Comparison of stiff tension fracture test and notched beam level II fracture tests

Walter Gerstle, Lary R. Lenke, Mahmoud Reda Taha, Jacob S. Hays & Anthony S. Cabrera
University of New Mexico, Albuquerque, NM, USA
Joseph M. Magallanes
Karagozian & Case, Burbank, CA
Ruben Martinez
Karagozian & Case, Albuquerque, NM

ABSTRACT: Using normal-strength concrete mixes, two methods for determining the cohesive traction versus crack opening displacement ($\sigma$-COD) relation of concrete are compared and contrasted. The first method, the “Stiff Tension Fracture Test” [Lenke & Gerstle 2001], uses a standard concrete cylinder, loaded in axial tension by a very stiff loading frame. The stiffness of the loading frame prevents snap-back and allows the tensile test to be conducted under open-loop load control in a standard universal testing machine. A shallow circumferential notch sawn into the surface of the cylinder at the central cross-section guides the crack formation. Three clip gages are employed to measure the crack opening displacement (COD), while the tensile load transferred across the crack plane is also monitored. An approximation of the complete normal traction versus crack opening displacement ($\sigma$-COD) relation is thus directly obtained. The second method, the “Level II (Closed Loop) Notched Beam Fracture Test”, [Jenq & Shah 1985, Guinea, Planas & Elices 1994], uses a centrally notched beam in three-point bending. This test method is currently being considered by the American Concrete Institute Committee 446 as a test standard. This test uses feedback from the crack mouth opening displacement (CMOD) clip gage to control the rate of loading. An inverse method proposed by Planas and Elices is used to deduce, based upon the tensile strength and the recorded CMOD, load, and load point displacement, an assumed bilinear $\sigma$-COD relation. The $\sigma$-COD relations from the two test methods are compared. In addition, the statistical variability of the two methods is discussed.

1 INTRODUCTION

1.1 Background

Measurement of the fracture toughness of concrete has been a challenging problem for at least 40 years. Part of the problem has been lack of agreement about what type of model (linear elastic fracture mechanics model (LEFM), two-parameter fracture model, cohesive crack model, etc.) should be employed to represent fracture of concrete. The other part of the problem is in deciding what type of specimen and test method to use in the laboratory given the significant effect of concrete specimen size on the extracted fracture parameters [Bažant & Planas 1997].

Many laboratory fracture toughness tests have been suggested in the literature to determine fracture toughness of concrete as a quasi-brittle material [e.g. Evans and Marathe 1968, Jenq and Shah 1985, Hillerborg 1985, Karihalloo & Nallathambi 1989, Bažant & Kazemi 1990, Lenke & Gerstle 2001]. Many of these test methods were developed with a specific fracture model assumed a priori and therefore they yield inconsistent results and methods of fracture characterization of concrete. Furthermore, these tests aim to extract different fracture toughness features (e.g. fracture energy $G_F$, Mode I fracture toughness $K_{IC}$, critical crack opening displacement $\text{COD}_{\text{crit}}$, and the brittleness length $l_j$). There is definitely a need for standardization of the fracture testing method to allow comparison between findings of different tests. The cohesive crack model appears to be gaining credibility as a reasonable fracture model for concrete. ACI Committee 446 has been considering standardizing two notched-beam tests – a Level I test (open-loop, to obtain only the initial linear part of the bilinear $\sigma$-COD curve) and a Level II test (closed-loop test, to obtain a bilinear approximation of the complete $\sigma$-COD curve) [ACI446 2009]. It appears that the fracture mechanics community generally agrees that a bilinear stress-COD curve is suf-
ciently simple, yet accurate, to represent fracture
toughness of plain concrete.
Lenke and Gerstle [2001] developed and reported
on a stiff tension fracture test (STFT) for obtaining
the complete $\sigma$-COD curve. The repeatability of this
test method appears to be good, as is evident from
the test results presented by Lenke and Gerstle
[2001] and also in this paper. On the other hand,
there has been long history of determining the frac-
ture toughness of concrete using the notched beam
standardized tests. Experiments by Guinea et al.
[1994] suggested the possible extraction of a bilinear
$\sigma$-COD curve from the notched beam test. The four
parameters defining the bilinear curve are dependent
upon the tensile strength of concrete, Young’s
modulus, the load point displacement versus load re-
lation, and the crack mouth opening displacement
versus load relation obtained from the notched beam
test. Comparisons between the fracture parameters
extracted using both methods are not available in
the literature. This paper evaluates the significance of
difference between the fracture parameters extracted
using both methods, including the fracture energies
and the shapes of the stress-COD curves.

