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Modelling concrete structures applying XFEM with a mixed mode constitutive model

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ABSTRACT: A mixed mode fracture model has been implemented within the framework of the eXtended Finite Element Method (XFEM). The applied mixed mode model is build on the basis of elastoplasticity combined with damage, and consists of a friction part and a cohesion part. The friction part is deformation state dependent, and is therefore capable of reducing the friction capacity when the crack opens, and rebuild it when it closes. The cohesion part is coupled with damage, which makes the model capable of exhausting the cohesion in an irreversible process when the crack opens. A generalized crack-tip element based on XFEM has been applied. The XFEM element is built on the basis of Linear Strain Triangular (LST) element. The element can be used as both fully and partly cracked, which makes it possible to model continuous crack growth. A double notched specimen in a mixed mode test setup has been modelled with the present formulation, and the results are compared with experimental work.

1 INTRODUCTION

In reinforced concrete beams cracks initiate in the tension part. In the beginning the cracks will be dominated by opening (Mode I). The initiated cracks will propagate towards a rebar, where they will initiate new cracks along the rebar (debonding). These cracks are mainly governed by sliding of the crack faces (Mode II). This means that the fictitious crack model by Hillerborg (1989) which only describes Mode I opening, is not sufficient when modelling cracks in reinforced concrete where both Mode I and II occur. A detailed constitutive model describing this mixed mode fracture process is needed. Work by Carol (1997) suggests such a mixed mode model. This model has a lagging capability of a realistic unloading of the crack faces. This is included in a new model by Nielsen (2009), so it not only can handle monotonic loading of the crack, but also different load combinations of the crack e.g. opening followed by closing and finally sliding of the crack faces.

When modelling cohesive crack growth the eXtended Finite Element Method has proven to be an efficient tool, see Belytschko (1999). Applying this method a discrete crack can freely propagate through the element mesh. Recently a lot of work has been put into formulations of partly cracked crack-tip elements. A crack-tip element is suggested by Zi (2003), which is appealing simple, but does not have the precision to give a realistic stress field within the tip element, and therefore a non smooth load-deflection response. Further enrichments within the crack-tip element have been introduced by Mougaard (2009). This new crack-tip element gives a rather precise solution in the near surroundings of the crack-tip. This means that the overall behaviour of the cohesive crack can be captured using relatively few elements along the crack. So far only a simple constitutive law including Mode I have been applied with the XFEM elements.

The overall scope of this work is to be able to model reinforced concrete. A reinforced concrete beam could with the present work be modelled using XFEM elements to formulate cracks in the pure concrete and interface elements to formulate cracks along rebars. The modelling presented in this work can be seen as preliminary tests before modelling reinforced concrete structures. Experiments with the intension to describe material point behaviour of concrete fracture will in the present work be modelled as a structure.

2 XFEM

The mixed mode specimen is modelled using Linear Strain Triangular (LST) elements. At crack initiation, XFEM elements are introduced.

Applying XFEM a set of additional shape functions are used within each cracked element. These additional shape functions (enrichments functions) are traditionally chosen identical to the original shape functions. This concept gives the possibility of a complete decoupling of displacements over the discontinuity (the crack) i.e. two separate continuums. One very strong benefit from this, is that cracks can be introduced without any re-meshing.

A generalized crack-tip element which can be used both as partly and fully cracked is used. The crack-tip element is presented in Mougaard (2009). The element is based on two symmetrical placed enrichment fields, which makes the element capable of reproducing equal stresses on both sides of the crack at the crack-tip. Applying this enhanced crack-tip element accurate results can be obtained even with rather coarse meshes as reported in Mougaard (2009).

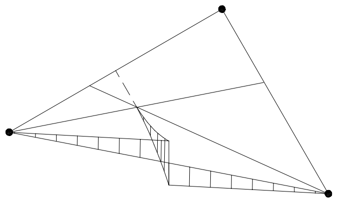

Figure 1. Displacement field for the crack-tip element. Displacements are shown perpendicular to the element plane.

So far, when modelling cohesive crack growth with XFEM a simple cohesive traction-separation-law has been applied. Here the tractions and stiffness are explicit given by the opening of the crack, see Hillerborg (1989). In the present work a more general constitutive law is introduced for the cohesive crack. In that context it is essential to know how the constitutive points (CP's) are positioned in the element.

