WEIBULL-STRENGTH SIZE EFFECT AND COMMON PROBLEMS WITH SIZE EFFECT MODELS

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Abstract: Size effect on quasi-brittle fracture of concrete-like materials is commonly investigated by testing geometrically-similar specimens with artificial notches. It can also be studied by testing plain specimens without any artificial notches to show the statistical influence of pre-existing defects/cracks using Weibull strength distribution. Those un-notched specimens can be further studied non-statistically, using the Fictitious Crack Model (FCM), which emphasizes the influence of crack-bridging within the fracture process zone (FPZ) on quasi-brittle fracture of concrete by adopting a single bridging-stress and crack-opening relation. This study discusses the merits and limitations of common size effect models, including Weibull strength and FCM, in dealing with quasi-brittle fracture of concrete specimens with and without notches, and shows it is necessary to adopt a tri-linear local fracture energy distribution, which considers the boundary influence from both the front and back specimen boundaries. The front boundary influence is particularly important to quasi-brittle fracture of un-notched specimens, as the limited stable crack growth before unstable fracture is well within the front boundary zone. Therefore, size effect models, based on constant fracture energy or constant local fracture energy distribution (suitable only to a long crack away from boundaries), cannot be used to reliably predict quasi-brittle fracture of un-notched concrete specimens.

1 INTRODUCTION

Study of size effect (SE) on quasi-brittle fracture of concrete is simply driven by the need of finding a suitable failure criterion, as a constant fracture toughness K_{IC} does not exist or cannot be used to characterize cracking-induced unstable fracture of concrete specimens commonly tested in laboratories. The problem is due largely to the coarse heterogeneous structures of concrete, and the sizable fracture process zone (FPZ) formed behind a crack tip during crack growth, where crack-bridging due to aggregate interlocking and frictional pullout, crack-branching and multiple-cracking generate extra resistance against crack growth, leading to the so-called quasi-brittle fracture behaviour.

A concise review of SE models on quasi-brittle fracture of notched concrete beams was provided by Karihaloo in 2003 [1], which is clear and to the point. Four different SE models were reviewed and compared, including Bazant's early SE model in 1984 [2], and Carpinteri's fractal-based SE model in 1994 [3,4]. The review pointed out there were still unresolved issues, despite of the fact that SE had been studied for nearly 20 years. For instance, although those SE models are different, they can all be used to fit the available experimental data within the limited specimen size range, typically from around 50 mm to 1,000 mm.

Since a fully-developed FPZ in a concrete specimen is not small in comparison with the specimen size, the formation and evolution of FPZ during stable crack growth, initiated from either an initial artificial notch or pre-existing surface defects/cracks, is inevitably influenced by the specimen size and specimen boundary conditions. Generally speaking, an initial notch including its length and width can be taken as part of the front boundary condition of a concrete specimen, which is particularly appropriate for shallow notches.

The evolution of FPZ during stable crack growth and its relative size in comparison to the crack and specimen size determine the quasi-brittle fracture behaviour. Therefore, fracture mechanics models emphasizing the role of FPZ on quasi-brittle fracture of concrete and its inevitable interaction with specimen boundaries were proposed by the Fracture Mechanics Group at the University of Western Australia from around 2000 [5-11], to consider various physical parameters such as FPZ length and width, fracture energy distribution, specimen thickness and size, and its front and back boundaries, and their subsequent influence on quasi-brittle fracture of concrete.

Concrete specimens used for SE study are typically measured from 50 mm to 1,000 mm, and they commonly contain aggregates from 5 to 30 mm. A typical fully-developed FPZ in concrete specimens is at least around 50 mm in length, or larger depending on both aggregate size and specimen size. Therefore, the physical size of a small concrete specimen around 50 mm is not big enough to accommodate a fully-developed FPZ. That is the reason why the boundary effect (BE) models [5-11] were proposed to study the interactions between FPZ and specimen boundaries, such as specimen thickness, front and back boundaries, and then the subsequent influence on quasi-brittle fracture of concrete.

A comparison between SE and BE models was reported by Bazant and his colleagues in 2010 [12]. Their first main conclusion was that the basic hypothesis of "Hu-Duan" BE model that the size effect is caused by interaction of the FPZ with the boundary is unjustified. Another major conclusion among others was that BE does not converge to the Weibull statistical theory for large un-notch specimens.