1.2 Scope of paper

In this paper, we seek to determine how the fracture
parameters obtained using the STFT and the
notched-beam Level II (denoted here as NB-LII) test
compare. In Section 2, we present the concrete mate-
rual. In Section 3, we present and discuss the test
methods using the STFT test and the NB-LII test. In
Section 4, we present the results of both tests on the
same concrete mix (at somewhat different ages). In
Section 5, we compare the results obtained from the
two test methods and discuss the sources of differ-
ence. Conclusions are drawn in Section 6.

2 MATERIALS

The concrete used in both tests is normal plain con-
crete. The aggregate is 19 mm nominal maximum,
limestone blend, dense graded. The water/cement ra-
tio by weight is 0.54 (the mix contains 310 kg of
ormal type cement per cubic meter of concrete). No
admixtures and no air-entraining admixture were
used. The concrete has unconfined compression
strength, $f_c$, of 33.7 MPa (4890 psi) at 56 days. The
flexural strength (modulus of rupture) is 5.63
MPa (817 psi) at 56 days. The split tensile strength,
$f_{st}$ is 3.48 MPa (505 psi) at 56 days. The average
Poisson’s ratio is 0.19 at 56 days. The average
Young’s modulus is 32.6 GPa (4,658 ksi) at 56 days.
Table 1 provides the basic characteristics of the con-
crete used in the fracture toughness tests.

<table>
<thead>
<tr>
<th>Test #</th>
<th>Density (kg/m$^3$)</th>
<th>$f_c$ (MPa)</th>
<th>$f_{st}$ (MPa)</th>
<th>E (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2365</td>
<td>34.6</td>
<td>3.21</td>
<td>31.0</td>
</tr>
<tr>
<td>2</td>
<td>2415</td>
<td>33.7</td>
<td>3.75</td>
<td>31.9</td>
</tr>
<tr>
<td>3</td>
<td>2410</td>
<td>33.7</td>
<td>3.75</td>
<td>31.9</td>
</tr>
<tr>
<td>4</td>
<td>2416</td>
<td>33.7</td>
<td>3.75</td>
<td>31.9</td>
</tr>
<tr>
<td>5</td>
<td>2416</td>
<td>33.7</td>
<td>3.75</td>
<td>31.9</td>
</tr>
<tr>
<td>6</td>
<td>2403</td>
<td>33.7</td>
<td>3.75</td>
<td>31.9</td>
</tr>
<tr>
<td>7</td>
<td>2411</td>
<td>33.7</td>
<td>3.75</td>
<td>31.9</td>
</tr>
<tr>
<td>8</td>
<td>2416</td>
<td>36.1</td>
<td>3.49</td>
<td>34.4</td>
</tr>
<tr>
<td>9</td>
<td>2420</td>
<td>36.1</td>
<td>3.49</td>
<td>34.4</td>
</tr>
<tr>
<td>10</td>
<td>2430</td>
<td>36.1</td>
<td>3.49</td>
<td>34.4</td>
</tr>
</tbody>
</table>

Mean 2410 34.5 3.62 32.6

Standard deviation ±17.3 ±1.1 ±0.19 ±1.3

3 METHODS

3.1 Stiff Tensile Fracture Test (STFT)

The STFT is designed to test a standard 6" (15.24
cm) diameter by 12" (30.48 cm) long concrete cy-
inder. The STFT test is composed of two steel end
caps into each of which is threaded a nominal 6" 
(15.24 cm) inside-diameter steel pipe jacket, into
which the concrete specimen is subsequently ep-
oxied. Three 1.25" (3.17 cm) diameter load rods
(ASTM Grade B7 threaded rod) are bolted to both
end caps in parallel with the specimen to provide the
stiffness necessary to prevent snap back. The STFT
loading frame is shown schematically in Figure 1.

