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Study on size effect in bending behavior of ECC

K. Asano
University of Tsukuba, Tsukuba, Ibaraki, Japan

T. Kanakubo
University of Tsukuba, Tsukuba, Ibaraki, Japan

T. Matsushima
University of Tsukuba, Tsukuba, Ibaraki, Japan

ABSTRACT: In this paper, the bending test of Engineered Cementitious Composites (ECC) with PVA fiber is conducted to investigate the size effect for bending behavior of ECC. The main parameter is the size of the specimens. The test results show that the bending strength increases as the size of specimen decreases. To express the wall effect of fibers, probability of fibers which cross the surface of specimen is solved theoretically. In the case of 40×40mm section specimen, 23% of fibers are influenced by specimen surface and considered to be oriented toward two-dimension. The coefficient of contribution to bending strength by two-dimensional orientation of fibers is proposed based on the probability.

1 INTRODUCTION

To use the feature of Engineered Cementitious Composites (ECC) (Li 1993) well, it is important to evaluate the tensile performance of ECC appropriately. Although a lot of methods to evaluate the tensile performance are proposed, uniaxial tension test is most proper. However, the shape, the size and the boundary condition of the specimens cause large influences on the test results. Therefore, it is difficult to perform the uniaxial tension test as a standard test. Although the tensile performance can be evaluated from the result of the bending test, the bending test results give a structural performance which is not a material performance. Therefore, it is necessary to understand a bending behavior and to clarify the relation between tensile behavior and bending one. Moreover, the size effect is caused by the difference of the size of the specimen. In addition, it is considered that the wall effect caused by two-dimensional orientation of fibers along the specimen surfaces affects the size effect on tensile and bending behavior. In this paper, the bending test of ECC with PVA fiber is conducted to investigate the size effect for bending behavior of ECC. The main parameter is the size of the specimens. After that, probability of fibers which cross the surface of specimen is solved theoretically to express the wall effect of fibers, The coefficient of contribution to bending strength by two-dimensional orientation of fibers is discussed based on the probability.

2 BENDING TEST

2.1 *Employed material*

The characteristics of PVA fiber are shown in Table 1. The compressive strength and the elastic modulus of ECC at the testing age are shown in Table 2. Moderate-heat Portland cement and fly ash (type II by JIS A 6201) are used as binder. Crushed limestone which specific surface area is 2500cm^2/g is used as the fine aggregate. The fiber volume fraction is 1.5% (PVA15) and 2.0% (PVA20). PVA15 and PVA20 were mixed by two batches each using a practical mixer with volume of 1m^3. The specimens were uniformly and continuously cast into the mold. After steam curing for 8 hours with 35 Celsius degree, the specimens were cured in atmospheric environment.

2.2 *Specimens*

Figure 1 shows the shape of the specimens. Table 3 shows the list of the specimens. The 4-point bending test is applied. The number of specimens for the same parameter is three.

2.3 *Loading and measurements*

The displacement controlled 2MN universal testing machine was used. The loading speed was set to 0.5mm/min. The applied load and the axial deformation at the test section were measured to calculate curvature. The position of displacement transducers was set to have the same dimensional ratio in each size of specimens. However, the gauge length of the B40 was 25mm due to the loading device restriction.

Table.1 Characteristics of PVA fiber.

Length (mm)	Diameter (mm)	Tensile strength (MPa)	Elastic modulus (GPa)
12	0.04	1690	40.6

Table.2 Compressive performance of ECC.

Batch	Fiber volume fraction (%)	Air content (%)	Compressive strength (MPa)	Elastic modulus (GPa)
PVA15-1	1.5	5.4	55.3	18.9
PVA15-2		5.6	56.9	19.9
PVA20-1	2.0	11.0	45.0	16.0
PVA20-2			47.1	16.4

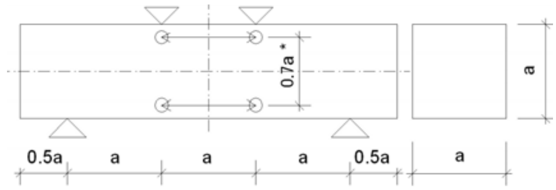

Figure 1. The shape of the specimens (*:25mm in B40).

