

Effectiveness of steel fibers as a crack controller: assessment using shear panel tests

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ABSTRACT: Ten 890×890×70 mm concrete panels were tested under in-plane pure-shear monotonic loading conditions to evaluate the effectiveness of steel fibers in controlling concrete cracking. The results of the tests indicate that concrete elements exhibiting ductile behavior, good shear resistance, and good crack control characteristics can be obtained with an adequate addition of steel fibers. Assessments were also made of the influence of fiber aspect ratio, fiber length, fiber tensile strength, fiber volume content, and concrete compressive strength on the crack control performance of fiber reinforced concrete (FRC). In addition, finite element (FE) analyses were performed to investigate whether currently available constitutive models can be utilized to accurately represent the behavior of the FRC panels tested. The analytical results indicate that improved constitutive models for tension response and crack spacing of FRC are required for successful finite element modeling.

Keywords: steel fiber, reinforcement, crack control, shrinkage, shear, panel tests, finite elements

1 INTRODUCTION

In concrete elements such as one-way slabs or precast bridge segments, secondary reinforcement is generally provided solely to ensure that concrete cracking due to shrinkage or temperature changes is controlled. This allows the crack widths at the serviceability limit state to remain within acceptable limits.

With the increasing use of high strength concrete, a reduction of the cross-sectional area of a concrete section is made possible due to the increased strength capacity, particularly when high compressive stress is present. However, the amount of reduction in the cross section dimensions realizable is limited by the minimum dimensions required to accommodate the reinforcement and to satisfy the requirements for concrete cover and reinforcement spacing. If a concrete material possessing good crack control characteristics were to exist, the need for shrinkage reinforcement could be eliminated. This would lead to a thinner and more optimized structure since the structure could then be designed to more effectively carry the load to which it was subjected.

Previous research on fiber reinforced concrete (FRC) has shown that the addition of steel fibers to concrete significantly improves the tensile behavior and the crack control characteristics of the concrete. Research has indicated that the use of FRC results in significantly reduced crack widths and crack spacing (Grzybowski & Shah 1990, Shah et al. 1998, Banthia et al. 1993, Ong & Paramasivam 1989). It is therefore appealing to study the feasibility of the use

of FRC to replace shrinkage and temperature reinforcement in order to obtain optimized designs of concrete structures.

In order to investigate the effectiveness of steel fibers in controlling concrete cracking, a series of panel tests were conducted at the structural laboratories at the University of Toronto. The tests were conducted using the Panel Element Tester. Although the crack control behavior of FRC is generally assessed using uniaxial tension tests, bending tests, or restrained shrinkage tests, panel tests enable a more thorough investigation of FRC behavior by also introducing the influence of shear stress mechanisms. Also, due to the larger size of the panel specimens relative to typical uniaxial tension test specimens, bending test specimens, or restrained shrinkage test specimens, conventional steel reinforcement can be incorporated and the interaction between the steel reinforcement and FRC can be assessed. Thus, various concrete behaviors such as tension stiffening, tension softening, and compression softening can be evaluated using the panel tests. Assessments were also made of the importance of fibre content, fibre aspect ratio, fibre length, fibre diameter, fibre tensile strength, and concrete compressive strength on the effectiveness of fibre reinforcement.

2 RESEARCH SIGNIFICANCE

Panel tests were conducted to investigate the effectiveness of hooked-end steel fibers to control con-

crete cracking. The use of panels enables a more thorough assessment of the crack control characteristics of FRC, particularly in the presence of conventional steel reinforcement. In addition, an assessment of the ability of the currently available constitutive models to accurately predict the behavior of the FRC panels was performed using VecTor2, a nonlinear finite element program based on the Modified Compression Field Theory (Vecchio & Collins 1986).

3 EXPERIMENTAL PROGRAM

Ten 890×890×70 mm panels were tested under in-plane pure-shear monotonic loading condition using the Panel Element Tester (see Fig. 1) to evaluate the crack control characteristics of FRC. Two panels served as control specimens, and as such were orthogonally reinforced with 40-D8 deformed wires in the longitudinal direction ($\rho_x = 3.31\%$) and with 10-D4 deformed wires in the transverse direction ($\rho_y =$

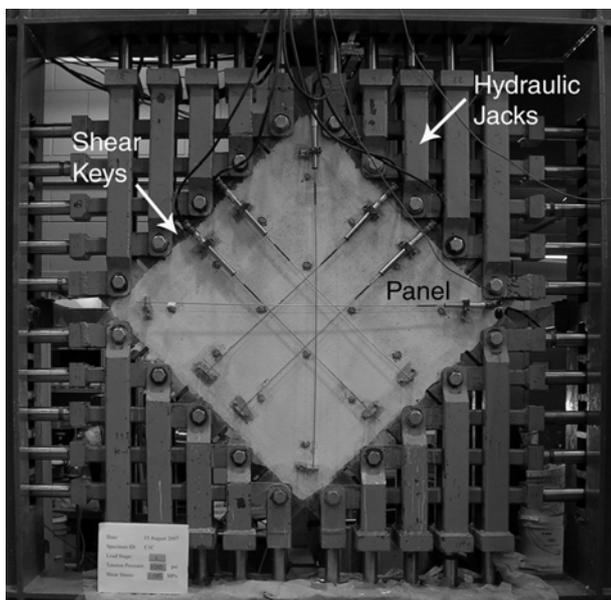


Figure 1. Panel specimen shown in Panel Element Tester.

