Rebar bond slip in diagonal tension failure of reinforced concrete beams

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ABSTRACT: The influence of modeling of rebar bond slip on diagonal tension failure of reinforced concrete beams in finite element analysis is examined through calculations with several modeling strategies for bond slip and bond splitting fracture. The elasto-plastic constitutive model with dilatancy in bond interface elements results in extensive bond splitting cracking and further propagation of the diagonal cracks connected to the bond splitting cracks. The partial debonding model between concrete and rebar beam elements can simulate the diagonal tension failure mode depending on the configuration of debonding nodes.

1 INTRODUCTION

Diagonal tension failure of reinforced concrete structures is influenced by many factors such as mixed mode fracture of concrete, localization and propagation of diagonal cracks, longitudinal splitting cracks along tension rebars, fracture bifurcation, constitutive relations of concrete, rebar bond slip, and dowel action of rebar. Those factors have to be taken into account to simulate the failure mechanism of the structures for rational shear design of reinforced concrete structures. In previous studies (Hasegawa 2004 and Hasegawa 2007a) finite element analysis of diagonal tension failure in a reinforced concrete beam was performed using the Multi Equivalent Series Phase Model (MESP Model; Hasegawa 1998), and the failure mechanisms were discussed by analyzing the numerical results. In the analysis, branch-switching for fracture bifurcation, influence of concrete crack models, mixed mode fracture, and mesh dependency were focused on and examined. Among the other influencing factors not examined in the previous studies, in particular rebar bond slip and longitudinal splitting fracture are considered to be quite important and difficult to examine in both experimental and numerical investigations. The bond slip of reinforcing bars is considered to influence the failure through the mechanism that the bond slip increases the width of diagonal cracks and changes the propagation of the cracks to unstable propagation. The bond slip also causes splitting cracks of concrete cover along the reinforcing bar, that connect with diagonal cracks, and it results in a trigger of diagonal tension collapse of the beam in an unstable manner. In this study (Hasegawa 2007b, 2008, and 2009) the influence of modeling of rebar bond slip on diagonal tension failure of reinforced concrete beams in finite element analysis is examined through calculations with several modeling strategies for bond slip and bond splitting fracture.

2 BOND STRESS-SLIP MODELS

Rebar bond slip is usually simulated by using interface elements with a bond stress $\tau$ - slip $S$ relationship such as Eq. 1 (Witte & Kikstra 2007) in finite element analysis of reinforced concrete structures. However, the bond stress-slip relationships obtained from pullout test experiments of reinforced concrete are known to be not unique but strongly depend on the boundary conditions in the experiments, such as embedded length of the rebar, distance of the measurement position

![Experimental cracking pattern of BN50 after failure.](image)

Table 1. Models for bond stress-slip relationship.

<table>
<thead>
<tr>
<th>Model</th>
<th>Length $L$ of pullout specimen (mm)</th>
<th>Location $2x/L$</th>
<th>$\tau$ - $S$ or $\varepsilon$ equation</th>
<th>Bond strength ($\text{Nm}^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>b-1</td>
<td>Non</td>
<td>Non</td>
<td>Eq. 1</td>
<td>3.80</td>
</tr>
<tr>
<td>b-2</td>
<td>120</td>
<td>0.50</td>
<td>Eq. 2</td>
<td>2.90</td>
</tr>
<tr>
<td>b-3</td>
<td>210</td>
<td>0.50</td>
<td>Eq. 2</td>
<td>4.81</td>
</tr>
<tr>
<td>b-4</td>
<td>210</td>
<td>0.50</td>
<td>quarter stiffness 4.81</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>of model b-3</td>
<td></td>
</tr>
<tr>
<td>b-5</td>
<td>120</td>
<td>0.75</td>
<td>Eq. 2</td>
<td>1.47</td>
</tr>
</tbody>
</table>
from the pullout end, and so on. To circumvent this problem the bond stress \( \tau - \text{slip} \ S - \text{rebar strain} \ \varepsilon_s \) relationship of Shima (Shima, Chou & Okamura 1987), Eq. 2 is utilized, which does not depend on the boundary conditions in pullout test of rebar. However, it is difficult to take the rebar strain into account in ordinary interface finite elements. To simulate diagonal tension failure of a reinforced concrete slender beam, experimental specimen BN50 tested at the University of Toronto (Podgorniak-Stanik 1998), the flexural tension part of the beam was considered to be capable of being modeled into pullout specimens of the lengths \( L = 1200, 210, 120 \) mm, see Figure 1. The integral equation for slip, Eq. 3, and the differential equation for bond stress, Eq. 4, are numerically solved for the assumed pullout specimens having their lengths \( L \) of 1200, 210, and 120 mm, together with the bond stress-slip-rebar strain relationship, Eq. 2. From the numerical analysis, bond stress \( \tau(x) \)-slip \( S(x) \) relationships at any location \( x \) are obtained, which take into account the lengths of the pullout specimens, i.e., crack spacings of the reinforced concrete beam.