Figure 1. Schematic of STFT [Lenke and Gerstle 2001].

The load rods are each instrumented with two
90° strain gage rosettes on opposite sides of the
rods in a full bridge configuration with bending
stress cancellation. Tensile load is applied via two
concentric load rods threaded into the end caps.
As the concrete begins to crack and ultimately
separate, the instrumented load rods support pro-
gressively more of the applied tensile load, prevent-
ing snap back of the concrete. As load is applied, the nominal 1” (2.54 cm) gap between the upper and lower pipe jackets is monitored by clip gages. The gap increase between these jackets is approximately the crack opening displacement (COD) of the developing crack in the concrete.

As the tensile strength of the concrete is reached, the stress in the concrete reduces and the COD continues to increase reaching a critical crack opening displacement (COD_{cr}). By simultaneously loading the STFT device, monitoring COD, and monitoring the forces in the three steel stiffening rods, it is possible to extract an approximation of the stress-COD relation. The area under this relation is a measure of the fracture energy, G_{F}, of the specimen.

The STFT test was performed at 56 days of age while the NB-LII test was performed at 180 days of age. All testing specimens were cured in water tanks at 23 °C up to the day of testing. Figure 2 shows the STFT test apparatus. Analysis of the data was performed using the cohesive crack model [Hillerborg 1985] and the approach described in more detail by Lenke and Gerstle [2001].

### 3.2 NB-LII test

On the other hand, the notched beam Level II test can be performed to obtain a bilinear approximation of the complete stress-COD curve following the procedure presented in ACI 446 [2009], which follows the original work of Guinea et al. [1994]. In this test a beam is notched and is loaded in three-point bending as shown schematically in Figure 3. Feedback is provided using COD measurement to control the loading rate and to allow recording of the descending part of the load-displacement relation. ACI 446 [2009] provides a detailed description of the test specimen preparation, loading set-up and test procedure. A photograph of the beam test set-up is shown in Figure 4.

![Figure 2. (a) STFT test setup used to extract cohesive crack model parameters. (b) Clip gauge measurements of crack opening displacement.](image)

![Figure 3. Schematic of NBL-II Test [ACI446 2009].](image)

Three beams were tested using the NB-LII test. All beams were tested at 180 days of age. Analysis of the data was performed using the suggested method by ACI 446 [2009]. The initial compliance of the load-CMOD relationship C_{i} is determined as

\[ C_{i} = \frac{\Delta \text{CMOD}}{\Delta P}. \]  

The initial compliance is then used to compute the elastic modulus of the concrete specimen as

\[ E = \frac{6S a_{0}}{C_{i}B D^{2}} V_{i}(\alpha_{0}) \text{ with } \alpha_{0} = \frac{a_{0} + h}{D + h}, \]  

where E is the elastic modulus in GPa, S is the loaded span, C_{i} is the initial compliance in μm N^{-1}, B is the beam thickness in mm, D beam depth in mm, a_{0} is the notch length, mm, h is the distance from the knife edges to the specimen surface, mm, and

\[ V_{i}(\alpha) = 0.8 - 1.7\alpha + 2.4\alpha^{2} + \frac{0.66}{(1 - \alpha)^{2}} \]  

\[ + \frac{4D}{S} \left( -0.04 - 0.58\alpha + 1.47\alpha^{2} - 2.04\alpha^{3} \right). \]  

The residual load P_{R} is determined for CMOD = 2 mm or nearest point denoted w_{MR}. The load was corrected using Equation 4 and the far end tail constant was determined per ACI 446 [2009] as follows:
\[ P'_t = \frac{P^*}{P_R} , \]  

where \( P' \) is the recorded load. The value of \( w_{MA} \) is determined as the intersection of the rising part of the corrected load \( P'_t \) versus CMOD curve with the CMOD axis. The far tail constant “\( A \)” is determined by least-square curve fitting of a quadratic relation of the load \( P'_t \) versus the quantity \( X \) derived from the points of record for which the corrected load is less than or equal to 5\% of the corrected load peak. \( X \) is computed as

\[ X = \left[ \frac{(4D)^2}{S} \right] \times \frac{1}{(w_M - w_{MA})^2} - \frac{1}{(w_{MH} - w_{MA})^2} . \]  