Table 1. Test Matrix.

ID	Specified f_c (MPa)	Fiber Content (%)	Fiber Type
C1C	50	-	-
C1F1V1	50	0.5	RC80/50-BN
C1F1V2	50	1.0	RC80/50-BN
C1F1V3	50	1.5	RC80/50-BN
C1F2V3	50	1.5	RC80/30-BP
C1F3V3	50	1.5	RC65/35-BN
C2C	80	-	-
C2F1V3	80	1.5	RC80/50-BN
C2F2V3	80	1.5	RC80/30-BP
C2F3V3	80	1.5	RC65/35-BN

Table 2. Properties of conventional reinforcement.

Rebar Type	d_b (mm)	A_s (mm ²)	E_s (GPa)	f_{ys} (MPa)	e_{ys} (me)	f_{us} (MPa)	e_{us} (me)
D4	5.72	25.81	187.2	446.9	2.41	548.9	57.6
D8	8.10	51.61	224.7	552.2	2.58	647.2	45.4

Table 3. Properties of fibers (Bekaert 2003).

Fiber Type	l_f (mm)	d_f (mm)	l_f/d_f	f_{uf} (MPa)
RC80/50-BN	50	0.62	81	1050
RC80/30-BP	30	0.38	79	2300
RC65/35-BN	35	0.55	64	1100

0.42%). The cross-sectional area of the D8 and D4 wires are 51.61 mm² (0.08 in²) and 25.81 mm² (0.04 in²), respectively.

The remaining eight panels, containing steel fibers, were reinforced in the longitudinal direction only with 40-D8 deformed bars ($\rho_x = 3.31\%$). Three types of Dramix hooked-end steel fibers (RC80/50-BN, RC80/30-BP, and RC65/35-BN) were used to investigate the influence of fiber aspect ratio, fiber length, and fiber tensile strength on the crack control performance of FRC. In addition, three different fiber volume contents (0.5%, 1.0%, and 1.5%) and two different concrete compressive strengths (50 MPa and 80 MPa) were investigated. The test matrix is given in Table 1. Details of concrete mix properties and construction methods can be found in Suresh (2009).

The transverse reinforcement provided in the control specimens represents the shrinkage and temperature reinforcement typically present in concrete structures. It was based on the requirement of the Canadian Highway Bridge Design Code (CHBDC) (CAN/CSA Standard S6-06 2006), which states that the “minimum area of shrinkage and temperature reinforcement in each face and in each direction shall be 500 mm²/m and the spacing of the bars shall not exceed 300 mm”. For an 890×890×70 mm panel, this translates to a reinforcement ratio of 1.43%, which was deemed to be too high for shrinkage reinforcement. It was assumed that the CHBDC requirement was for a typical concrete deck with a thickness of approximately 250 mm. In such a case, the CHBDC requirement translates to a reinforcement ratio of 0.40%, which is the value used in the experimental program.

The properties of the conventional reinforcement and the fibers used in the panels are listed in Table 2 and Table 3, respectively. Due to the cold-forming of the deformed wires, the yield plateau could not be determined; the yield strength and the yield strain were thus determined from the proportionality limit of the wire steel.

4 EXPERIMENTAL RESULT

The results of the panel tests are summarized in Table 4. As indicated in the table, all of the FRC panels failed due to aggregate interlock failure. Data on reinforcement stresses indicate that at the onset of

failure, the conventional longitudinal reinforcement provided within the FRC panels had not yielded and the concrete had not reached its ultimate compressive strength. The failures of the control panels, on the other hand, were initiated by yielding or rupturing of the transverse reinforcement. Note, however, that the stresses in the longitudinal reinforcement of the control panels remained at levels comparable to

Table 4. Summary of panel tests.

ID	f'_c (MPa)	v_{cr} (MPa)	γ_{cr} (mε)	v_u (MPa)	γ_u (mε)	w_m (mm)	s_m (mm)
C1C	65.7	2.01	0.086	5.77	6.01	0.55	57.2
C1F1V1	51.4	2.09	0.197	3.53	2.77	0.55	114.4
C1F1V2	53.4	2.65	0.139	5.17	5.27	0.45	54.7
C1F1V3	49.7	1.83	0.055	5.37	5.10	0.45	57.2
C1F2V3	59.7	1.85	0.070	6.68	6.20	0.45	38.1
C1F3V3	45.5	2.24	0.118	5.59	4.27	0.50	57.2
C2C	90.5	2.57	0.099	6.40	7.00	0.50	66.2
C2F1V3	79.0	2.17	0.131	6.90	5.25	0.70	36.0
C2F2V3	76.5	1.59	0.110	6.31	4.35	0.65	46.6
C2F3V3	62.0	2.10	0.222	5.57	4.97	0.60	40.6

Table 4-cont. Summary of panel tests.

