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An analytical study on the impact of hollow shapes in bi-axial hollow slabs

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ABSTRACT: This paper presents optimal hollow sphere shapes in a biaxial hollow slab. To derive optimal hollow shapes, numerical simulations using nonlinear Finite Element Methods were executed by the nonlinear finite element program 'LUSAS'. Recently, various types of slab systems which can reduce self-weight of slabs have been studied as the height and width of building structures rapidly increase. A biaxial hollow slab system is widely known as one of the effective slab system which can reduce self-weight of slab. A biaxial hollow slab has hollow spheres within slab in order to reduce self-weight of slab. Because of reducing self-weight of slab by hollow spheres, size of vertical elements like walls and columns can be smaller and slabs span can be longer. A capacity of biaxial hollow slabs is influenced by the shape and volume of hollow spheres. Therefore, in this study, several biaxial hollow slabs which have different shapes of hollow spheres were analyzed by using the finite element method program in order to derive optimal hollow sphere shapes.

1 INTRODUCTION

1.1 *Preface*

In building, the slab is very important structural member to make a space. And Slab is one of the largest member consuming concrete. In a general way, the slab was designed only to resist vertical load. However, deflection and vibration of slab are also considered recently because people are getting more interest of residential environment. In addition, when span of the building is increasing, deflection of slab is more important. Therefore, the slab thickness is on the increase. The increasing of slab thickness makes slab heavier, and it leads to increase column and base size. Thus, it makes building consume more materials such as concrete and steel. Moreover, the increasing of weight is harmful for building when earthquake occur.

To avoid these disadvantages which were caused by increasing of self-weight of slabs, the biaxial hollow slab system, also known as void slab, was suggested. This slab system could optimize the size of vertical members like walls and columns by lightening the weight of slabs. Therefore, it got attention because it made efficient and economical building design possible. A capacity of biaxial hollow slab is influenced by hollow sphere shapes. However, the researches about hollow sphere shape have been insufficient. So, in this study, several kinds of hollow slabs which have different hollow sphere shapes were analyzed by using finite element method program in order to derive the optimal hollow sphere shapes and to verify the impact of hollow shapes in biaxial hollow slabs.

1.2 *Literature review*

A biaxial hollow slab system was developed in 1990s. In 21C, hollow slab systems which have same concept and different hollow shapes were invented over USA, Europe and Japan. (See Table 1.)

According to existing hollow slab systems, self-weight reduction ratio of slab was 25~30%. And their flexural strength was similar to solid slab.

1.3 *Research objective*

The aims of this research were to grasp the relationship between hollow sphere shapes and slab's

Table 1. The existing Hollow slab systems.

	Name	System	Void Material	Void Shape	Weight reduction	Strength Flexure	Strength Shear
Europe	Cobiax		Plastic	Sphere Ellipse	30%	100%	50%
	U-Boot			Cuboid	35%	100%	45%
USA	Filigree wide slab	Void Slab			25%	100%	65%
Japan	Mom-slab		Styro-foam	Sphere Ellipse	30%	100%	50%
	EJ Void				30%	100%	50%

* Compared to Solid slab

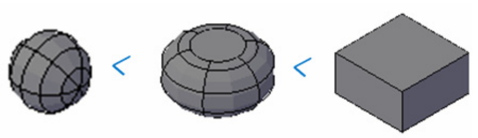
capacities. And development of the optimal hollow sphere shape was another aim of this research.

To do these, 3 stages of analytical research process were performed by using finite element method program named 'LUSAS'.

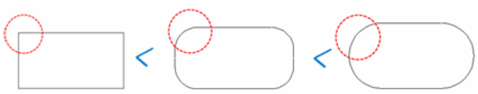
1) Finding out the parameters of hollow shape.
2) Grasping the impact of hollow sphere shapes in biaxial hollow slabs.
3) Developing the optimal hollow sphere shape.