This study is to response the questions raised by Bazant and his colleagues [12]. Weibull strength distribution [13,14] and Fictitious Crack Model (FCM) [15,16] are included in the discussion as they are often taken as yardsticks of SE models when fracture statistics and stable crack growth due to frictional crack bridging within FPZ are considered. Pro and cons of both BE and SE models will be discussed together with recent development in SE study, so that readers interested in the field of size effect on quasi-brittle fracture of concrete-like materials can benefit from the discussions.

2 WEIBULL STRENGTH & FCM

Caution is still required in applications of Weibull strength distribution and FCM, although they have been widely accepted for size effect study on quasi-brittle fracture of concrete specimens without sharp notches.

2.1 Weibull strength distribution

Unfortunately, an essential feature of Weibull strength distribution of a brittle material is often overlooked. In principle, Weibull strength distribution of an idea brittle homogenous material, such as glass, can be traced precisely back to the pre-existing defect/crack distribution contained in the material, described by Pareto distribution [17,18]. Without this crucial link to the pre-existing crack distribution, measurement of

Weibull strength distribution is unfortunately degraded down to a mere empirical curve-fitting exercise.

Fracture of an ideal brittle material with distributed micro-cracks follows the weakest-link theory, i.e. crack growth from any pre-existing cracks leads to unstable fracture. The failure probability (or percentage failure) F can be defined by both the failure stress, σ, and pre-existing flaw/crack, a, distributions, i.e.

$$F = 1 - \exp\left\{ -V\left(\frac{\sigma}{\sigma_V}\right)^M \right\} = 1 - \exp\left\{ -V \int_{a(\sigma)}^{\infty} q(a) \cdot da \right\} \quad (1)$$

$$q(a) = \rho_f \frac{M}{2a_0}\left(\frac{a_0}{a}\right)^{(M+2)/2} \qquad a \geq a_0$$

As far as fracture is concerned, there is no need to consider all pre-existing cracks, as only those pre-existing cracks that contributed to fracture need to be considered. Here, $\boldsymbol{a_0}$ is the smallest flaw/crack size, which is responsible for the strength measurements. After a_0 is defined, the flaw density, ρ_f, for all pre-existing flaws/cracks ≥ a_0 is thus well defined for a given material. In fact, there is no need to determine a_0 and ρ_f separately, as a combined parameter σ_V is determined experimentally. $a(\sigma)$ is linked to the fracture toughness K_{IC} of an ideal brittle homogeneous material. The familiar Weibull strength distribution shown in Eq. (1) for pure tension, can also be derived for bending cases.

Concrete with coarse aggregates, various defects/cracks and different phases is highly heterogeneous, so microscopic stable crack growth is expected. As a result, Weibull strength distribution from concrete fracture cannot be traced back to the pre-existing defect/crack distribution. However, such a strength distribution can still be linked statistically to the “critical” crack distribution when unstable concrete fracture occurred. Here, we define a “critical” crack, a_C, as the initial crack plus the stable crack growth increment, i.e. $a_C = a_i + \Delta a$. In other words, crack growth from any critical cracks is unstable, and thus the weakest link theory still applies [17]. In principle, the critical crack distribution is determined by the pre-existing defect/crack distribution and relevant material structure characteristics, such as the aggregate size of concrete.

It is anticipated that small concrete specimens and large concrete structures significantly different in size have to be prepared in different ways. As a result, they will experience different curing conditions during concrete preparation, influenced by casting method, temperature, humidity and shrinkage etc. Those different conditions will lead to different pre-existing defects/cracks and thus different critical crack distributions when fracture occurs. As far as Weibull strength distribution is concerned, small concrete samples and large concrete structures are “different” materials, although they have identical material compositions, aggregates, and the same water/cement ratio. Different defect/crack distributions, either in terms of the pre-existing defect or critical crack distributions, will lead to different quasi-brittle fracture behaviors. Therefore, Weibull strength of a large concrete structure cannot be reliably linked to that of small concrete specimens tested in laboratories by a single SE relation. Any Weibull strength SE prediction for quasi-brittle fracture of concrete specimens and structures vastly different in size, such as those of Bazant’s SE models [12,19,20], is merely academic, as they are not supported by experimental evidence.