ID	$f_{c1,cr}$ (MPa)	$f_{c1,max}$ (MPa)	$f_{c1,u}$ (MPa)	$f_{c2,u}$ (MPa)	f_{sx} (MPa)	f_{sy} (MPa)	Fail Mode
C1C	2.05	2.87	1.43	-11.70	250	501	y-bar yield
C1F1V1	2.21	2.83	1.85	-6.73	148	-	Shear
C1F1V2	2.59	3.04	2.82	-9.46	201	-	Slip
C1F1V3	1.85	3.13	2.97	-9.70	204	-	Shear slip
C1F2V3	1.85	3.89	3.69	-12.12	256	-	Shear Slip
C1F3V3	2.23	3.85	3.08	-10.12	213	-	Shear slip
C2C	2.53	2.55	1.23	-14.53	341	512	y-bar yield
C2F1V3	2.10	3.66	3.44	-13.83	315	-	Shear slip
C2F2V3	1.57	3.75	3.62	-10.99	224	-	Shear slip
C2F3V3	2.08	2.93	2.89	-10.75	238	-	Shear slip

those in the FRC panels, and did not reach the reinforcement yield strength at the onset of panel failure.

4.1 Crack control characteristics

An important property of FRC is its ability to control crack propagation. In conventionally reinforced concrete (RC), crack control is provided by the bonded steel reinforcement; without it, the concrete exhibits a brittle behavior. In FRC, the fibers act as crack controllers and, therefore, the behavior of the concrete is significantly improved.

Figure 2 compares the maximum crack width of all panels. It can be observed that for normal strength concrete panels, the FRC panels exhibited smaller crack widths than did the control panel un-

der the same shear stress. This exemplifies the ability of fibers to control crack propagation. Due to the smaller crack widths, high stresses could still be transmitted across the crack and, thus, the integrity of the cracked panels could be maintained. An exception was Panel C1F1V1. Although it initially

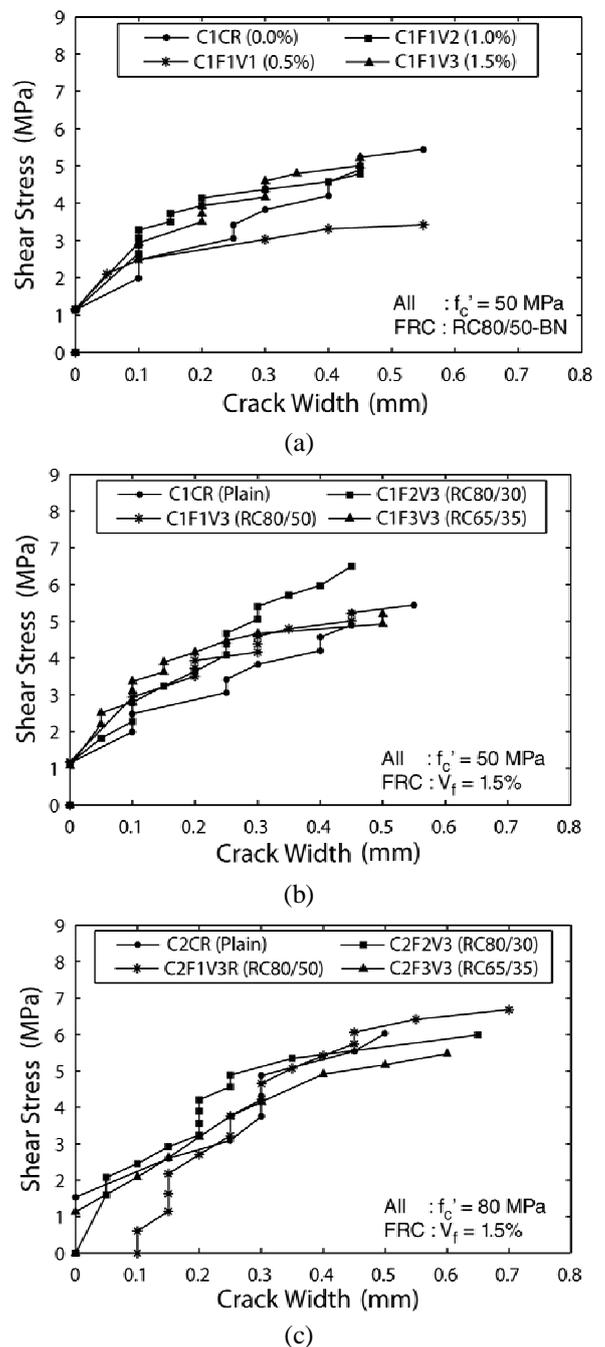


Figure 2. Crack width of the panels.

exhibited smaller crack widths than did the control panel, it was incapable of controlling the cracks at later load stages. The large crack widths limited the magnitude of shear stress that could be transmitted across the cracks, and hence led to the poor performance of Panel C1F1V1.