2 THE PARAMETERS OF HOLLOW SPHERE SHAPE

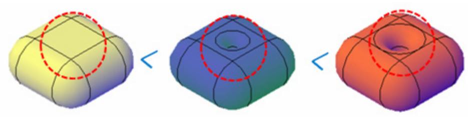
For finding out the parameters of hollow shape, the existing hollow shapes were compared and analyzed each other. As a result, three parameters were derived such as typical shapes, corner radius and hole diameter. (Fig. 1)



a) Typical shapes of hollow sphere (Hollow volume)



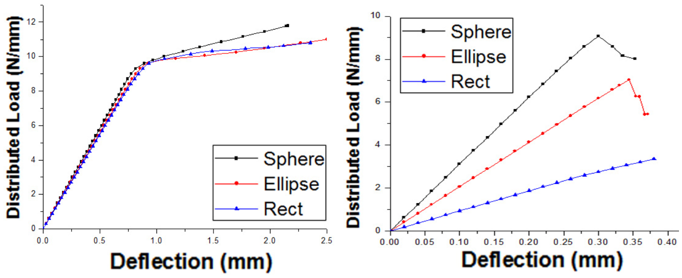
b) Corner radius of hollow sphere



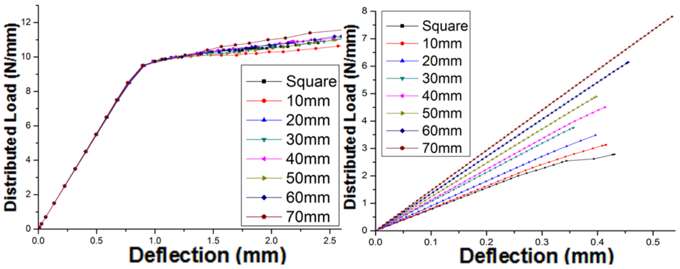
c) Hole diameter of hollow sphere

Figure 1. The Parameters of hollow sphere shape.

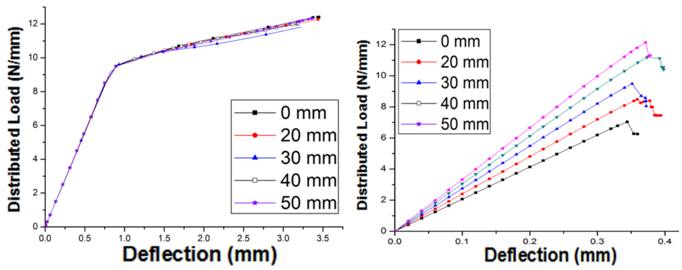
The section geometries of hollow spheres were studied to limit the field of the three-dimensional hollow sphere shapes, before the derivation of the three-dimensional hollow sphere shapes. To do this, the hollow slabs were modeled in two-dimensional plane, changing 3 parameters such as typical shapes, corner radius and hole diameter. And numerical simulations of two-dimensional hollow slab model were performed. It is meaningless to perform quantitative analysis of two-dimensional hollow slab model about above parameters. Because it cannot consider change of hollow shapes and concrete web parts between hollow spheres. But it is meaningful to qualitative analysis of two-dimensional hollow slab model about the parameters to find qualitative effects on hollow slab such as crack propagation or concentration.



a)Typical shape : Load-Deflection Curve (Flexural/Shear)

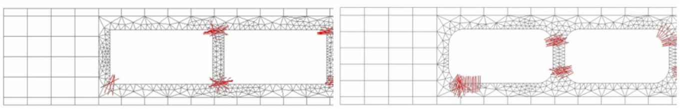


b) Corner radius : Load-Deflection Curve (Flexural/Shear)

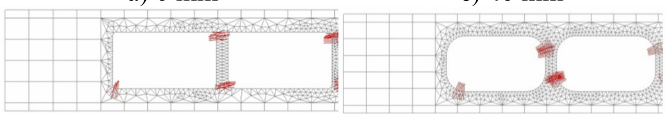


c) Hole diameter : Load-Deflection Curve (Flexural/Shear)

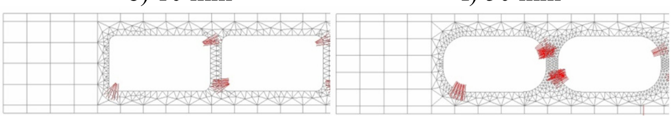
Figure 2. 2D FEM Load-Deflection Curve.



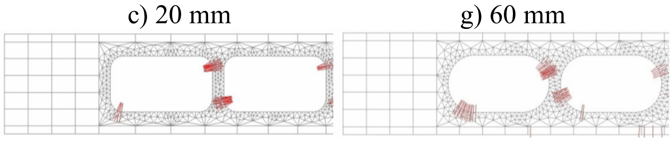
a) 0 mm e) 40 mm



b) 10 mm f) 50 mm



c) 20 mm g) 60 mm



d) 30 mm h) 70 mm

Figure 3. Crack pattern by corner radius.