For the fully cracked element the CP's are located in each end of the crack, where they are fixed. For the partly cracked element one CP is placed where the crack enters, and one is placed at the crack-tip see Figure 2. The CP at the crack-tip will move when the crack propagates, in general when a CP is moved it needs to be updated from the old constitutive state. Here it is a little simpler, since the CP is placed at the crack-tip, where the constitutive state is known, and cannot change.

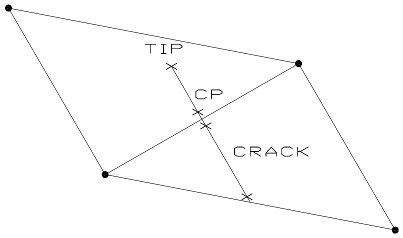

Figure 2. Location of constitutive points in the partly and fully cracked XFEM element.

Using one CP at each end of the crack in the XFEM element a linear stress distribution can be modelled. This is a sufficient approximation remembering that the stresses only can be of linear variation within the other parts of the element.

3 MIXED MODE MODEL

Cyclic testing of concrete in compression and tension shows irreversible strains and degradation of stiffness. These effects are essential when modelling structural concrete, and can be handled combining elastoplasticity with damage. The concept of the applied constitutive model for concrete cracking is shown in Figure 3. Details in the model are presented in the accompanying paper by Nielsen (2009). The model couples elastoplasticity with damage. When a crack initiates a) the cohesion is damaged and the friction capacity is reduced with the crack opening. When the crack faces is completely separated b) no stresses can be transferred i.e. the yield surface is shrunken, and tends to a point. When the crack faces meat again the friction rebuilds due to the roughness of the crack faces c) i.e. the yield surface rebuild but without cohesion.

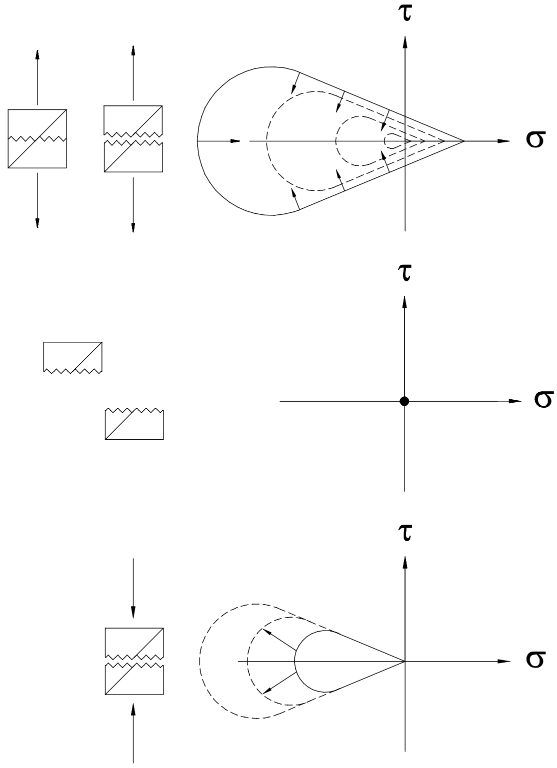

Figure 3. Concepts of the mixed mode model illustrated with the evolution of the yield surface in the three regimes (crack initiation, complete separation and unloading).

The rheological buildup of the model is shown in the scheme in Figure. It consists of a friction part and a cohesion part in a parallel coupling. In both models an associated flow rule is applied, the combined model captures the effects which are typically accounted for by a non associated flow rule in e.g. Mohr Coulomb. The friction submodel is build with reversible deformation state dependence which degrades the friction capacity with the crack opening, and rebuilds the friction when the crack closes. The cohesion submodel introduces a damaging cohesion as function of the crack opening.

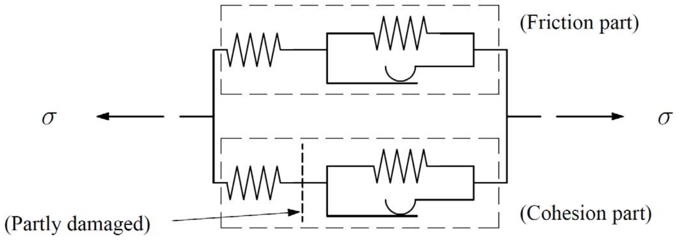

Figure 4. Rheological buildup of the mixed mode model. It consists of a friction and a cohesion submodel coupled in parallel.