\[
\begin{align*}
0 \leq S < S^0 : & \\
\tau &= f_r \left[ 5 \left( \frac{S}{S^0} \right) - 4.5 \left( \frac{S}{S^0} \right)^2 + 1.4 \left( \frac{S}{S^0} \right)^3 \right] \\
S \geq S^0 : & \\
\tau &= 1.9 f_r \\
\end{align*}
\]

\[
\tau = \frac{0.73}{f'_c} \left[ \ln \left( 1 + 5000 S/D \right) \right]^3 \quad \text{for} \quad S \leq S^0
\]

\[
S(x) = \int_{x_0}^{x} \varepsilon_s(x) \, dx + S(x_0)
\]
Table 2. Analysis cases.

<table>
<thead>
<tr>
<th>Analysis case</th>
<th>Tension model</th>
<th>Bond interface element</th>
<th>Number of disconnected nodes for rebar</th>
<th>Bond slip constitutive model</th>
<th>$\tau - S$ relationship</th>
<th>$\tan \phi$</th>
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<td>A05</td>
<td>t-3</td>
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<td>Non</td>
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<td>Non</td>
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<td>G01</td>
<td>t-5</td>
<td>Used</td>
<td>Non</td>
<td>$\tau - S$ relationship only</td>
<td>b-2</td>
<td>Non</td>
</tr>
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<tr>
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<td>t-8</td>
<td>Used</td>
<td>Non</td>
<td>$\tau - S$ relationship only</td>
<td>b-2</td>
<td>Non</td>
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<tr>
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<td>t-5</td>
<td>Used</td>
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<td>$\tau - S$ relationship only</td>
<td>b-5</td>
<td>Non</td>
</tr>
<tr>
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<td>t-3 and t-9</td>
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<td>3</td>
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<td>41</td>
<td>Non</td>
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<td>30</td>
<td>Non</td>
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<td>13</td>
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<td>Non</td>
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<td>Non</td>
<td>Coulomb friction model</td>
<td>b-3</td>
<td>2.0</td>
</tr>
</tbody>
</table>

$$\tau(x) = \frac{D}{4} \frac{d\sigma_s(e_s(x))}{dx}$$  \hspace{1cm} (4)

where $\tau(x) = \text{bond stress at } x$; $S(x) = \text{slip at } x$; $e_s(x) = \text{rebar strain at } x$; $f_t = \text{tensile strength of concrete}$; $f_c' = \text{compressive strength of concrete}$; $D = \text{diameter of rebar}$; $x = \text{location coordinate of pullout specimen with its origin } x_0$ at the center of the specimen.