The effective peak load \( P_{\text{max}} \) is then computed as

\[ P_{\text{max}} = P_{\text{max}} + \frac{A}{(w_{MR} - w_{MA})^2} , \]  

where \( P_{\text{max}} \) is the corrected peak load. The plastic flexural strength of the beam is computed as

\[ f_p = \frac{P_{\text{max}} S}{2Bb^2} , \]  

where \( B \) is the beam thickness and \( b \) is the ligament length equal to \( D-a_0 \) and \( S \) is the test span. The ratio of the tensile strength (determined from the splitting tension test) to the plastic flexure strength denoted \( x = f_t/f_p \) is used to compute the brittleness length \( l_1 \) as

\[ l_1 = \kappa D \left( \frac{11.2}{x^2 - 1} + \frac{2.365}{x} \right) , \]  

where \( \kappa = 1 - \omega_0^{1.7} \) and \( \omega_0 = a_0/D \) is the notch-to-depth ratio. The brittleness length \( l_1 \) is used to determine the horizontal intercept of the softening curve \( w_{G} \) as

\[ w_{G} = 1000 \frac{2f_t}{110} l_1 , \]  

The total work of fracture \( W_F \) is computed as

\[ W_F = W_{Fm} + \frac{2A}{\delta_R - \delta_A} , \]  

where \( W_{Fm} \) is the measured work of fracture calculated as the area under the load versus the load-point displacement. \( A \) is the far end constant determined early, \( \delta_R \) is the load-point displacement at the end of the test and \( \delta_A \) is the load-point displacement at zero corrected load \( (P_t) \). The fracture energy \( G_F \) is then computed as

\[ G_F = \frac{1000 W_F}{B b} , \]  

where \( B \) is the beam thickness in mm and \( b \) is the ligament length equal to \( D-a_0 \). The Mode I fracture toughness \( K_{IC} \) is extracted using the fracture energy, \( G_F \), modulus of elasticity, \( E \), and Poisson’s ratio, \( v \), as

\[ K_{IC} = \sqrt{\frac{E G_F}{(1-v^2)}} . \]  

The center of gravity of the softening curve \( w_{G} \) can be determined using the far tail constant \( A \) and the fracture energy \( G_F \) as

\[ w_{G} = \frac{4A}{BSG} \times 10^6 . \]  

The bilinear approximation of the softening curve is then determined using the mean value of the fracture energy \( G_F \), the brittleness length \( l_1 \) and the horizontal intercept of the softening curve \( w_{G} \). The mean values denoted \( l_{1,m} \), \( G_{Fm} \) and \( w_{Gm} \) are determined from the three fracture toughness testing specimens. The characteristic crack opening \( w_c \) is determined as

\[ w_c = \frac{G_{Fm}}{f'_t} , \]  

where \( G_{Fm} \) and \( f'_t \) are the mean fracture energy and the mean tensile strength respectively. The characteristic crack opening \( w_c \) is used to determine the critical crack opening of the bilinear approximation curve \( w_c \) as

\[ w_c = \frac{w_{Gm} + w_{I,m}}{2w_{Gm} - w_{I,m}} \left[ 1 - \frac{2w_{I,m} (2w_{Gm} - 2w_{Gm} + 2w_{I,m})}{w_{Gm} (3w_{Gm} - w_{I,m})^2} \right] . \]  

The stress at the kink point of the bilinear approximation curve denoted \( \sigma_k \) is computed as

\[ \sigma_k = f'_t \frac{2w_{Gm} - w_{I,m}}{w_c - w_{Gm}} . \]  