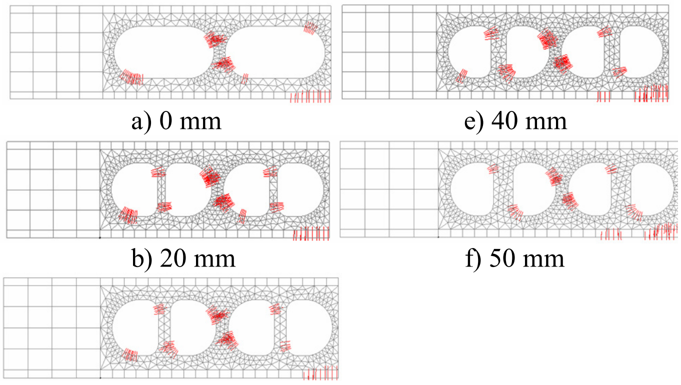


Figure 4. Crack pattern by hole diameter.

The results of 2D FEM analysis were shown in Figure 2, 3 and 4.

As shown Figure 2, the parameters of two-dimensional hollow slab model does not affect bending strength and stiffness of slab.

However, Figure 2 shows that the parameters affect shear strength and stiffness. And Figure 3 shows that the corner radius becomes smaller, the cracks are concentrated more. Because of that, the early destruction occurs by the progress of cracks in shear failure mode. And Figure 4 shows that the hole diameter becomes larger, the shear cracks are prevented.

3 SETTING UP THE HOLLOW SPHERE SHAPE

3.1 *Target structure*

The hollow slab system is effective when self-weight of slab is a high rate. In other words, this system is most effective when it is applied to thick flat plate slabs. Therefore, target structure system was set up ‘wall + flat plate slab’ system which had 250mm thick slab. (See Fig. 5.)

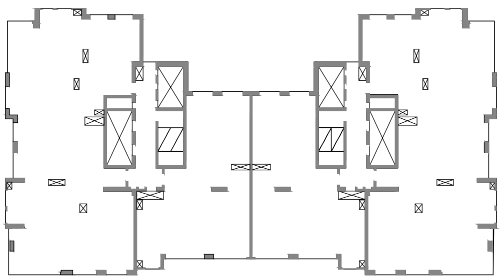


Figure 5. Target structure system (wall + flat plate slab).

3.2 *Size of hollow sphere*

The height of hollow sphere was set up, according to height of compressive stress block in target structure slab. As assuming hollow shape was pipe type, the height of compressive stress block derived by using strain compatibility method.

The strain compatibility method is the rigorous method of estimating the flexural strength. Concrete section is based on the compatibility of strains and equilibrium of forces acting on the section at the stage of failure such as reinforced bar strain reaches 0.002 or concrete strain reaches 0.003. Strain of concrete and reinforced bar could be derived by using equations (1) ~ (4).

$$\varepsilon_s = \frac{d_p}{c}(0.003) - 0.003 \tag{1}$$

$$f_s = \varepsilon_s E_s \tag{2}$$

$$T = A_s f_u \tag{3}$$

$$C = 0.85 f_c^{'} \beta_1 cb \tag{4}$$

In target structure, the height of compressive stress block was changed by location. As shown Figure 6, the height of hollow sphere had to be less than 160mm to avoid development of compressive stress block in hollow parts. However, as considered anchorage of reinforced bar, 140mm was adequate height of hollow sphere in 250mm thick flat plate slabs. Considered architectural module, adequate width of hollow sphere was 270mm.

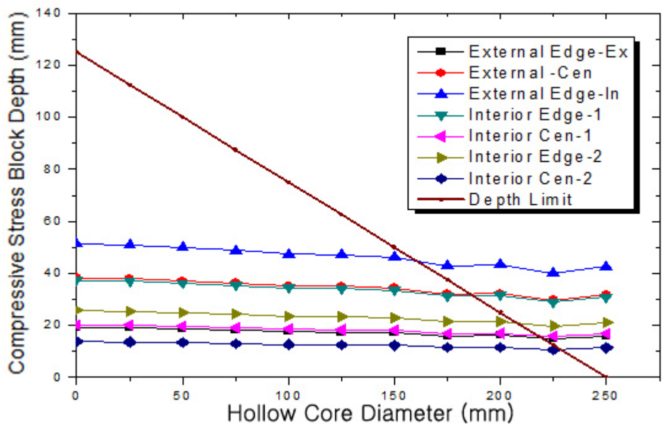


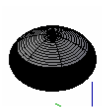
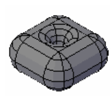
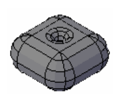
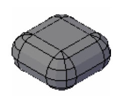
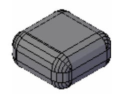
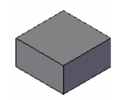
Figure 6. The height of compressive stress block.