Table 3. The list of the specimens.

Name of specimens	Batch	*a* (mm)	Test section (mm)
B40-15	PVA15-1	40	50
B100-15		100	100
B200-15		200	200
B400-15	PVA15-2	400	400
B40-20	PVA20-1	40	50
B100-20		100	100
B200-20		200	200
B400-20	PVA20-2	400	400

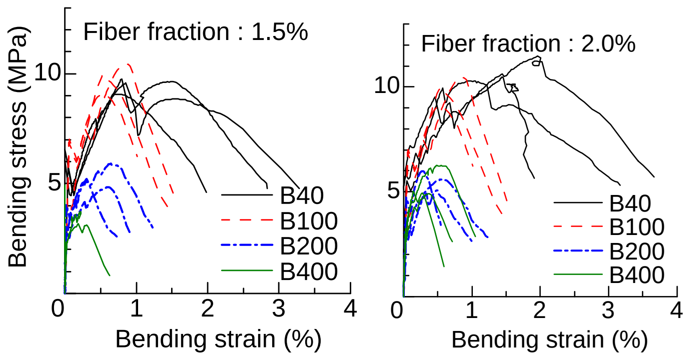

Figure 2. The relation between bending stress and bending strain.

2.4 *Test results*

Table 4 shows the test results. The bending stress is defined as a value of bending moment divided by the modulus of section and the bending strain is the curvature multiplied by specimen depth. These factors are defined to consider the differences of the size of specimens. Figure 2 shows the relationship between the bending stress versus the bending strain. In spite of the difference of fiber volume fraction and the size of specimen, the deflection hardening behavior in which the load increases after first crack can be recognized. The average value and the coefficient of variation (COV) of the maximum bending stress and the bending strain at the maximum are shown in Table 5 and Table 6. There are a lot of the specimens which COV is lower than 10% for the maximum bending stress. However, the range of the COV is from 20% to 30% for the bending strain. A clear relationship can not be confirmed between the size of the specimens and the COV.

3 QUANTIFICATION OF THE SIZE EFFECT

3.1 *Probability of fibers which cross the surface of specimen*

In this paper, the idea of "Buffon's Needle" (Rozanov & Sobor 1976) enhanced to three-dimension is discussed. "Buffon's Needle" treats the needle of a certain length that is distributed at random in the limited plane area, and expresses the calculation of the probability of crossing the boundary in the area by the algebraic formulation. In this paper, it is enhanced to three-dimension, and the probability of the fiber crossing the surface is calculated from the boundary in the spatial area. When the fibers are distributed in a three-dimensional space, the fibers which are actually restrained by the mold and a free boundary (casting surface) are thought to be oriented along two-dimension as shown in Figure 3. These fibers do not cross the surface and exists in the neighbor. This phenomenon is defined as a probability event in which the originally random oriented fibers are influenced by the boundary (wall effect).

3.2 *Theoretical probability*

The size of the specimen and the length of the fiber are treated as the parameters. The probability that the fiber crosses the boundary is constructed. To simplify the construction, the diameter of the fiber is disregarded. At first, the complementary event of the target probability is considered. The left figure in

Table 4. Test results.