The crack opening at the kink point of the bilinear approximation curve, \( w_k \), corresponding to the stress \( \sigma_k \) is determined as

\[ w_k = \frac{w_{I,m} + 2w_{Gm} - 2w_{Gm}}{w_c - w_{I,m}} . \]  

Using the above procedure the bilinear approximation curve can be established using the three distinct points of the mean tensile strength \( f_t \), the mean horizontal intercept of the softening curve \( w_{Gm} \) and the kink point stress and crack opening \( \sigma_k \) and \( w_k \) respectively. The area under the linear approximation curve is equal to the mean fracture energy \( G_{Fm} \). Verification proposed by ACI 446 [2009] to ensure the validity of the above analysis is to be performed.

699  
3.3 Statistical analysis

The fracture toughness parameters extracted from both tests were statistically analyzed. The fact that the results of the STFT test were extracted from 10 specimens and results from the NB-LII test were extracted from 3 specimens precludes using just the mean values for comparison. We therefore performed the student t-test considering a two tailed distribution for unequal variance samples to compare between the two means. A 95% level of confidence was assumed sufficient to judge the significance of difference between the two means.

4 RESULTS

A typical stress-COD relation obtained from the STFT test is shown in Figure 5. A summary of the fracture parameters of the concrete extracted from the STFT test and NB-LII test are presented in Tables 2 and 3 respectively. A typical load versus CMOD relation obtained from the NB-LII test is shown in Figure 6. A typical load versus load-point displacement curve is shown in Figure 7. The bilinear approximation curve for the cohesive crack extracted from the NB-LII test is shown in Figure 8.

Table 2. Fracture toughness characteristics including COD_{crit} (µm), fracture toughness K_{IC} (MPa.m^{1/2}) and fracture energy G_{F} (N.m/m^{2}) extracted from the STFT test.

<table>
<thead>
<tr>
<th>Specimen #</th>
<th>COD_{crit} (µm)</th>
<th>K_{IC} (MPa.m^{1/2})</th>
<th>G_{F} (N.m/m^{2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>432</td>
<td>2.81</td>
<td>245</td>
</tr>
<tr>
<td>2</td>
<td>318</td>
<td>2.19</td>
<td>145</td>
</tr>
<tr>
<td>3</td>
<td>470</td>
<td>3.00</td>
<td>271</td>
</tr>
<tr>
<td>4</td>
<td>419</td>
<td>2.36</td>
<td>168</td>
</tr>
<tr>
<td>5</td>
<td>330</td>
<td>2.48</td>
<td>186</td>
</tr>
<tr>
<td>6</td>
<td>406</td>
<td>2.75</td>
<td>229</td>
</tr>
<tr>
<td>7</td>
<td>406</td>
<td>2.36</td>
<td>168</td>
</tr>
<tr>
<td>8</td>
<td>546</td>
<td>2.95</td>
<td>244</td>
</tr>
<tr>
<td>9</td>
<td>458</td>
<td>2.88</td>
<td>233</td>
</tr>
<tr>
<td>10</td>
<td>533</td>
<td>2.90</td>
<td>236</td>
</tr>
</tbody>
</table>

| Mean       | 432             | 2.67                 | 212.5             |
| Standard deviation | ±74.7          | ±0.3                 | ±42.1             |

Figure 5. Typical stress-COD relationship obtained from the STFT test.

Table 3. Fracture toughness characteristics including COD (µm), Critical energy release rate K_{IC} (MPa.m) and fracture energy G_{F} (N.m/m^{2}) extracted from the NB-LII test.

<table>
<thead>
<tr>
<th>Specimen #</th>
<th>K_{IC} (MPa.m)</th>
<th>G_{F} (N.m/m^{2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.47</td>
<td>180.9</td>
</tr>
<tr>
<td>2</td>
<td>2.67</td>
<td>210.9</td>
</tr>
<tr>
<td>3</td>
<td>2.81</td>
<td>233.1</td>
</tr>
<tr>
<td>Mean</td>
<td>2.65</td>
<td>208.3</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>±0.17</td>
<td>±26.2</td>
</tr>
</tbody>
</table>

Figure 6. Typical corrected load-CMOD relationship obtained from the NB-LII test.

