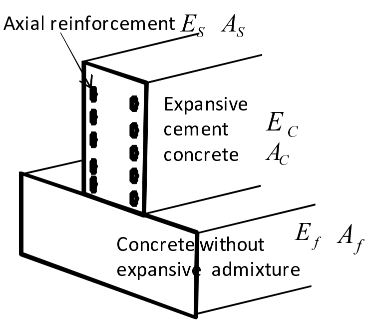


Figure 6. Section containing expansive cement concrete.

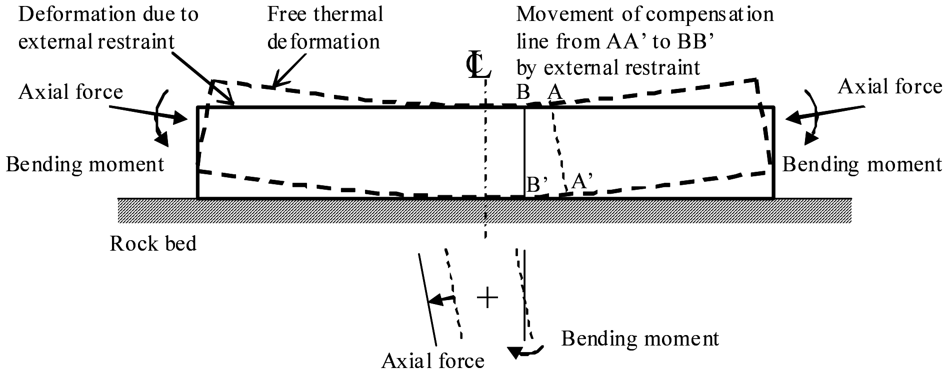


Figure 7. Externally restrained condition.

$$EA = E_f A_f + E_C A_C + E_S A_S \quad (10)$$

$$EI = E_f \int_{A_f} (y - y_g)^2 dA_f + E_C \int_{A_C} (y - y_g)^2 dA_C + E_S \int_{A_S} (y - y_g)^2 dA_S \quad (11)$$

in which E and A denotes Young's modulus and area, respectively, $\Delta\varepsilon^t$ and $\Delta\varepsilon^{che}$ are free thermal and free chemical expansive strains, respectively, y is a normal coordinate axis and y_g is a center of gravity of the section. The subscript f, C and S denote concrete without expansive admixture, expansive cement concrete and axial reinforcement region, respectively. The internally restrained initial stress increments $\Delta\sigma_{(I)}$, which induces zero axial force and zero flexural moment to the section, can be written as

$$\begin{aligned}
\Delta\sigma_{f(I)} &= E_f \left\{\Delta\bar{\varepsilon} + (y - y_g)\Delta\bar{\phi} - \Delta\varepsilon^t\right\} \\
\Delta\sigma_{C(I)} &= E_C \left\{\Delta\bar{\varepsilon} + (y - y_g)\Delta\bar{\phi} - \Delta\varepsilon^t - \Delta\varepsilon^{che}\right\} \\
\Delta\sigma_{S(I)} &= E_S \left\{\Delta\bar{\varepsilon} + (y - y_g)\Delta\bar{\phi} - \Delta\varepsilon^t\right\}
\end{aligned} \quad (12)$$

Once the plane is obtained for external restraint free condition, the compensation plane moves according to the degree of the external restraints.

The external restraints act to restrict axial movement of the plane as well as to restrict flexural rotation of the plane. This situation is shown in Figure 7.

Therefore, the external restraining actions are two folds. One is the axial restraint and the other is the flexural restraint. Then the externally restrained initial stress increment $\Delta\sigma_{(R)}$ is given in analogy with a beam column problem as

$$\begin{aligned}
\Delta\sigma_{f(R)} &= -E_f \left\{R_N \Delta\bar{\varepsilon} + R_M (y - y_g)\Delta\bar{\phi}\right\} \\
\Delta\sigma_{C(R)} &= -E_C \left\{R_N \Delta\bar{\varepsilon} + R_M (y - y_g)\Delta\bar{\phi}\right\} \\
\Delta\sigma_{S(R)} &= -E_S \left\{R_N \Delta\bar{\varepsilon} + R_M (y - y_g)\Delta\bar{\phi}\right\}
\end{aligned} \quad (13)$$

in which R_N and R_M are external restraining coefficients. These coefficients have been fully investigated by the Technical Committee on Thermal Stress of Massive Concrete of JCI.