4 THE MIXED MODE TEST SETUP

This paper focuses on the implementation of realistic mixed mode model for fracture in concrete within the framework of XFEM. In order to validate the modelling results are compared with experimental work done by Jacobsen (2009). The specimen used is a double notched with dimensions as shown in Figure 5.

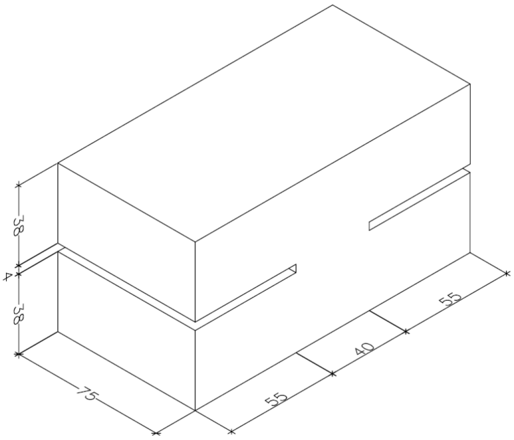

Figure 5. Geometry of the double notched mixed mode test specimen.

First the specimen is loaded in pure tension in order to ensure a fully propagated crack over the ligament. Figure 6 shows the applied local coordinate system, where the opening is denoted Δu_n and the sliding is denoted Δu_s. The mixed mode angle α is introduced as the angle between the relative displacement vector $\Delta u = [\Delta u_s \quad \Delta u_n]$ and the s axis as shown in the figure.

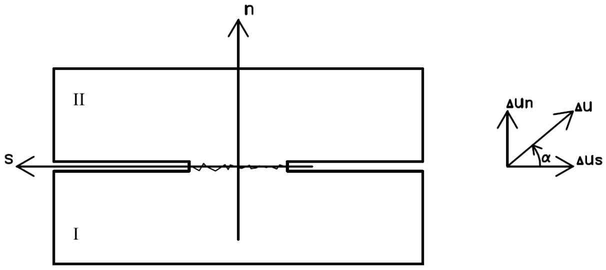

Figure 6. Orientation of the crack coordinate system, and definition of the mixed mode angle α.

In the experimental work it is argued that the results are representative for a straight crack between the two notches. Figure 7 shows the formed crack between the two notches, the roughness of the crack faces is given by the aggregate size. There is no other structural effect coursing the crack path to deviate from the straight line between the two notches. Therefore the results may be considered as representative for a material point.

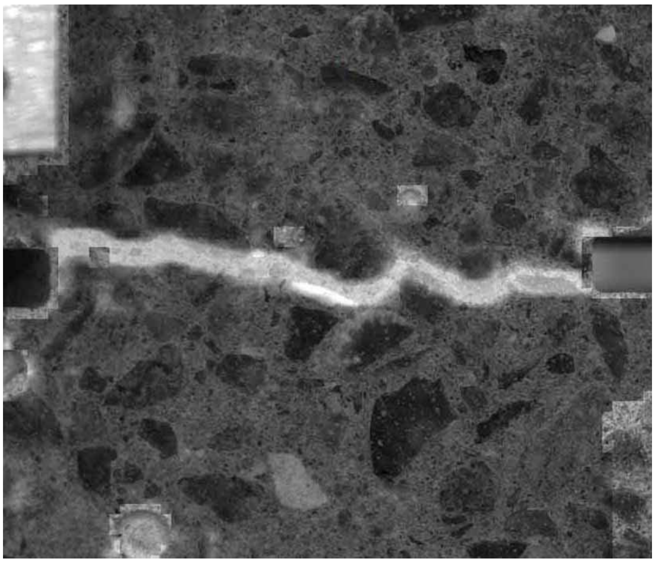

Figure 7. The formed crack between the two notches, found in the experimential work by Jacobsen (2009).

The results from Jacobsen (2009) have been used to calibrate the mixed mode model as described in Nielsen considered as a material point. In the following section a structural example is used to demonstrate that the mixed mode model for a material point can be implemented in a XFEM element. This is as mentioned earlier preliminary modelling tests before modelling reinforced concrete. The structural example used is this mixed mode test, where the structural effects are known, and therefore straight forward to compare with.