The influence of fiber volume content on the crack control characteristics can be observed in Figure 2a. A fiber volume of 0.5% was deemed too low to guarantee adequate crack control, as illustrated by

the wide cracks of Panel C1F1V1. Increasing the fiber content to 1.0% significantly reduced the crack widths by a factor of 3. However, not much additional improvement was obtained in increasing the fiber content from 1.0% to 1.5%.

The influence of the fiber type on the crack control characteristics is also illustrated in Figure 2b and Figure 2c. As indicated in the figures, the panels containing high aspect ratio fibers exhibited smaller crack widths than did panels containing low aspect ratio fibers.

The effectiveness of the fibers to control the crack width was less pronounced in high strength concrete panels than in normal strength concrete panels. Although the fibers in the FRC panels still exhibited a remarkable crack control ability, there was no indication of a significant improvement in their crack control characteristics over those of the conventional RC panel, as indicated in Figure 2.

4.2 Influence of fiber content

Figure 3 shows the responses of panels having similar concrete strengths and fiber type, but different amounts of fiber. Panel C1C-R was the control panel with zero fiber content, whereas Panel C1F1V1, Panel C1F1V2, and Panel C1F1V3 contained 0.5%, 1.0%, and 1.5% by volume of RC80/50-BN Dramix® steel fibers, respectively.

A comparison of the shear stress-shear strain responses depicted in Figure 3a indicates strong similarity in behavior prior to cracking. However, the similarity diminished as the panels were subjected to higher shear stresses. All FRC panels except Panel C1F1V1 exhibited a gradual softening in the response until failure occurred. The control panel C1C-R also exhibited softening in the response, but at a lower stiffness. Panel C1F1V1, having only 0.5% of fibers by volume, exhibited a plateau in the response soon after the first crack appeared. Its response remained flat until the panel failed at a considerably lower shear resistance and ductility level than those of the control panel.

It is evident that an increase in the fiber content led to an improvement in concrete performance. Doubling the fiber volume content from 0.5% to 1.0% resulted in a 46% increase in the shear resistance and a 94% increase in the maximum attainable shear deformation. However, less than 4% added improvement in performance was obtained from increasing the fiber volume content from 1.0% to 1.5%. It is suspected that the optimum fiber addition threshold for this particular concrete mix might have been exceeded with the 1.5% fiber addition, as suggested by Rossi (1998), leading to the minor improvement.

Examinations of the principal tensile stress-principal tensile strain response of the panels further

clarified the influence of the fiber content on the behavior of FRC. As indicated in Figure 3b, although the response of Panel C1F1V1 indicated improvements over the response of the control panel, it still exhibited a strain softening behavior immediately after the maximum principal tensile stress was attained. By increasing the fiber volume content to at least 1.0%, substantial improvements in the concrete tensile behavior were observed, as indicated by the plateau in the principal tensile stress responses and by the high residual post-cracking tensile strengths. Again, only a marginal increase in the tensile stresses that could be transmitted across the crack was obtained from increasing the fiber content from 1.0% to 1.5%, leading to the minor performance improvement of Panel C1F1V3 over Panel C1F1V2.