After all, total stress increment $\Delta\sigma$ in the section can be expressed as summation of internally and externally restrained stress increments, i.e.

$$\begin{aligned}
\Delta\sigma_f &= \Delta\sigma_{f(I)} + \Delta\sigma_{f(R)} \\
\Delta\sigma_C &= \Delta\sigma_{C(I)} + \Delta\sigma_{C(R)} \\
\Delta\sigma_S &= \Delta\sigma_{S(I)} + \Delta\sigma_{S(R)}
\end{aligned} \quad (14)$$

In the conventional CP method, total stresses can be obtained directly from the compensation plane change, which can be calculated from explicit free initial strain such as temperature change. However, according to Equation (7), calculation should be based on the chemical energy conservation law, i.e. it should keep the free chemical expansive strain multiplied stress constant at every point in the section, which can be given as

$$\left\{\sigma_{C,0} + \Delta\sigma_C\right\}\Delta\varepsilon^{che} = -\Delta U^{che} \quad (15)$$

in which $\sigma_{C,0}$ is stress of young concrete at previous step and ΔU^{che} is a chemical expansion energy increment. So, this is not an initial strain problem. The free chemical expansive strain $\Delta\varepsilon^{che}$ will be given as the results of the calculation and as the function of restraints. The equation enabled to reduce the problem into an iteration problem to find the free chemical expansive strain that satisfies the chemical energy conservation law.

However, it should be noted that the conservation law of chemical energy can be available under non-zero compressive stress field of concrete and one needs the critical limit value for it; otherwise the initial strain will go up to the value of infinity when the compressive stress is very small.

In this study, the critical limit value is assumed to be -0.1 N/mm^2, which is positive for tension. As for this point, more detail investigation should be carried in the future.

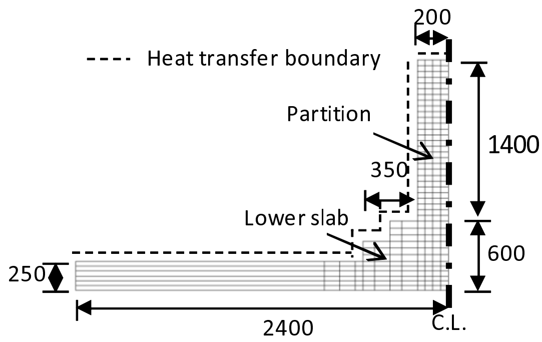


Figure 8. Analytical model of RC girder structure.

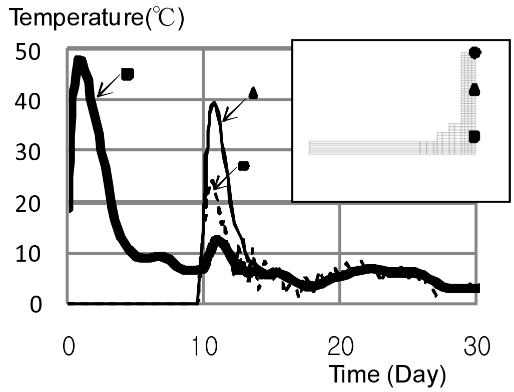


Figure 9. Temperature histories obtained from the CP method.

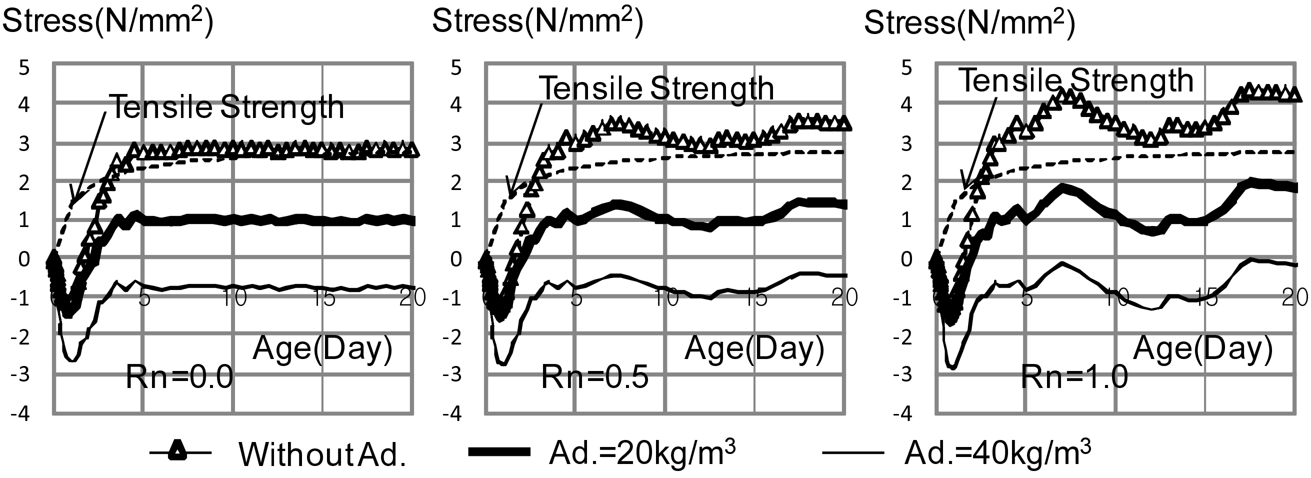


Figure 10. Stress variation calculated by the extended CP method.