It has been shown experimentally that Weibull strengths of micro- and nano- silicon samples with size measurement varying by up to 5,000 times (thus different methods were required to prepare those silicon samples) cannot be linked by a single Weibull strength distribution due to different surface defect/crack distributions [21,22]. In summary, as pointed out by the flaw statistics in Eq. (1), a single Weibull strength distribution cannot be used to link SE of concrete specimens and large concrete structures as they do have different defect/crack populations. Therefore, criticism that BE does not converge to the Weibull statistical theory for large un-notch specimens [12] is unfounded.

2.2 FCM & tri-linear local fracture energy

Like J-integral [23], FCM [15,16] relies on a well-defined cohesive-stress and crack-opening relation, or unique tensile-softening curve, to describe crack bridging within the FPZ behind a fictitious crack in concrete. The area under the softening curve, or the specific fracture energy G_F, is thus a material constant.

The irony is that although the experimentally-measured G_F is size dependent, a constant intrinsic G_F is always assumed when FCM is used to simulate the stable crack growth in concrete, either initiated from a smooth specimen surface or a sharp notch. A well-defined tensile-softening curve used in FCM is inseparable from a constant intrinsic G_F. If the experimentally-measured G_F is specimen-size dependent, FCM based on a single tensile-softening relation has to be modified.

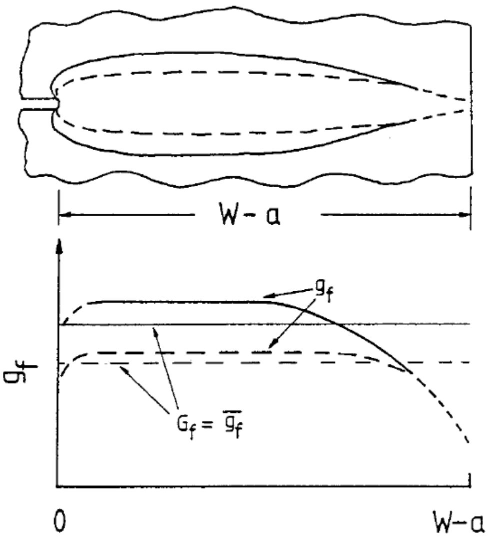

Figure 1: The local fracture energy g_f is constant if away from both front and back boundaries. The averaged G_f is specimen-size dependent.

A constant intrinsic G_F implies the fracture energy consumption is constant along the crack path, which is equivalent to a constant fracture energy distribution over the entire crack area. Bearing this in mind, the local specific fracture energy g_f distribution, as shown in Fig. 1, was first proposed by Hu and Wittmann in 1992 [24].

The key assumption is that the local specific fracture energy g_f is constant away from the specimen back boundary and the initial notch, which can also be taken as part of the specimen front boundary. Taking an artificial notch as an integrated part of the specimen front boundary becomes even more logical for concrete specimens with very shallow notches. Since g_f is constant within the inner region, the classic FCM and J-integral apply and a well-defined tensile softening relation can be established with a constant specific fracture energy G_F (= g_f = constant).

The local specific fracture energy g_f distribution, shown in Fig. 1, resolves the dilemma that FCM requires a constant intrinsic G_F to simulate concrete fracture while the experimental G_f averaged over the entire fracture area is specimen-size dependent. The RILEM recommended G_f [25], averaged over the entire crack area, contains the constant intrinsic G_F in the inner zone, and the reduced local specific fracture energy g_f in the boundary regions. If the inner zone with constant instrisic G_F and boundary regions with reduced g_f are comparable, SE on G_f is inevitable.

It has been shown recently from 2010 by four independent research groups [26-30] that the local specific fracture energy g_f distribution postulated in Fig. 1 can be adequately described by a simplified tri-linear g_f distribution. One acoustic emission (AE) confirmation of the tri-linear relation g_f is shown in Fig. 2 [30]. The added linear relation close to the initial notch defines the front boundary region of around 20 mm for this concrete specimen with aggregate size of 5 mm, which is much shorter than the back boundary region of around 60 mm.