Name of specimens	Size of the cross section (mm)	At the maximum load Bending moment M_{max}(kNm)	Curvature φ_{max}(1/m)	Bending stress σ_{max}(MPa)	Bending strain ε_{max}(%)
B40-15-1		0.100	0.215	9.58	0.85
B40-15-2	40×40	0.095	0.185	9.06	0.73
B40-15-3		0.105	0.201	9.76	0.80
B100-15-1		1.525	0.548	9.08	0.55
B100-15-2	100×100	1.670	0.539	9.97	0.54
B100-15-3		1.780	0.847	10.45	0.85
B200-15-1		7.911	0.032	5.92	0.64
B200-15-2	200×200	6.465	0.030	4.82	0.60
B200-15-3		6.803	0.015	5.06	0.30
B400-15-1		52.740	0.005	4.93	0.18
B400-15-2	400×400	40.276	0.005	3.72	0.21
B400-15-3		34.168	0.005	3.18	0.19
B40-20-1		0.124	0.487	11.49	1.96
B40-20-2	40×40	0.110	0.247	10.30	0.99
B40-20-3		0.109	0.367	10.47	1.45
B100-20-1		1.863	0.677	11.20	0.68
B100-20-2	100×100	1.728	0.726	10.44	0.72
B100-20-3		2.160	1.170	12.85	1.18
B200-20-1		7.591	0.027	5.58	0.55
B200-20-2	200×200	8.048	0.014	5.98	0.28
B200-20-3		6.763	0.025	5.08	0.49
B400-20-1		67.002	0.012	6.27	0.49
B400-20-2	400×400	52.358	0.009	4.91	0.38
B400-20-3		54.202	0.004	5.03	0.14

Table 5. Maximum bending stress.

Specimen	Average of the maximum bending stress (MPa)	Coefficient of variation (%)	Specimen	Average of the maximum bending stress (MPa)	Coefficient of variation (%)
B40-15	9.47	3.1	B40-20	10.75	4.9
B100-15	9.83	5.8	B100-20	11.50	8.7
B200-15	5.27	8.9	B200-20	5.54	6.7
B400-15	3.94	18.6	B400-20	5.40	11.4

Table 6. Bending strain at the maximum load.

Specimen	Average of bending strain at the max. (%)	Coefficient of variation (%)	Specimen	Average of bending strain at the max. (%)	Coefficient of variation (%)
B40-15	0.79	6.2	B40-20	1.47	27.2
B100-15	0.65	22.5	B100-20	0.86	26.2
B200-15	0.52	29.2	B200-20	0.44	26.5
B400-15	0.19	6.0	B400-20	0.34	43.3

Figure 4 shows the cross-section of the specimen with shows the cross-section of the specimen with b(mm)×D(mm). Four fibers are expressed by straight lines in the figure. As for two fibers, the both ends of the fiber touch the boundaries with the orientation angle of θ. The other two fibers touch the corner at the end also with the orientation angle of θ. These fibers indicate the maximum area in which the fiber has possibility to cross the boundary, in other words, possibility to be influenced by the surface of the cross section. The grayed rectangle in the figure expresses the area in which when the center of the fiber exists, the fiber is not influenced by the boundary. The central figure in Figure 4 shows the expansion the above mentioned consideration to 3-dimensional fiber existence toward the axial direction of the specimen. The fiber inclines with the direction of φ by the axial direction keeping the angle of θ to the cross-sectional direction. Then the grayed rectangle in the left figure expands by considering 3-dimensional orientation as shown in the right figure. If the center of the fiber exists in the grayed rectangle, that fiber does not cross the boundary.

Next, the above-mentioned probability event is formulated. It is described step by step from the left figure in Figure 4.

1) Left figure

The probability in two-dimension is given by calculation of the ratio of the grayed inner rectangle area to cross section area of the specimen with varying of angle θ. When the length of the fiber is defined as l_f, inner rectangle area is given by the following formula.

$$\left(b-l_f\left|\cos\theta\right|\right)\left(D-l_f\sin\theta\right) \tag{1}$$

where, $\cos\theta$ is expressed as the absolute value. This is because the range of θ is from 0 to π, and the projection of the fiber length on a horizontal axis can be expressed.