3.3 *Hollow sphere shape*

The hollow sphere shapes varied with the limitation of size. And the range of hollow volume was set up 20% to 45% of one slab module volume (300mm x 300mm x 250mm). Considered the height of hollow sphere, a range of corner radius of hollow sphere was set up 0mm to 70mm. And a range of hole diameter was set up more than 30mm, as considered concrete construction.

As compared the 3 parameters of hollow shape, 8 types of hollow spheres which were divided into 3 classes. The first group varied with the typical sphere

Table 2. The properties of hollow spheres.

	Solid	Sphere	Mushroom	Ellipse	Rect Donuts (D=50mm)	Rect Donuts (D=30mm)	Round Rect (R=70mm)	Round Rect (R=50mm)	Square
Shape	-								
Volume(cm^3)	-	1436	5625	6300	7380	7650	7785	8910	10125
Diameter(cm)	-	14	27	27	27	27	27	27	27
Height(cm)	-	14	14	14	14	14	14	14	14
Weight reduction (%)	0%	20%	25%	28%	32.8%	34%	34.6%	39.6%	45%

shapes such as Sphere, Mushroom, Ellipse and Round Rect(R=70mm). The second group varied with the corner radius such as Square, Round Rect(R=50mm) and Round Rect(R=70mm). And the last group varied with the hole diameter such as Round Rect(R=70mm), Rect Donuts(D=50mm) and Rect Donuts(D=30mm). Further information and properties of the hollow spheres are shown Table 2.

4 3D FEM ANALYSIS OF HOLLOW SLAB

4.1 *Modeling*

To derive the optimal hollow shape, numerical simulations using nonlinear Finite Element Methods were performed by the nonlinear finite element program LUSAS. To perform numerical simulations of the slabs by using finite element method, two dimensional model was often used when slabs were uniform in transverse axis. However, in the case of biaxial hollow slabs, it was impossible to use two-dimensional model because of the unequally section geometry along the longitudinal and transverse axis by extraordinary hollow sphere shape. So, it was modeled using three-dimensional model which could be considered concrete web parts between hollow spheres like Figure 7. And to generate a finite element mesh of the hollow slab which has extraordinary shapes of hollow sphere inside, tetrahedral elements which have four nodes were used because other mesh elements such as Pentahedral or Hexahedral were not able to generate the geometry of the hollow slab.

Two material models were used to perform nonlinear finite element method analysis. The one, which was used reinforced steel bar, is bi-linear model of steel. It is assumed that steel behavior will be totally elasto-plastic in tensile and compressive loading condition. (See Fig. 8-a) And another material model, which was used concrete, is ‘LUSAS concrete model 94’. It can consider multi-cracks of concrete and strength softening of concrete.(Fig. 8-b)

The convergence study was performed about mesh size ranged 20mm~150mm to verify the numerical simulation result. The result converged when mesh size was 30mm into 27.1kN. This is theoretical value by using equations (5), (6).

$$M_n = (A_s f_y - A_s^{'} f_s^{'})(d - \frac{a}{2}) + A_s^{'} f_s^{'}(d - d^{'}) \quad (5)$$

$$P_n = \frac{2M_n}{l} \quad (6)$$

In this study, the hollow slab model was generated

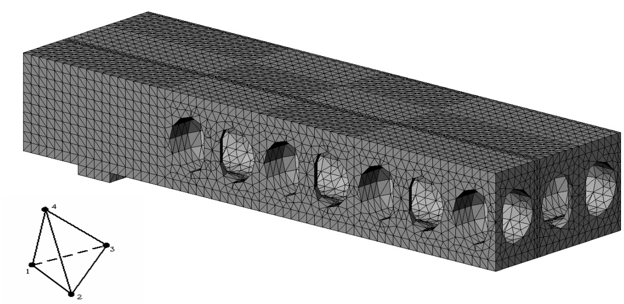
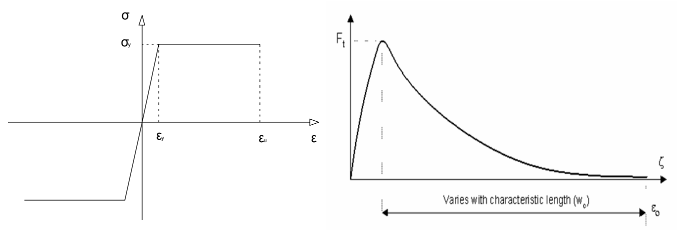


Figure 7. Finite element mesh modeling (In the view of the cross-section).



a) Bi-linear model b) LUSAS Concrete Model 94

Figure 8. Nonlinear material model.