5 MODELLING OF THE MIXED MODE TESTS

In the following section the mixed mode test specimens will be modelled as a structure. The specimens are assumed in plane stress. Triangular elements of LST type are used. At the load level of cracking XFEM elements are introduced. The crack is assumed to start at one of the notches, and propagate perpendicular to the largest principal stress direction. Only one crack is allowed in the model, which might be somewhat unrealistic. Structurally two cracks should be initiated symmetrically at the two notches. Due to the deviation in the material strength, it is often observed that a crack penetrates from one side. When the crack is propagated ¾ of the ligament it is set to propagate to the opposite notch.

5.1 *Material parameters*

The material outside the crack is assumed linear elastic with parameters as reported from the experimental results.

Table 1. Material parameters for the elastic parts outside the crack.

E [GPa]	ν [-]
40	0,2

Inside the crack the material parameters describing the mixed mode model is divided in two groups, one for the cohesion submodel, and one for the friction submodel.

Table 2. Material parameters for the cohesion submodel.

E_c [MPa/mm]	G_c [MPa/mm]	μ_c [-]	c_c [MPa]	f_t [MPa]
600	250	0,75	7,5	3
Δu_f [mm]	Δu_0 [mm]			
0,02	0,005			

Parameters are all for the cohesion submodel to avoid inconsistence subscript c is used on some parameters. E_c and G_c is the initial normal/shear stiffness μ_c is the friction coefficient, c_c is the cohesion, f_t is the tensile strength, Δu_0 is a threshold for the damage initiation, Δu_f is a scaling factor for the damage evolution.

Table 3. Material parameters for the friction submodel.

E_f [MPa/mm]	G_f [MPa/mm]	μ_f [-]	c_f [Mpa]	β [Mpa]
600	250	1,30	7,5	5,22
ρ_0 [MPa]	α [-]	W [mm]	w_e [mm]	
20	1,5	0,15	0,15	

E_f and G_f is initial normal/shear stiffness, μ_f is the friction coefficient, c_f is the cohesion, β controls the yield surface gradient discontinuity at the cusp (controls the angle interval where friction is not activated) ρ_0 is the initial value of the deformation state parameter, w is a scaling parameter for the evolution of the displacement state parameter. α and w_c is parameters controlling the evolution of the deformation state dependency in the elastic regime.

5.2 *FE- mesh and crack path*

The applied mesh is shown in Figure 8, Figure 9 shows the crack path in detail.

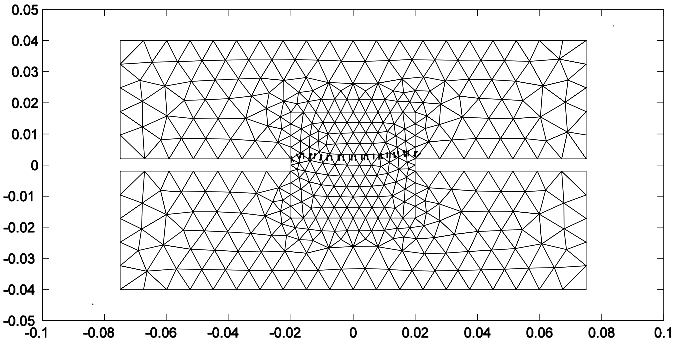

Figure 8. The applied FE-mesh and the achieved crack path.

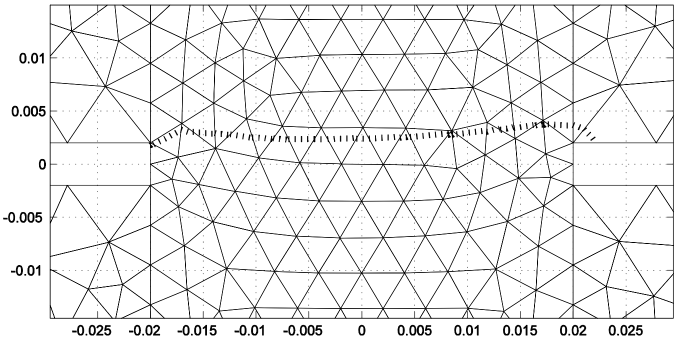

Figure 9. Zoomed view of the crack path.