2) Central and right figure

The $\cos\varphi$ is multiplied to Formula (1) including l_f when the fiber inclines by the axial direction with the angle of φ as shown in the central figure. When assuming that the surface exists toward the axial direction at certain distance L, it shall be considered that the $l_f \sin\varphi$ expresses the influenced region of fibers in the axial direction. The axial length of the non-influenced solid can be described as $L - l_f \sin\varphi$. Therefore, the volume of the solid is given by the following formula.

$$\left(b-l_f\left|\cos\theta\right|\left|\cos\phi\right|\right)\left(D-l_f\sin\theta\left|\cos\phi\right|\right)\left(L-l_f\sin\phi\right) \tag{2}$$

The probability that the fiber is not influenced by the boundary is given by the following formula.

$$\frac{\left(b-l_f\left|\cos\theta\right|\left|\cos\phi\right|\right)\left(D-l_f\sin\theta\left|\cos\phi\right|\right)\left(L-l_f\sin\phi\right)}{bDL} \tag{3}$$

Formula (3) gives the probability in case of an arbitrary orientation angle, θ and φ. It is necessary to calculate the probability in all the orientation angles. The probability that θ varies with random angle $d\theta$ from 0 to π is given by the following formula.

$$\frac{1}{\pi}\cdot d\theta \tag{4}$$

Similarly, for the case of φ,

$$\frac{1}{\pi}\cdot d\phi \tag{5}$$

The events expressed by formulas (3), (4), and (5) are the independent events. The probability that the fiber is not influenced by the boundaries ($\overline{p}$) can be calculated by the double integral of these product. However, it is not necessary to treat axial direction when only the pure bending section is considered. Therefore, a part of formula (3) is given as follows.

$$\frac{\left(L-l_f\sin\phi\right)}{L}=1 \tag{6}$$

Finally, $\overline{p}$ is given as follows.

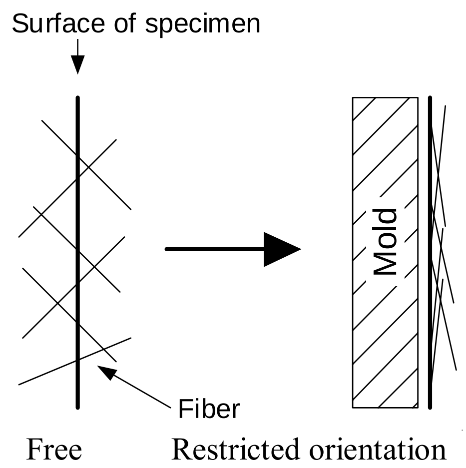

Figure 3. Fiber influenced by the surface.

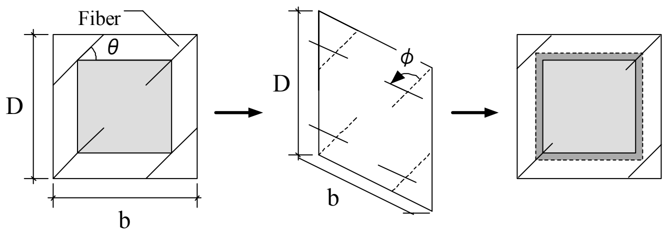

Figure 4. The non-influenced area for center position of fiber.

$$\int_0^{\pi}\int_0^{\pi}\frac{\left(b-l_f\left|\cos\theta\right|\left|\cos\phi\right|\right)\left(D-l_f\sin\theta\left|\cos\phi\right|\right)\left(L-l_f\sin\phi\right)}{bDL}\frac{d\theta d\phi}{\pi^2}$$
$$=\int_0^{\pi}\int_0^{\pi}\frac{\left(b-l_f\left|\cos\theta\right|\left|\cos\phi\right|\right)\left(D-l_f\sin\theta\left|\cos\phi\right|\right)}{bD}\frac{d\theta d\phi}{\pi^2}$$
$$=4\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\frac{\left(b-l_f\cos\theta\cos\phi\right)\left(D-l_f\sin\theta\cos\phi\right)}{bD}\frac{d\theta d\phi}{\pi^2}$$
$$=1-\frac{4l_f}{\pi^2}\left(\frac{1}{b}+\frac{1}{D}\right)+\frac{{l_f}^2}{2\pi bD}$$

Because $\bar{p}$ is a complementary event of the target probability that the fiber crosses the boundaries, the probability p that the fiber crosses the boundaries is given as the following equations.

$$p=1-\bar{p}$$
$$=\frac{4l_f}{\pi^2}\left(\frac{1}{b}+\frac{1}{D}\right)-\frac{{l_f}^2}{2\pi bD} \qquad (7)$$

In case of this study, for B40 specimen, 23% of fibers are influenced by specimen surface and considered to be oriented toward two-dimension.