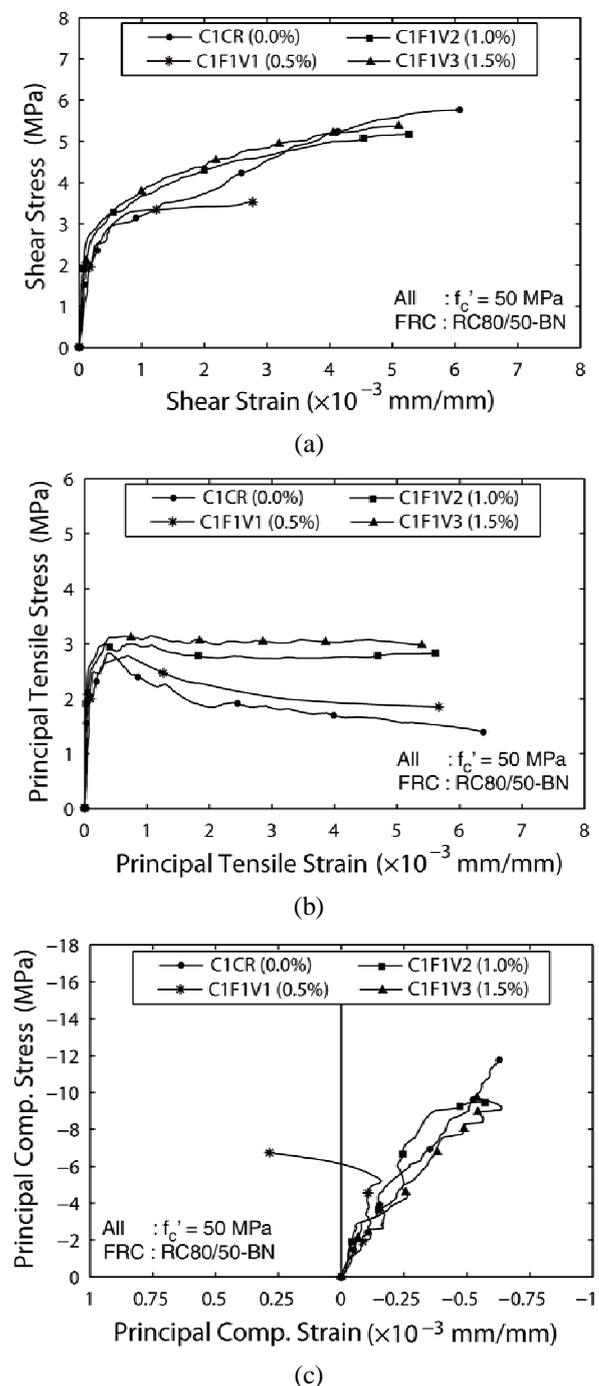
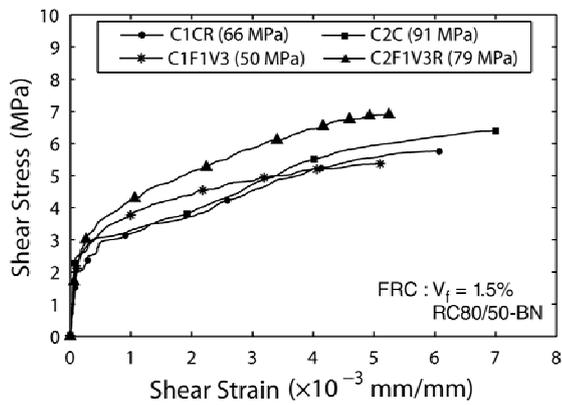
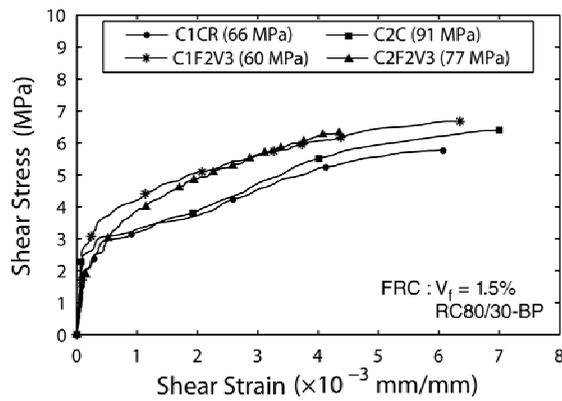


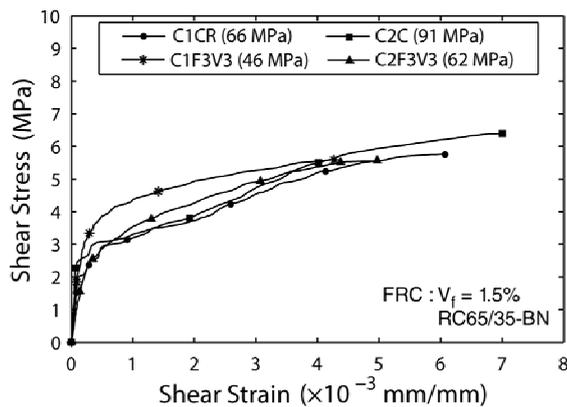
Figure 3. Influence of fiber content on panel responses.



(a)



(b)



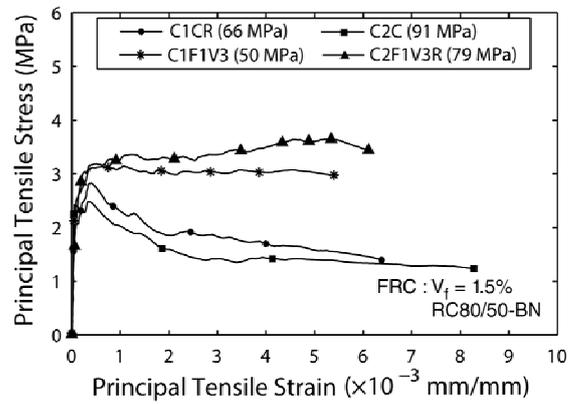
(c)

Figure 4. Influence of concrete strength on the shear stress-strain responses of the panels.

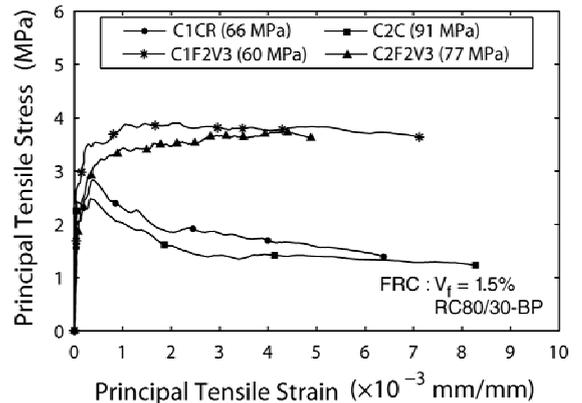
The comparison of the principal compressive stress-principal compressive strain responses of the panels is depicted in Figure 3c. As illustrated in the figure, the responses resembled each other. This supported the hypothesis that fiber addition minimally affected the pre-peak compressive behavior of the concrete, particularly at small compressive stress and strain levels (Naaman & Homrich 1987, Hsu & Hsu 1994).