Figure 5 compares the calculated shear response in analysis cases A05, A06, G02, and G03 with the experiment. In Figure 6 the calculated shear response in analysis cases A05, G01, and G05 are shown. While in analysis case A05 the calculation is done for perfect bonding between concrete and rebar, analysis case G01 assumes the bond model b-2 that is obtained for $2x/L = 0.50$ and has a higher bond strength (2.90 N/mm$^2$). On the other hand, in analysis case G05 the bond model b-5 is assumed, that is calculated for $2x/L = 0.75$ and has a lower bond strength (1.47 N/mm$^2$). In all those analysis cases we can not obtain the expected unstable and brittle diagonal tension failure mode due to widening of diagonal crack widths resulted from rebar bond slip, but flexural failure mode. Figures 7, 8 and 9 plot the lines of maximum principal strain $e_r \geq 5e_{r0}$ with the thickness proportional to its value in analysis cases A05, G02, and G05. This represents the crack strain and crack direction at maximum shear load $v'$, and is a good measure of crack width ($e_{r0} = \text{the tensile strain corresponding to tensile strength}$). Comparing cracking patterns of analysis cases A05 with perfect bond and G05 with the weak bond model obtained for the position near the pullout end, longitudinal cracks do not occur in the latter case since the force transfer from rebar to concrete is not enough because of weak bond strength. In analysis case G05 diagonal cracking does not develop so much, and cracks concentrate in the middle part of the specimen span, which resembles the tied arch load-carrying mechanism observed in reinforced concrete beams without bond between the rebar and concrete. Compressive failure of arch crown concrete is confirmed at the maximum shear load in analysis.
case G05. On the other hand, in analysis case G02, the bond model b-3 with higher bond strength (4.81 N/mm²) produces a good prediction of the experimental cracking pattern with longitudinal and diagonal cracks.

Figure 10 is the slip distribution in bond interface elements at the step before small decrease in shear capacity (Fig. 5), along with indications of the intersection of diagonal cracks and rebar in analysis case G02. Figures 11 and 12(a) show the bond stress-slip response of interface element i (Fig. 8) and the stress-strain response of concrete element a (Fig. 8), which are located at the intersection of the main diagonal crack and the rebar in analysis case G02. On the other hand Figure 12(b) shows the stress-strain response of concrete element a (Fig. 7) in analysis case A05 without bond slip. The element a is located at a similar point to that in analysis case G02. These results suggest that the rebar bond slip does not increase the width of a certain dominant diagonal crack resulting in localized diagonal tension failure with instability, but tends to homogenize the strain of all cracks along the rebar, which causes stable flexural failure. Use of bond interface elements decreases the force transfer from the rebar to concrete elements, and therefore, suppresses the growth of longitudinal splitting cracks. This results in failure to simulate the complete diagonal tension collapse mechanism.

3 PARTIAL DEBONDING MODEL

In analysis case H some parts of rebar beam elements are arranged to be not connected to concrete elements, simulating partial debonding between concrete and rebar. To trigger an unstable propagation of a dominant diagonal crack, concrete elements in the vicinity of rebar beam elements need to increase their crack strain by being disconnected from the rebar beam elements. For that purpose some parts of concrete elements are arranged not to have
The moisture mass balance requires that the variation in time of the water mass per unit volume of concrete (water content \( w \)) be equal to the evaporable water as one obtains

\[
\frac{\partial w}{\partial t} = \frac{\partial}{\partial h} \left( J_h \right) = -h \nabla J_h
\]

where \( J_h \) is the moisture flux, \( h \) is the relative humidity, and \( \alpha \) is a proportionality coefficient.

The relation between the amount of evaporable water and the non-evaporable water per unit volume held in the gel pores at 100% relative humidity, and it can be expressed (Norling Mjornell 1997) as

\[
\alpha = \alpha_c + \alpha_s + \alpha_n = \alpha_c + \alpha_s + \alpha_{ns}
\]

where \( \alpha_c \) is the degree of silica fume reaction, \( \alpha_s \) is the degree of hydration, \( \alpha_r \) is the degree of SF. The coefficient \( \alpha_{ns} \) is the moisture capacity. The following, "sorption isotherm" will be used with reference to both sorption and desorption conditions.

By the way, if the hysteresis of the moisture isotherm would be taken into account, two different relations, evaporable water vs relative humidity, must be used according to the sign of the variation of the degree of hydration.

The proportionality coefficient \( D(h,T) \) is called moisture permeability and it is a nonlinear function of the relative humidity.