Figure 7. Typical load versus load-point displacement of the NB-LII test.

Figure 8. Bilinear cohesive curve extracted using the NB-LII test results.
5 DISCUSSION

Both tests (STFT and NB-LII) showed very similar behavior. Fracture energies, $G_{IC}$, extracted from both tests have very close mean values 212.5 ($\pm$42.1) N. m/m² for the STFT test versus 208.3 ($\pm$26.2) N. m/m² for the NB-LII test. Statistical analysis using the student t-test showed the probability of being significantly different was about 16%. This is a low probability compared with a 95% level of confidence. Therefore it can be concluded that the two mean fracture energies extracted from the STFT test and the NB-LII test are not significantly different. We used the mean value of Young’s modulus of elasticity extracted from the modulus of elasticity test to compute $K_{IC}$ for both tests, therefore the $K_{IC}$ from both tests were also not significantly different.

To extract the bilinear curve approximation for the NB-LII test, an estimate for the elastic modulus of the notched beam is computed using Equation (2). We note that the mean value for that elastic modulus was 44.7 ($\pm$2.3) GPa which was found to be significantly different than the directly measured elastic modulus of elasticity 32.6 ($\pm$1.3) GPa. While this difference did not result in changing the value for the fracture energy it affected the final shape of the bilinear curve. The difference in Young’s modulus of elasticity might be attributed to the fact that the value extracted from the NB-LII test is based on a linear approximation of the initial slope extracted from the ascending part of the load displacement curve and therefore is prone to inaccuracy. It can also be attributed to the fact that all beams tested were of similar size; therefore, the extracted Young’s modulus of elasticity and bilinear curve might be non-unique for incorporating a size effect. Further research is needed to examine that issue.

The cohesive curves extracted from both experiments were similar. This can be observed in Figure 9 where the two bilinear curves are compared. The NB-LII test showed a concrete critical crack opening displacement ($COD_{crit}$) of 511 mm. This value is derived from the three tests and is based on the approximation of the bilinear curve. This value is compared with the $COD_{crit}$ of 432 mm computed as the average critical crack opening displacement from the 10 STFT tests. The $COD_{crit}$ extracted from the NB-LII test was therefore 20% higher than that from the STFT tests. This can be explained by the fact that the $COD_{crit}$ from the NB-LII test is an approximated value extracted from the average testing of the three specimens.

6 CONCLUSIONS

The fracture toughness parameters extracted from the stiff tension fracture test (STFT) were surprisingly similar to those extracted from the notched beam level II (NB-LII) test. The loading procedure and analysis suggested by ACI 446 [2009] for the NB-LII test were followed and produced a bilinear curve that is very similar to the cohesive bilinear curve extracted from the STFT test. The similarity of the results from the two different test methods is striking but could be coincidence. More testing is required to definitively determine whether or not the cohesive relations obtained from both test methods are indeed objective.

The uniqueness of the bilinear curve approximation extracted from the NB-LII test needs to be examined. There might be a need to consider multiple size specimens from the NB-LII test to extract a truly non-size dependent bilinear cohesive relation.

This scoping exercise has shown that both methods (STFT and NB-LII) tests appear to be meaningful tests that provide meaningful and very similar results. More testing is needed to verify this tentative conclusion.

ACKNOWLEDGEMENTS

The financial support to the graduate students by Defense Threat Reduction Agency (DTRA) grant # HDTRA1-08-1-0053 to the third author is greatly appreciated. Special thanks to Karagozian & Case, Burbank for their support for all the concretes used in these experiments. Special thanks shall go to M. Jalalpour and E. H. Gheitanbaf for their help in performing the NB-LII experiments.
REFERENCES


Hillerborg, A. 1985."The theoretical basis of a method to determine the fracture energy GF of concrete", Materials and Structures, 18, pp. 291-296


Lenke, L. and Gerstle, W., 2001, Tension Test of Stress Versus Crack Opening Displacement Using Cylindrical Concrete Specimens, ACI SP201, pp. 189-206.