4.3 Influence of concrete matrix strength

Figure 4 and Figure 5 compare the shear stress-strain and principal tensile stress-strain responses of the panels, respectively, to show the influence of concrete strength on panel behavior.



(a)



(b)

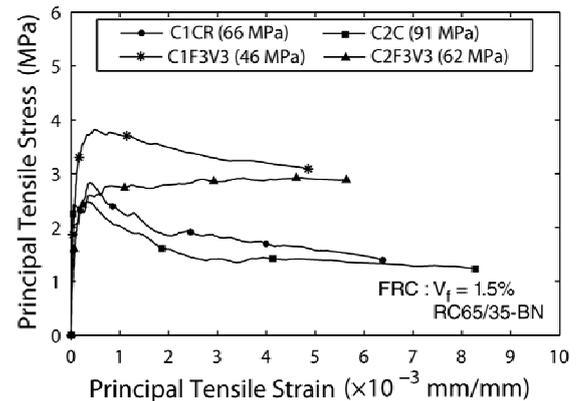


Figure 5. Influence of concrete strength on the principal tensile stress-strain responses of the panels.

As indicated in the shear stress-shear strain responses plotted in Figure 4, concrete strength did not seem to have a substantial influence on the shear stress response of the panels, as no significant correlations could be discerned from the observed behaviors. In the case of panels containing the RC80/50-BN fibers, a higher concrete strength resulted in a higher shear resistance. However, in panels containing the RC65/35-BN fibers, the result was opposite: a higher concrete strength resulted in a lower shear resistance. It appeared that the type of fibers used in the panel was more influential than the strength of the concrete matrix.

Another indication that the concrete strength did not significantly influence the FRC behavior was given by the principal tensile stress responses, plot-

ted in Figure 5. The influence of concrete strength on the panel behavior varies with the type of fibres used. Nevertheless, it is evident that the FRC panels substantially outperformed conventionally reinforced concrete panels in terms of concrete tensile behavior, as indicated by the high residual post-cracking tensile stress.

4.4 Influence of fiber type

Figure 6 plots various responses of panels having a similar concrete strength and containing 1.5% of fibers by volume for the case of FRC panels. It is evident from the plots that varying the type of fibers used altered the response of the panels. The plots also indicate that the properties of the fibers used were important in determining the shear performance of the fiber reinforced concrete panels. Nevertheless, regardless of the type of fibers used, fiber addition was found to considerably improve the behavior of the concrete.

The shear stress responses of the normal strength concrete panels are plotted in Figure 6a and Figure 6b. It can be observed from the figures that the panels containing short fibers (C1F2V3) exhibited a higher shear resistance than the panels containing long fibers with a similar aspect ratio (C1F1V3). The principal tensile stress responses plotted in Figure 6c and Figure 6d also suggest similar conclusions. It is presumed that this is due to the actual number of fibers present in the panels. Despite the same fiber content, the number of individual fibers was larger for short fibers than for long fibers due to the shorter length and the smaller diameter, leading to an increased possibility of the fibers intersecting microcracks. The development of microcracks into macrocracks could then be well controlled, and the concrete performance would be improved.

A further examination of the responses plotted in Figure 6 reveals the influence of the fiber aspect ratio on the panel responses. High aspect ratio fibers appear to be more effective than low aspect ratio fibers in improving the behavior of the concrete. The shear stress responses of Panel C1F1V3 and Panel C1F2V3, which contained fibers with aspect ratios of 81 and 79, respectively, showed better post-cracking ductility than did Panel C1F3V3, which contained fibers with an aspect ratio of 64. Furthermore, the principal tensile stress responses of Panel C1F1V3 and Panel C1F2V3 indicated a plateau after the maximum principal tensile stresses had been attained, whereas the response of Panel C1F3V3 showed a strain softening behavior. These findings emphasized the important influence of the fiber aspect ratio, more than the fiber length, on the effectiveness of fiber reinforcing, and agrees well with