3.2 *Numerical example of effect of expansive cement concrete on thermal cracking prevention*

In this section, the effect of expansive cement concrete on thermal cracking prevention will be numerically discussed using the really constructed example of a girder structure.

A section of the girder is roughly shown in the Figure A1 of Appendix. It consists of upper and lower slab connected with five partitions, i.e. four box section.

The lower slab is simply supported by the hardened RC piers and the lower slab is casted at Nagoya region in December 15th and 10 days later the five partitions are casted on the lower slab. A part of the lower slab and partition are the target portion of crack prediction by the extended CP method shown in Figure 8 considering the sequence of construction. Reinforcement ratio of the target portion is assumed to be zero, although the effect of reinforcement is considered in the proposed method. Heat and material properties used in the analyses are listed in Table 2 in which t denotes concrete age (day). Variation of compressive strength, compressive and tensile strength relationship, and compressive strength and Young's modulus relationship are also substantially based on the JSCE standard (2002). Temperature histories of representative points at the section are shown in Figure 9.

Table 2. Thermal and material properties used in the analyses.

Heat conductivity(W/m K)	2.7
Specific heat(J/kg K)	1100
Density(kg/m^3)	2350
Adiabatic Temperature rise Q (K)	$Q = 72.98(1-\exp(-1.19t))$
Heat transfer coefficient(W/m^2 K)	14
Variation of compressive strength(N/mm^2) f_c'	$f_c' = \frac{42.8t}{2.9+0.95t}$
Tensile strength(N/mm^2) f_t	$f_t = 0.44\sqrt{f_c'}$
Young's modulus(N/mm^2) E	$E = 4700\sqrt{f_c'}$
Thermal expansion coefficient (1/K)	0.00001

In the extended CP method, externally restraining coefficients must be estimated. Determination of these coefficients has been already established for wall or slab structure casted on ground by the Technical Committee on Thermal Stress of Massive Concrete of JCI. However, these coefficients has been not clarified yet for structures in which girder is casted on support such as pier and foothold. In this study, flexural restraining coefficient is assumed to be 1.0 because the length of RC girder is very large compared to its height. Then, the values of axial restraining coefficient (Rn) are assumed to set 0.0, 0.5 and 1.0 because real axial restraining coefficient will exist within the range from 0.0 to 1.0.

In the proposed method, one needs variation of the chemical expansive energy ΔU^{che} as well as the parameters of the conventional CP method. ΔU^{che} can be determined by experimental investigation previously mentioned in section 2.2. The chemical energy U^{che} is assumed to be given as the following functions of concrete age corresponding to unit expansive admixture content (Ad.)

$$U_{che} = 6.0\times10^{-5}(1-\exp(-t)) \quad \text{Ad.=20kg/m}^3 \qquad (16)$$

$$U_{che} = 6.0\times10^{-4}(1-\exp(-t)) \quad \text{Ad.=40kg/m}^3 \qquad (17)$$

in which unit of ΔU^{che} is N/mm^2 and t is concrete age. According to the JSCE standard, Ad.=20kg/m^3 means the unit amount which can reduce initial cracking width and Ad.=40kg/m^3 means that adequate chemical prestress will be given to RC members.

The results of the extended CP method is shown in Figure 10, in which dotted line represents variation of tensile strength, while the stress development with varying unit expansive admixture content are shown by two solid lines and one triangular mark. These results indicate that thermal cracking will occur in the partition within a few days if chemical expansive admixture is not used. It can be also con