3.3 *Evaluation of Bending Test Results*

The bending strength (maximum of bending stress) observed in bending test is evaluated using previously discussed probability. The fiber distributed along the surface is considered to be in two-dimensional and has a big influence to the bending strength. In the literature (Li 1990), the strength of pull out test of the single fiber is expressed according to the orientation angle. It is assumed that the probability given by Eq. (7) has the certain influence to the bending strength. In the bending test, it is assumed that bending behavior has the same tendency with the behavior of pull out test. Eq. (8) is introduced referring to the proposed model by exponential formula in order to express the size effect considering the probability. Where, α_b is introduced as a contribution coefficient that expresses the influence of two-dimensional oriented fiber to bending strength. Figure 5 shows the relation between the bending strength of each size specimen normalized by that observed by B400 specimen and the probability given by Equation (7). The curve in the figure indicates Equation (8). The value of α_b is obtained from the least squares method. α_b is 4.68 in case of fiber volume fraction of 1.5%, and 3.59 in case of 2.0%. The size effect becomes smaller as the fiber fraction increases.

$$\frac{\sigma_{\max}}{\sigma_0}=\exp(\alpha_b\cdot p) \qquad (8)$$

σ_{max} : bending strength of each size specimen
σ_0 : bending strength of the largest size specimen
α_b : contribution coefficient (α_b>1)
p : probability that fiber crosses boundaries

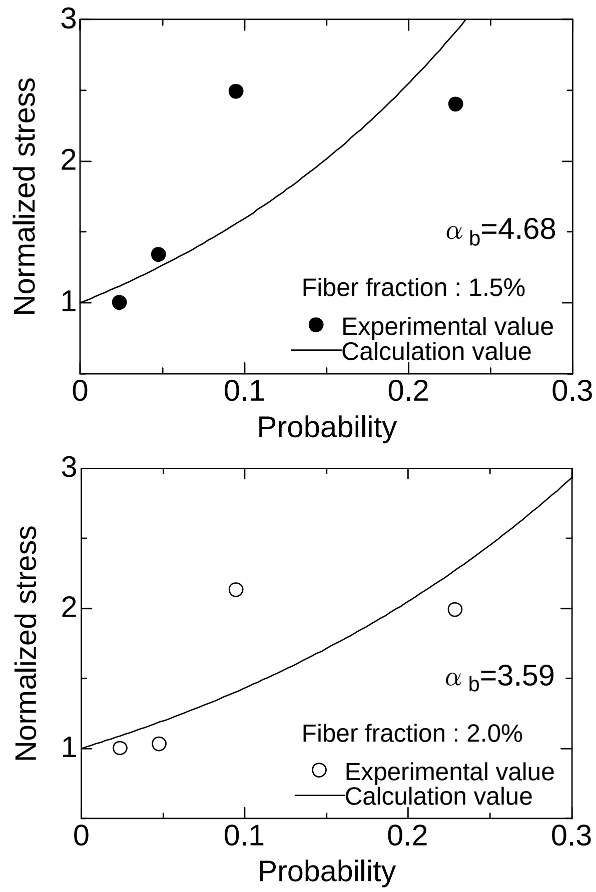

Figure 5. The relation between probability and bending strength.

4 CONCLUSION

To quantify the size effect in bending behavior of ECC, the bending test is carried out using the specimen with various sizes as the main parameter. The probability that the fiber crosses the boundary is constructed. The followings are concluded in this study.

(1) The bending test results show that the bending strength increases as the size of specimen decreases.

(2) To express the wall effect of fibers, probability of fibers which cross the surface of specimen is solved theoretically. In the case of 40×40mm section specimen, 23% of fibers are influenced by specimen surface and considered to be oriented toward two-dimension.

(3) The coefficients of contribution to bending strength by two-dimensional orientation of fibers are proposed based on the probability.

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