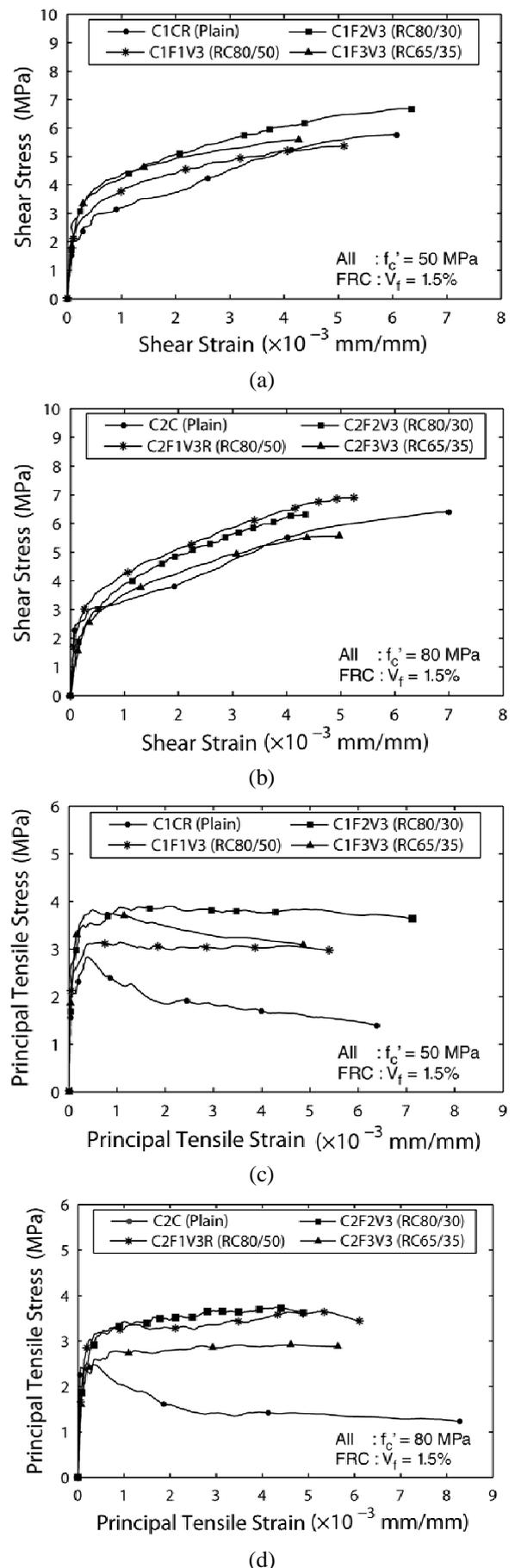


Figure 6. Influence of fiber type on the responses of the panels. the conclusions drawn by Johnston & Skarendahl (1992).

The fiber tensile strength did not seem to have a significant influence on the responses of the FRC panels tested in this research program. It appeared that the tensile stresses that were developed in the fibers were lower than the fiber ultimate tensile strength since no fiber fracture was observed. All activated fibers appeared to experience a ductile fiber slippage instead of the brittle fiber fracture, accounting for the ductile behavior of the FRC panels. Note that the tensile strength of the fibers does influence the moment capacity of the fibers, which affects the required force to straighten the hooks and pull the fibers out of the concrete.

5 MODELLING OF THE PANELS

Nonlinear finite element (FE) modeling of the panels was performed using VecTor2 to investigate whether the currently available constitutive models could accurately predict the behavior of the FRC panels. Consider the FE modeling performed for Panel C1F1V3. The panel was modeled as one four-node plane stress rectangular concrete element with a dimension of 890×890×70 mm, as indicated in Figure 7. The element was restrained against movements in the x- and y-directions at the lower left corner, and against movements in the y-direction at the lower right corner. The element was loaded in pure shear, with the load applied as a monotonically increasing load at the corners of the element. The bonded conventional reinforcement was modeled as smeared reinforcement embedded within the concrete element.

Two tension softening constitutive models for FRC are available in VecTor2: custom tension softening model, in which four points representing the tension softening curve are provided as input to the program; and the Variable Engagement Model (VEM) (Voo & Foster 2003).

The FE analysis results for Panel C1F1V3 using the custom tension softening curve and the VEM are given in Table 5. The shear stress-strain and principal tensile stress-strain responses are plotted in Figure 8. It is evident from the results that the FE analysis could not accurately capture the behavior of the FRC panel using the currently available constitutive models. Although it managed to reasonably predict the responses of the panels up to the failure stresses obtained from the experiments, the FE analysis significantly overestimated the strength and the deformation capacity of the FRC panels.

A closer inspection of the FE analysis results reveals a possible explanation for the overestimation in the predicted response. In the formulation of the Modified Compression Field Theory (MCFT) (Vecchio & Collins 1986) and the Disturbed Stress Field Model (DSFM) (Vecchio 2000), on which VecTor2 is based, the concrete average post-cracking tensile

stress is determined as the maximum of the stress calculated using the tension stiffening model and that calculated using the tension softening model.

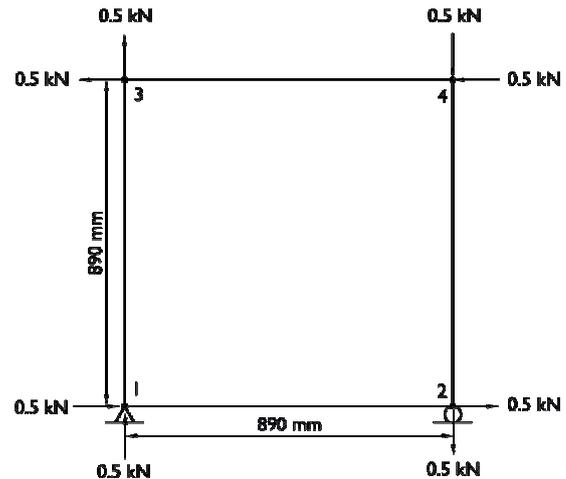


Figure 7. Finite element modeling of the panels.

Table 5. FE results of Panel C1F1V3.