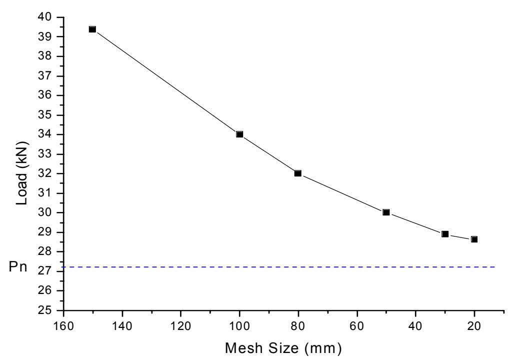


Figure 9. Nonlinear material model.

a finite element mesh with 30mm-sized. Because the use of small mesh size less than 30mm did not change the results significantly, but led to an increased computation time. The relationship between mesh size and convergent tendency in this hollow slab model is shown Figure 9.

4.2 *Hollow slab analysis model*

To perform 3D nonlinear FEM analysis, the properties of hollow slab model was idealized. It had 8.9m length, 300mm width. The reasons of performing idealize process are like this

It was possible to estimate the whole capacity of slab with the capacity of one module slab model because hollow spheres were located with uniform gap toward width direction of slab.

So, to compare the impact of hollow shapes on flexural capacity of slabs, it was enough to perform FEM analysis with one module model of slab. And this way was time-efficient than using whole slab model.

Table 2. Properties of hollow slab model.

Width		300 mm
Height		250 mm
Length		8900 mm
Reinforced bar		Upper D10 x 2 Lower D13 x 2
Boundary condition		Fixed end
Load	Self-weight	330~600kg/m^2
	Dead load	185kg/m^2
	Live load	200kg/m^2 ~
fck		24 MPa
Fy		400 MPa

The conditions of support were fixed. Because, In FEM Analysis, simple supported condition might bring about error such as stress concentration. The reason of using distributed load was the same. Loads were imposed on slabs in order likes self-weight, dead load and live load until the slab was destroyed.

Further information and properties of hollow sphere is shown Table 3.

5 RESULTS AND ANALYSIS

5.1 *Results of numerical simulations*

The 3D hollow slab models, applied above hollow spheres (Table 2), were analyzed by numerical simulations using nonlinear Finite Element Method program.

Table 4 is shown that results of computation about 8 cases of hollow slabs which can be distinguished by hollow sphere shapes and 1 case of solid slab.

In the results of computation, 8 cases of hollow slabs were judged safe in design load. And some of hollow slabs show a good load bearing capacity and stiffness, compared with solid slab,

The results, analyzed more concretely, are as follows:

As compared difference design load and ultimate load, hollow slab applied 'Rect Donuts(D=50mm)' was shown the largest difference, 26.24kN. It means additional load bearing capacity of slab after dead load. Because a dead load varied with hollow volume, it was reasonable to compare real capacity of hollow slab.

The deflections in design load were varied with weight reduction of slabs. Round Rect(R=50mm)

Table 4. Numerical simulation result of three-dimensional hollow slab model.

	Solid	Sphere	Mushroom	Ellipse	Rect Donuts (D=50)	Rect Donuts (D=30)	Round Rect(R=70)	Round Rect(R=50)	Square
Self-weight (kN/m^2)	5.9	4.7	4.4	4.2	3.9	3.9	3.8	3.5	3.2
Dead Load (kN/m^2)	9.7	8.5	8.2	8	7.7	7.7	7.6	7.3	7
Ultimate Load (kN/m^2)	34.36	33.80	33.35	33.97	33.94	33.50	33.12	32.08	28.50
$\frac{Dead-Load}{Ultimate-Load}$	28.2%	25.1%	24.6%	23.6%	22.7%	23.0%	22.9%	22.8%	24.6%
Deflection at D.L (mm)	2.16	1.86	1.72	1.68	1.57	1.58	1.61	1.56	1.61
Deflection at U.L (mm)	22.42	21.32	22.60	23.71	23.33	24.02	24.23	24.68	22.30
Failure mode at Design Load	F	F	F	F	F	F	F	F	F
Failure mode at U.L	F	F	F	F	F	F	F	F	F+S

* F : Flexural Crack Occur, S : Shear Crack Occur

and Rect Donuts(D=30mm) were the smallest deflection at dead load about 72% of solid slab. And the deflections in ultimate load were varied with its load-bearing capacity and stiffness.