The shape of the sorption isotherm for HPC is influenced by many parameters, especially those that influence extent and rate of the chemical reactions and, in turn, determine pore structure and pore size distribution (water-to-cement ratio, cement chemical composition, SF content, curing time and method, temperature, mix additives, etc.). In the literature various formulations can be found to describe the sorption isotherm of normal concrete (Xi et al. 1994). However, in the present paper the semi-empirical expression proposed by Norling Mjornell (1997) is adopted because it is the only one that explicitly accounts for the evolution of hydration reaction and SF content. This sorption isotherm is age-dependent sorption/desorption isotherm, i.e.

\[
\sigma = f(h) = f \left( \frac{w}{v} \right)
\]

where \( w/\) is the water content, \( v \) is the specific volume of concrete, and \( f \) is an empirical function. The moisture mass balance and the governing equation (Equation 3) must be completed by substituting Equation 1 into Equation 2 one obtains

\[
\frac{\partial^2 w}{\partial h^2} + \frac{\partial q}{\partial h} = 0
\]

where \( q \) is the heat flux, \( T \) is the absolute temperature, and \( \lambda \) is the heat conductivity; in this case \( \lambda = \lambda_c + \lambda_s + \lambda_{ns} \). The proportionality coefficient \( \propto \) is a function of the relative humidity, and it can be expressed (Norling Mjornell 1997) as

\[
\propto = \propto_c + \propto_s + \propto_n = \propto_c + \propto_s + \propto_{ns}
\]

where \( \propto_c \) is the degree of hydration, \( \propto_s \) is the degree of SF, \( \propto_r \) is the degree of SF content, and \( \propto_{ns} \) is the moisture capacity.

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where \( \propto_c \) is the degree of hydration, \( \propto_s \) is the degree of SF, \( \propto_r \) is the degree of SF content, and \( \propto_{ns} \) is the moisture capacity.
the primary causes of diagonal tension failure, do not develop much in analysis case H04 due to the weak bond between concrete and rebar beam elements. These discrepancies result in the overestimation of maximum shear load in analysis case H04.

4 BOND SPLITTING FRACTURE MODELING

Since bond splitting cracks or longitudinal splitting cracks, which occur at the level of the tension rebar, usually connect with the dominant diagonal crack and accelerate its propagation, the bond splitting cracks or longitudinal splitting cracks are considered an important trigger of diagonal tension collapse. The bond splitting cracks occur in the concrete cover due to the tensile hoop stress around rebar, which is caused by the radial force from the rib of rebar when the rebar is pulled out from concrete and slips. The rational simulation of the mechanism for bond splitting cracking needs truly three-dimensional meso-level mechanics or micromechanics analysis in which detailed geometrical modeling of the ribs of rebar and surrounding concrete has to be done together with identification of meso-level mechanics properties or micromechanical properties of the materials. The three-dimensional analysis is not that easy since experimental information is lacking. In this study we pursue an expedient method to simulate the mechanism of bond splitting cracking in conjunction with diagonal tension failure under two-dimensional plane stress conditions for simplification. In tension model t-10, concrete elements, rebar plane elements, and bond interface elements are arranged to be connected as shown in Figure 3(e) so that the bond slip in interface elements causes dilatancy in the normal direction of rebar, which induces tensile stress and cracking in the longitudinal direction of concrete elements. For this phenomenological modeling of bond splitting crack, a Coulomb friction elasto-plasticity constitutive model is utilized in the bond interface elements, which is described as in Eqs. 5, 6, and 7.

\[
\frac{dt}{du} = \begin{bmatrix} D_{nn} & D_{nt} \\ D_{tn} & D_{tt} \end{bmatrix} du = \begin{bmatrix} D^e - \frac{\partial g}{\partial \gamma^T} \frac{\partial f}{\partial \gamma} \\ h + \frac{\partial f^T}{\partial \gamma} D^e \frac{\partial g}{\partial \gamma} \end{bmatrix} du \tag{5}
\]

\[
f = \sqrt{t_n^2 + t_n \tan \phi - c(\kappa)} = 0 \tag{6}
\]

\[
g = \sqrt{t_n^2 + t_n \tan \psi} \tag{7}
\]

where \( t = \) normal traction \( t_n \) and bond traction \( t_i \); \( u = \) normal relative displacement \( u_n \) and slip displacement \( u_i \) or \( S \); \( D^e = \) elastic stiffness matrix; \( f = \) yield function; \( g = \) plastic potential function; \( \phi = \) friction angle; \( \psi = \) dilatancy angle; \( c(\kappa) = \) cohesion; \( \kappa = \) internal parameter or effective plastic strain; \( h = \) plastic hardening modulus.