5.3 *Loading schemes*

All the experimental tests are initially loaded in pure tension with the purpose of establishing a fully penetrated crack over the ligament. In the modelling the crack is after propagation opened to a level of 0,025mm, followed by the a mixed mode displacement with the angles [40,45,50,60,70] degrees

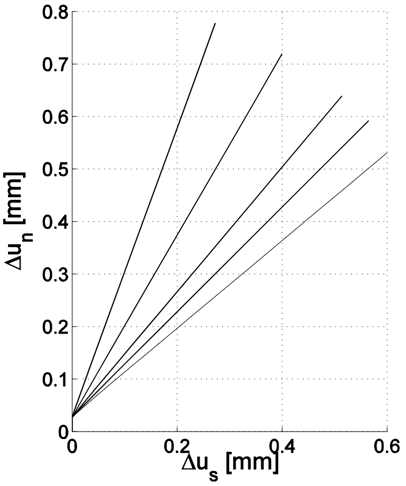

Figure 10. Loading scheme for the mixed mode tests. The opening and sliding are measured between two fix points, one at each side of the crack. The points are chosen in accordance with Jacobsen (2009).

The load is applied as prescribed displacements on the top and bottom surface. The crack deformations are measured in accordance with the measurement rails used in the experiments. The crack deformation history for the five loading series can be seen on Figure 10.

5.4 *Modelling results*

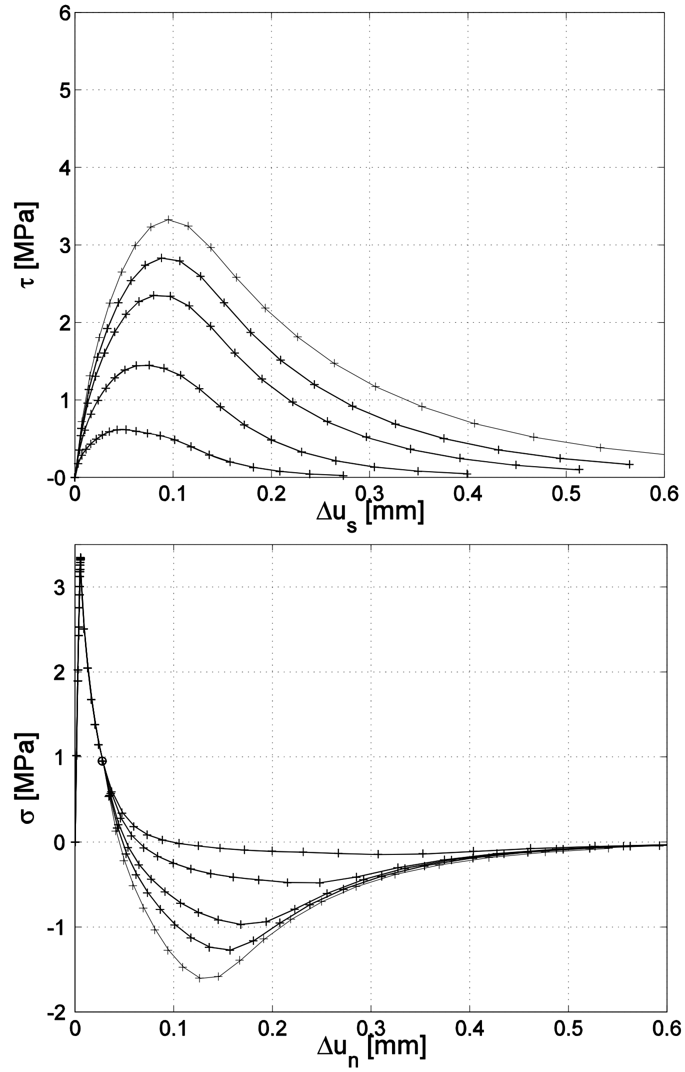

Figure 11. Modelling results obtained using XFEM combined with the mixed mode model. At the top: shear stresses versus the sliding in the crack. At the bottom: normal stresses versus the opening of the crack.