Looking at the failure mode, all slabs were shown flexural crack in design load. When load reaches ultimate load, the failure mode was shown flexural failure behavior except Square shape. The Square shape had flexural and shear failure behavior by judging with strain of reinforced bar.

5.2 *Impact of hollow sphere shape*

To find out the impact of hollow sphere shape, the results of each group which were divided into 3 classes in above chapter 3 were analyzed. And the impact of each parameter is as following.

5.2.1 *Impact of typical hollow shape*

To compare impact of typical hollow shape, 5-types hollow spheres having difference shape and volume were used because a typical hollow shape correlates with a hollow volume. The relationship between a typical hollow shape and hollow volume was not linear but staircase type.

Figure 10-a) was shown the load-deflection curve of the typical sphere shapes such as Solid, Sphere, Mushroom, Ellipse and Round Rect(R=70mm). It showed different behaviors of slab at dead load. When volume of hollow sphere was increased, slab stiffness and deflection tended to be decreased. However, reduction of stiffness was not according to changing of hollow shape. As seeing the crack pattern, it was according to reduction of section by hollow volume.

Therefore, when corner radius and hole diameter were same and hollow volume was increasing, stiffness and deflection of slab tended to decrease at dead load. At ultimate load, an amount of influence by typical hollow shape was about 1.6~3.6% of solid slab, with the aspect of strength.

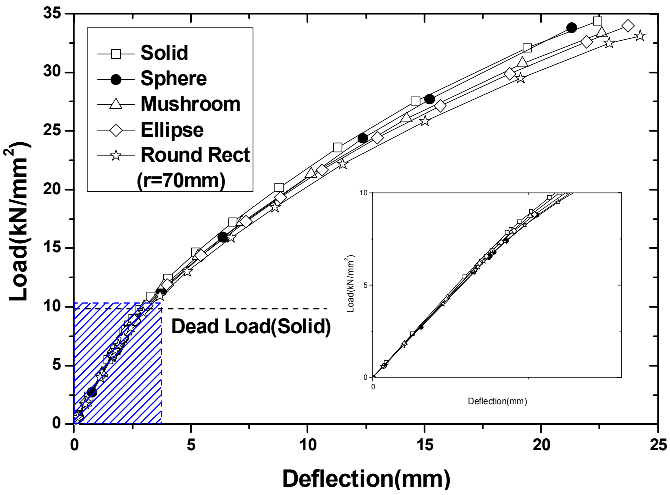
5.2.2 *Impact of corner radius*

To compare impact of corner radius, 4-types hollow spheres having different corner radius were used. Figure 10-b) was shown the load-deflection curve of the having different corner radius such as Solid, Round Rect(R=70mm), Round Rect(R=50mm) and Square. It showed as following. Like a tendency of 2D FEM analysis, when corner radius was increased, slab stiffness and strength tended to be increased. However, as seeing the deflection at dead load, stiffness was not only according to changing of corner radius but also changing of hollow volume by corner radius.

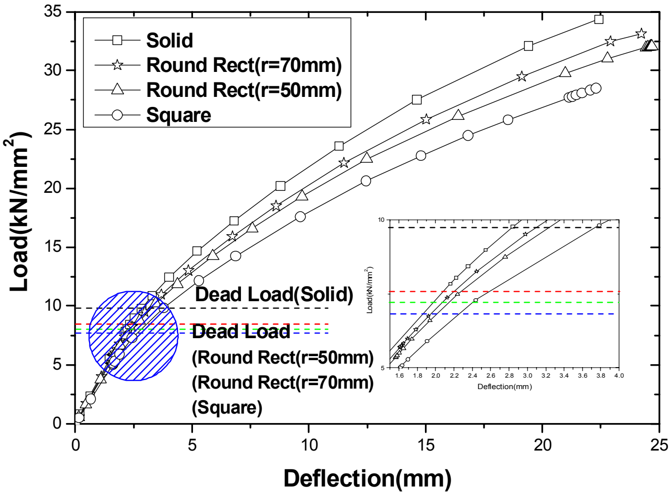
Therefore, when corner radius was increasing and the other conditions were same, stiffness and strength of slab tended to increasing. At ultimate load, an amount of influence by corner radius was about 4~17% of solid slab, with the aspect of strength.