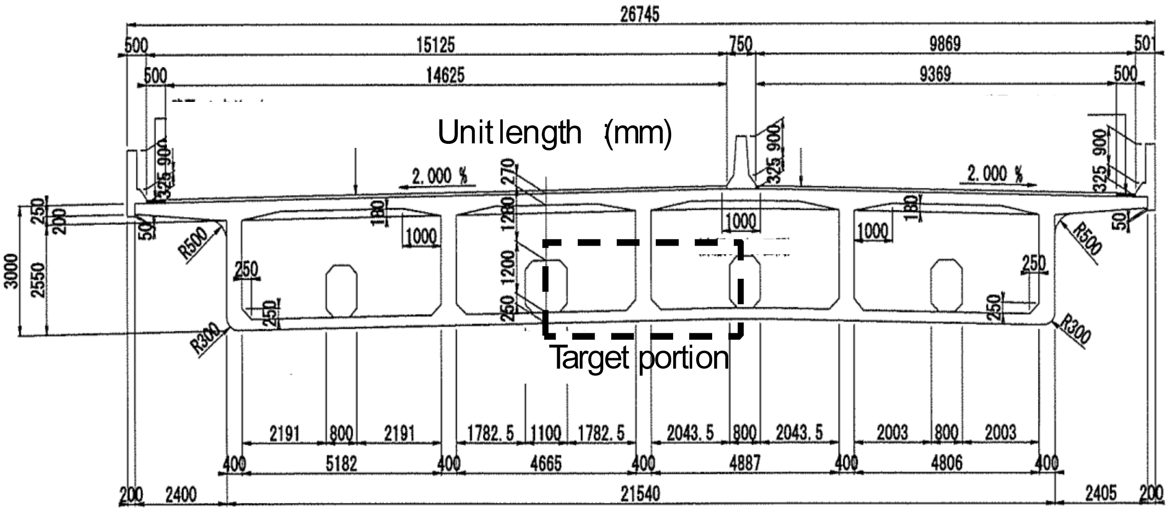


Figure A1. A section of the girder structure.

firmed from the results obtained from three dimensional FEM as shown in the Appendix. In the extended CP method, the effect of chemical admixture can be easily considered if only the chemical energy is given. Moreover, the extended CP method can give rational reduction of tensile stress in accordance with chemical admixture content.

4 CONCLUSION

In this study, the CP method is extended in order to give more precise prediction of the expansive cement effects on of thermal cracking prevention for early age concrete. The key concept of the chemical energy conservation law is newly presented for the prediction. Then, thermal stress analysis using the extended CP method is carried and discussed. The effect of expansive cement on reducing thermal stress is numerically examined and the results are found to be reasonable. However, more detail investigation about the stress range in which the conservation law of chemical energy is valid should be carried.

AKNOWLEDGEMENT

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APPENDIX: THE EXAMINED PC HIGH WAY BOX GIRDER WHICH IS CONSTRUCTED RECENTLY

A section of the girder structure is roughly shown in A1 in which an analytical region for the CP method is surrounded by dotted line. Analytical model for 3DFEM is also shown in Figure A2.
The girder connects two piers with rubber supports.
Heat and material properties used in 3DFEM are quite the same as those of the CP method.
The results without chemical admixture of the extended CP method, i.e. the results of the CP method are compared with the results of three dimensional finite element method (3DFEM).
Variation of stresses at a partition calculated by the two methods is shown in Figure A3 Stress means the maximum principal stress in 3DFEM and the stress normal to the section in the CP method. It is found that stress variation in 3DFEM is quite close to the stress variations in case that the axial restraining coefficients are set 0.0 and 0.5.

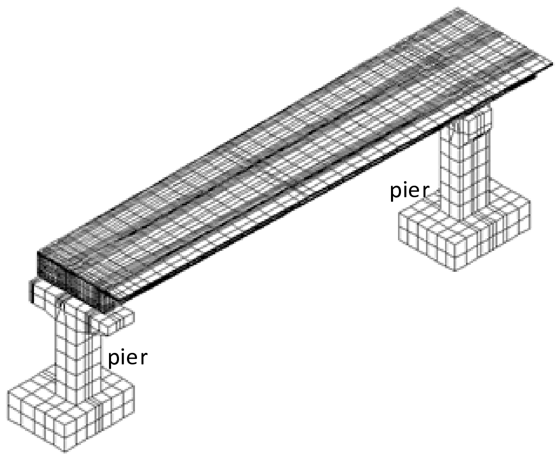

Figure A2. Analytical model for 3DFEM.

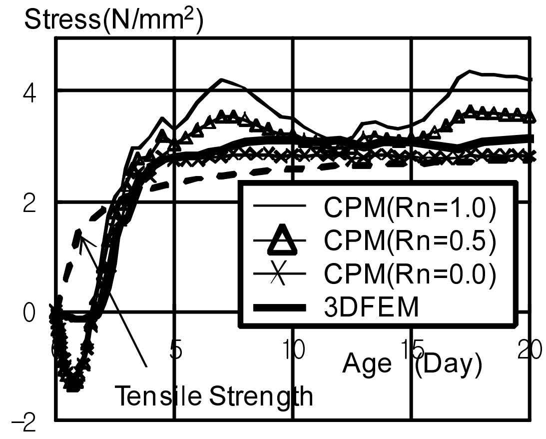

Figure A3. Stress variation calculated by the CP method(CPM) and 3DFEM.