	Experiment	Custom Tension Softening	VEM
v_u (MPa)	5.37	7.77	6.10
γ_u (me)	5.10	20.70	18.85
Max f_{c1} (MPa)	3.13	4.31	2.38
Max f_{c2} (MPa)	-9.70	-19.20	-16.20
Max f_{sx} (MPa)	204	501	439
w_m (mm)	0.45	1.63	2.19
s_m (mm)	57.2	57.2	78.1

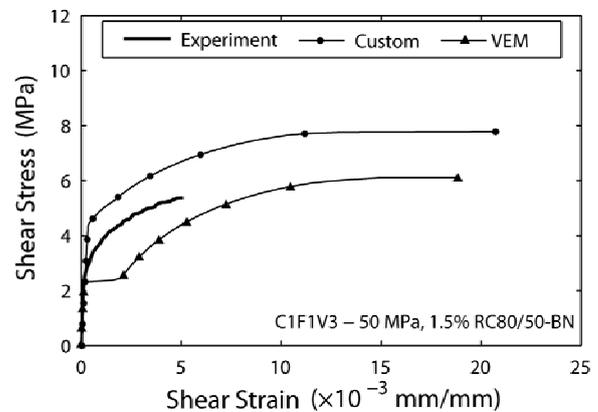


Figure 8. Predicted shear response of Panel C1F1V3.

Whereas the concrete principal tensile stress due to the tension stiffening is assumed to be zero at the crack, the concrete principal tensile stress due to the tension softening is assumed to continue to exist at the crack. In an FRC member, the addition of the fiber significantly increases the post-cracking tensile stresses and ductility of the concrete. Thus, the concrete principal tensile stresses due to tension softening remain consistently higher than those due to tension stiffening throughout the loading history. As a consequence, the average tensile stresses could be transmitted across the crack without any increase in the local reinforcement stresses. This resulted in no local shear stresses arising at the crack and, hence,

no crack slip occurred. The panel would continue to carry the applied load until another failure mechanism, either crushing of the concrete or yielding of the x-direction reinforcement, took place.

Additional FE analyses were performed using various crack spacings to investigate the influence of

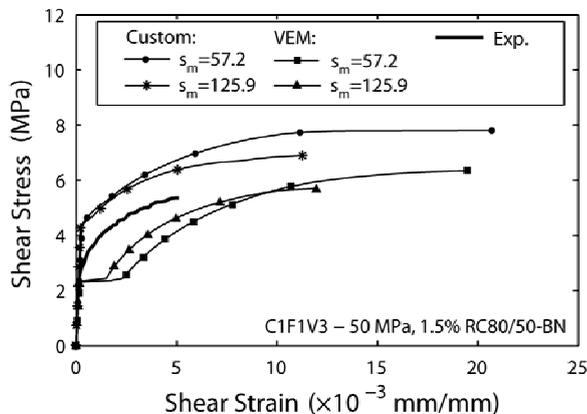


Figure 9. Influence of crack spacing on prediction results.

the crack spacing on the predictions. Average crack spacing is used to correlate the average concrete principal tensile strain required by the constitutive models to the crack width used in the VEM and in custom tension softening curve.

The analysis results are plotted in Figure 9. As the results indicate, larger crack spacings resulted in wider crack widths. With the custom tension softening curve, a wider crack width caused the concrete principal tensile stress to degrade at a faster rate. With the VEM, a wider crack width resulted in fibers being engaged sooner in carrying the tensile stress, leading to an early strength gain in the panels. In both cases, reductions in the strength and deformation capacity of the panel were observed.

6 CONCLUSIONS

Panel test results indicated that the FRC panels containing a fiber volume content of at least 1.0% managed to withstand at least 87% of the maximum shear stresses sustained by the conventionally reinforced concrete panels, and were able to undergo at least 62% of the maximum shear deformations of the conventionally reinforced concrete panels. The fiber volume addition of 0.5% was found to be insufficient to guarantee adequate shear resistance and shear deformation of the panel.

The amount of fiber addition significantly influenced the behavior of the panels. A fiber volume content of 0.5% was too low to control crack propagation. Increasing the fiber content to 1.0% resulted in a 46% increase in the shear resistance, a 94% increase in the maximum shear deformation, and a re-

duction in the crack widths by a factor of 3. However, only a minor improvement was obtained by increasing the fiber volume content from 1.0% to 1.5%, possibly due to fiber saturation.

The concrete compressive strength was not found to significantly influence the shear response of the FRC panels. The fiber type was found to be more influential than the concrete compressive strength. The use of fibers with a high aspect ratio resulted in much improved post-cracking deformation capacities and crack control characteristics compared to when fibers with a low aspect ratio were used. The panels containing short fibers were found to exhibit a higher shear resistance and a higher maximum concrete principal tensile stress than the panels containing long fibers with a similar aspect ratio due to the presumably larger number of fibers for the same fiber volume content.

Significant overestimations of the shear strength and the deformation capacity of all FRC panels were obtained from nonlinear finite element analyses using the available constitutive models. They are believed to be the result of the failure to properly recognize the correct tension stiffening behavior and the aggregate interlock mechanism. In addition, crack spacing was found to significantly influence the analysis results.

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