5.2.3 *Impact of hole diameter*

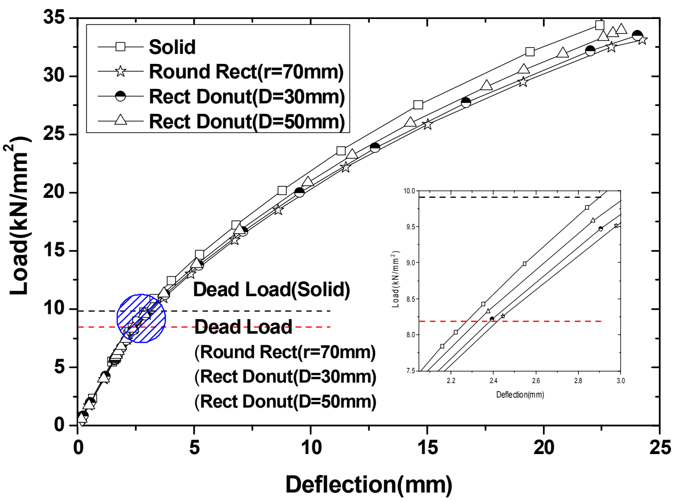
To compare impact of hole diameter, 4-types hollow spheres having different hole diameter were used. Figure 10-c) was shown the load-deflection curve of the having different hole diameter such as Solid, Round Rect(R=70mm), Rect Donuts(D=50 mm) and Rect Donuts(D=30mm). It showed as following. When hole diameter was increased, slab strength



a)Typical shape : Load-Deflection Curve

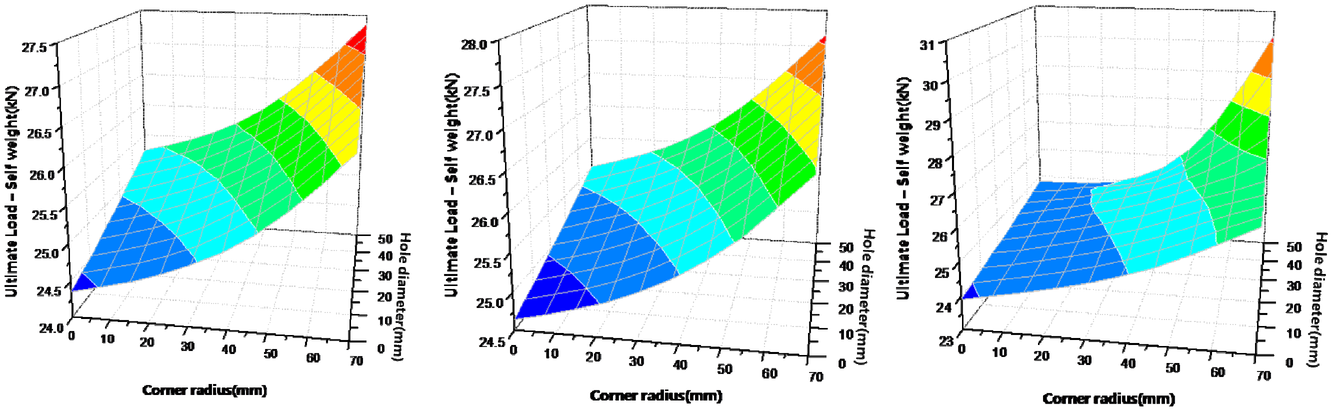


b) Corner radius : Load-Deflection Curve



c) Hole diameter : Load-Deflection Curve

Figure 9. 3D FEM Load-Deflection Curve.



a)Typical shape : Sphere b) Typical shape : Elliptical Sphere c) Typical shape : Cuboid

Figure 10. Parametric analysis.

tended to be increased a little, 1.3~4% of solid slab.

However, as seeing the deflection and stiffness, hollow slabs, applied Rect Donuts(D=50mm) and Rect Donuts(D=30mm), showed bigger stiffness and smaller deflection than hollow slab, applied Round Rect(R=70mm) which had no hole. In addition, volume of Round Rect(R=70mm) was more than volume of Rect Donuts(D=50mm) and Rect Donuts (D=30mm). Therefore, the hole of hollow sphere functions as preventing a deflection of slab.

5.3 *Parametric analysis*

Based on the results of impact of hollow shape (chapter 5.2), parametric analysis was performed. The unknown results, which were not performed numerical simulations, were assumed by linear interpolation.