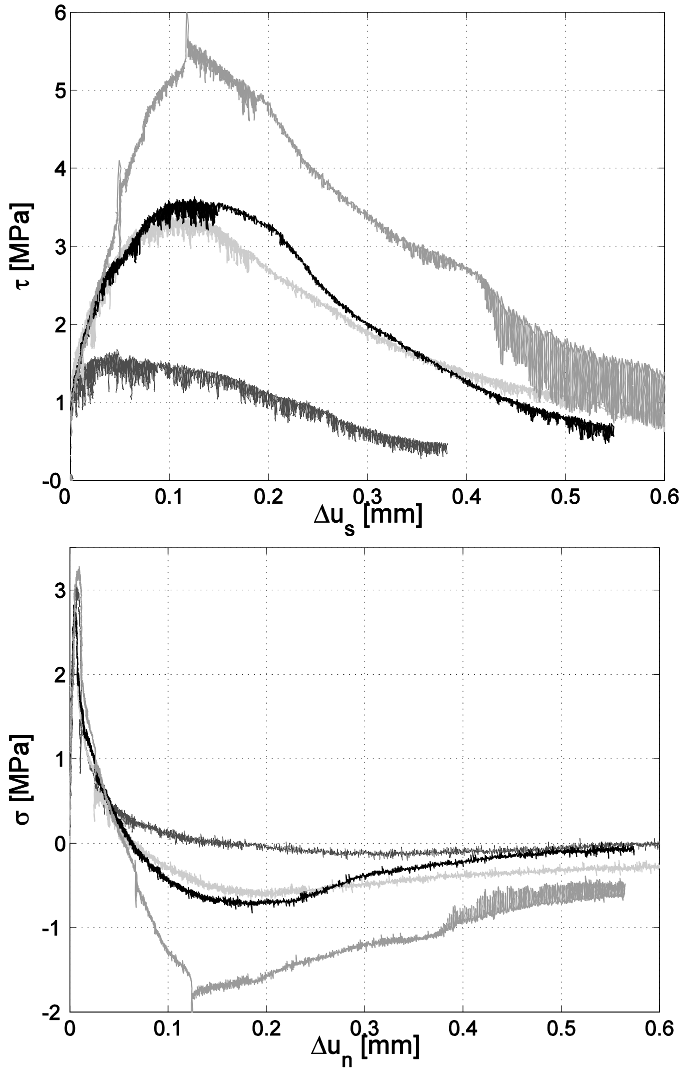

Figure 12. Experimental results from Jacobsen (2009). At the top: the measured shear stresses versus the sliding in the crack. At the bottom: normal stresses versus the opening of the crack.

Figure 11 shows the modelling results obtained using XFEM combined with the mixed mode model. For comparison experimental results from Jacobsen (2009) are shown, these results are arranged in similar plots in Figure 12.

As mentioned in section 3 the mixed mode model is calibrated against these results as a point model. Since we are modelling this mixed mode specimen, where other structural effects can be neglected, as earlier argued in this paper and by Jacobsen (2009). We should not expect large deviations from the implemented material point behavior.

In the present work the entire load path is modelled from elastic behavior to crack initiation over crack growth to fully crack penetration. From Figure 8 the determined crack path can be seen. It is observed that the crack does not follow a complete straight line between the two notches. Due to the stress concentration at the notches the crack initiates in a direction at about 45 degrees measured from a horizontal line. It is further observed that the crack propagation quickly stabilizes in a horizontal direction, and that it only deviates a few millimeters from the straight line. This can be seen as a small effect compared to the natural roughness of the crack faces which is of same scale as the maximum aggregate size (8mm).

The results show an overall good agreement with the experiments. For small angles α, larger deviation is observed. In the experimental work it is concluded that there is a tendency to some secondary shear failure, which may explain the increased dilatation for this 40 degree experiment.

6 CONCLUSION

In the present work a mixed mode fracture model for concrete has been implemented in a XFEM element. The mixed mode fracture model by Nielsen (2009) is used, which is based on elastoplasticity coupled with damage and deformation state dependency. The XFEM element applied is presented in Mougaard (2009), and is a generalized crack-tip element, which can be used both as a fully and partly cracked element. The element is of LST type, therefore two constitutive points are placed in the crack to allow the crack stresses to vary linear matching the continuum stresses in the surrounding parts of the element. The element and the fracture model have been used to model a mixed mode fracture test of a double notched concrete specimen. Experiments by Jacobsen (2009) have been used as reference for the modelling. The specimens used by Jacobsen (2009) are designed so a straight crack is formed between the two notches with only a small deviation. Therefore a structural modelling of the specimen applying the present formulation is straight forward to verify because the structural behavior can be scaled to the material point behavior which is used as input for each constitutive point in the crack. The results from the modelling confirms these conclusions.

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