In the case of the bond stress \( \tau - \) slip \( S \) relationship utilized for bond interface elements in most finite element analysis of reinforced concrete, the off-diagonal terms of Eq. 5 are all zero, which means that neither dilatancy nor cross effect is taken into account. The associated flow rule (\( f = g \)) is assumed for stable calculation. The plastic hardening or effective plastic stress-strain relationship is identified by using the bond model b-3.

In analysis case I the tension model t-10 is used together with the Coulomb friction elasto-plasticity constitutive model in bond interface elements. Figure 20 shows the finite element mesh in analysis case I. Since the dilatancy of the Coulomb friction elasto-plasticity constitutive model influences bond splitting cracking, the analysis is done by assuming various dilatancy angles \( \psi \) for each analysis case as shown in Table 2. Figure 21 shows the slip and normal relative displacements obtained in shear test simuation using the Coulomb friction elasto-plasticity constitutive model with various dilatancy angles. It is obvious that normal relative displacement or the dilatancy can be adjusted by the dilatancy angle.

In Figure 22 the calculated shear response in analysis case I is compared with the experiment. Taking into account the dilatancy induced by bond slip does not result in a good estimate of the maximum shear load of the experiment. Figures 23, 24, 25, 26, and 27 are cracking patterns at maximum shear load in analysis cases 101, 102, 103, 104, and 105. In Figures 28, 29, 30, 31, and 32 the incremental displacement at the maximum shear load in analysis case I is shown together with maximum principal strain. The bond splitting cracking mechanism was expected such that the dilatancy induced by plastic slip in bond interface elements causes tensile stress and splitting cracks in concrete elements at the back of rebar plane elements. However, in analysis case I most of the splitting cracks occur at concrete elements just above the upper bond interface elements. Comparing the cracking pattern of analysis case 101 (Figs. 23 and 28) in which no dilatancy is taken into account with the one for analysis case 104 (Figs 26 and 31) assuming dilatancy, extensive bond splitting cracking and further propagation of diagonal cracks connected to the bond splitting cracks are observed in the latter case. The position of the dominant diagonal crack is much closer to the supporting plate in analysis case 104, which resembles the experiment (Fig. 1) much better than analysis case 101. The inclination angle of the dominant diagonal crack in analysis case 104 is relatively smaller compared with analysis case 101, which decreases aggregate interlock action, and results in the slight reduction of maximum shear load of the beam. In the typical diagonal tension failure experiment, the collapse mechanism completes by penetration of the diagonal crack beneath into the flexural compression part under the loading plate. A similar propagation of the diagonal crack is observed in analysis case 104.
5 CONCLUSIONS

The influence of modeling of rebar bond slip on diagonal tension failure of reinforced concrete beams in finite element analysis was examined through calculations with several modeling strategies for bond slip and bond splitting fracture. In the first series of analysis the bond between concrete finite elements and rebar beam elements was modeled by using interface elements with average bond stress-slip relationships which are determined by solving bond slip differential-integral equations of pullout specimens with the constitutive model for bond stress-slip-rebar strain relationship. The analysis of a reinforced concrete beam showed that the bond slip modeling does not result in either widening of diagonal cracks or diagonal tension failure, but in crack dispersion and flexural failure. In the second series of analysis, another type of bond modeling used concrete and rebar beam elements disconnected partially by adopting dual nodes for them. The modeling can simulate the diagonal tension failure mode depending on the configuration of dual nodes. In the third series of analysis, the elasto-plastic constitutive model with dilatancy was assumed for bond interface elements so that the bond slip causes tensile hoop stress in concrete elements, and bond splitting cracking. Extensive bond splitting cracking and further propagation of diagonal cracks connected to the bond splitting cracks were observed in the analysis.

REFERENCES