A parametric analysis was performed as following conditions. The typical shape parameters were fix and the other parameters such as corner radius and hole diameter were changing. As compared the ultimate load-bearing capacity, the effects of corner radius and hole diameter were evaluated in combination. Figure 10 was shown the tendency of the each parameter in combination as following.

1) When the typical hollow shape is same, and the corner radius or the hole diameter is increasing, the capacities of hollow slab is increasing.
2) When the typical hollow shape is cuboid type, the increase by corner radius or hole diameter will be larger than the other typical hollow shape.

6 DEVELOPMENT OF THE OPTIMAL HOLLOW SPHEAR SHAPE

Based on the results of numerical simulation, the optimal hollow sphere shape was derived. To derive the optimal hollow sphere shape, three aspects of criterions which were safety, strength (additional load bearing capacity of slab after dead load) and deflection (at same loading condition) were considered.

1) Safety

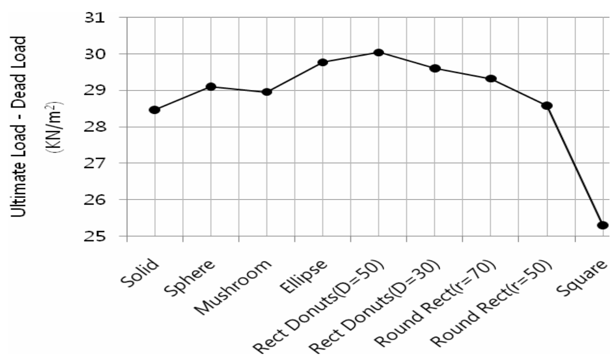
Judged from safety, all hollow slabs were safe in design load. However, failure mode of square type hollow slab was shear in ultimate load. The shear failure mode is dangerous because it is very sudden and brittle. Therefore square shape hollow sphere is not suitable for the hollow slab system.

2) Strength

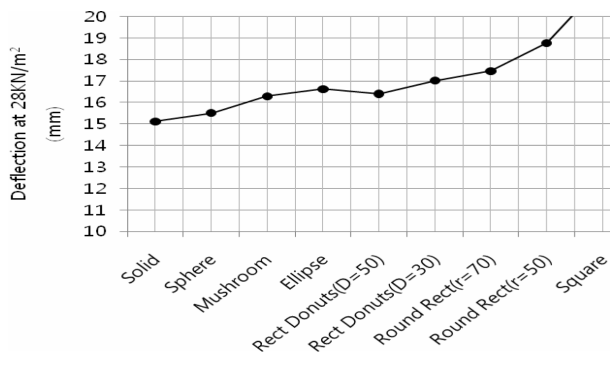
As Compared to solid slab, merely 3 cases of hollow sphere shapes, 'Rect Donuts(D=50mm)', 'Rect Donuts(D=30mm)' and 'Round Rect(R=70mm)' were good load-bearing capacity without decreasing of weight reduction ratio. (See Fig. 11-a)

3) Deflection

The hollow slab stiffness tended to decrease because of reduction of section area. So, as comparing



a) Ultimate Load-Dead Load capacity



b) Deflection at same load

Figure 11. Derivation of the optimal hollow sphere shape.

the deflection at same load, the optimal hollow sphere shape was derived. (See Fig. 11-b)

After all the analysis, 'Rect Donuts(D=50mm)' shape which had hole inside was judged as the optimal hollow sphere shape.

7 CONCLUSION

In this study, several hollow slabs which had different hollow sphere shapes were analyzed to compare with structure capacity and failure mode by using finite element method program. Based on the results of numerical simulation, the impacts of hollow sphere and the optimal hollow sphere shape could be derived. There are conclusions about this study.

1) Capacity of hollow slab and failure mode was related with hollow sphere shape. Especially, these are proved that corner radius, hole diameter and volume of hollow sphere were closely related with them.

2) As corner radius of hollow sphere was smaller, crack caused by concentrated stress was developed earlier. The hollow sphere shapes having corner radius more than 50mm was appropriate in 250mm thick hollow slab.

3) The hole of hollow sphere functions as preventing a deflection of slab. And as hole diameter of hollow sphere was smaller, the deflection of slab was bigger.

4) The hollow slab having optimal hollow sphere, Rect donuts(D=50mm), that demonstrated in this study showed more than 99% of load resisting capacity and less than 72% deflection at design load which compared normal slab. Therefore, it is proved that the Rect donuts(D=50mm) is optimal hollow sphere shape in 250mm thick hollow slab.

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