While consideration of the back boundary influence is more important to SE on RILEM $\boldsymbol{G_f}$, the front boundary influence is more critical to the SE on quasi-brittle fracture of plain concrete specimens without artificial notches as the stable crack growth is limited well within the front boundary region.

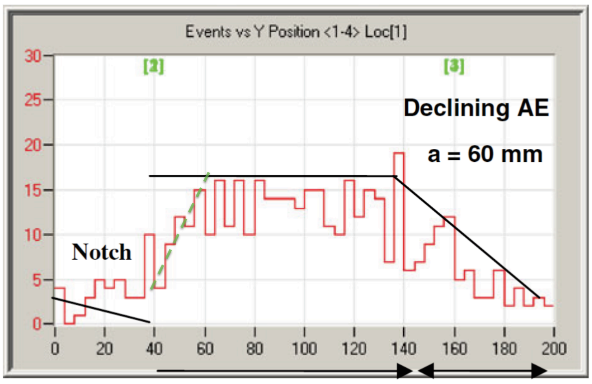

Figure 2: A tri-linear local fracture energy g_f can be assumed, as suggested by the AE measurements [30].

Our recent analysis [28] of experimental results [31] shows that the influence of the front boundary region is more profound for specimens with shallow notches, indicating the tri-linear g_f distribution shown in Figs. 1 and 2 is notch-length dependent. This observation is expected if the initial notch is considered as part of the specimen front boundary. In reality, complete separation of a notch from the front boundary is impossible, particularly if a very shallow notch is considered.

3 SE & BE MODELS

3.1 Bazant's SE models

The early work in 1984 [2] is by far the most widely-used SE model, in comparison to Bazant's more elaborate variants proposed in later years. The relation between a specimen representative size, D (e.g. width or height), and the nominal strength, σ_N (e.g. tensile or bending strength determined by the critical load P_{cri}, ignoring the presence of a crack/notch) is given as follows:

$$\sigma_N = \frac{B \cdot f_t}{\sqrt{1+\frac{D}{D_0}}} = \frac{B_t}{\sqrt{1+\frac{D}{D_0}}} \tag{2}$$

The tensile strength f_t and experimental constant B can be combined into one constant B_t as shown in the review of SE models given by Karihaloo [1]. The most common application of Eq. (2) is for empirical curve-fitting, i.e. the relation between σ_N and D is measured experimentally by testing a set of notched "geometrically-similar" specimens with different size D, but constant notch/size a_0/D ratio. The scaling parameters B_t and D_0 are then determined through curve-fitting of experimental results. It is convenient to use Eq. (2) to show the influence of size D on the nominal strength σ_N, or SE on quasi-brittle fracture, as long as both the scaling parameters B_t and D_0 are treated as empirical experimental constants. Therefore, empirical curve-fitting to experimental results thus becomes the most common usage of Eq. (2).

To explain the physical meanings of the two scaling parameters, detailed expressions of B_t and D_0 in Eq. (2) have been provided by Bazant [e.g. 12], i.e.

$$D_0 = c_f \cdot g'(\alpha_0)/g(\alpha_0)$$
$$B_t = \sqrt{EG_{F-ini}/g'(\alpha_0)c_f} \tag{3}$$

D_0 is the transitional size, and the material constant $c_f \propto$ FPZ $\propto EG_F/f_t$ is introduce to show FPZ plays a part in D_0. The initial notch/size ratio $a_0/D = \alpha_0$. At the point of fracture, $a = a_0 + c_f$, and $\alpha = a/D$. $g(\alpha)$ is the dimensionless energy release function of linear elastic fracture mechanics (LEFM) at the point of fracture, i.e. $g(\alpha) = K_{IC}^2 D(b/P_{cri})^2$. The derivative of $g(\alpha_0)$, $g'(\alpha_0)$, is also included in Eq. (3). G_{F-ini} is the initial fracture energy, or the initial part of one single softening curve with constant G_F, assumed to be applicable to the entire fracture area.

It is not practical to use Eq. (3) for curve-fitting in conjunction with Eq. (2) in order to find out the details of those physical parameters. The modification in Eq. (3) is merely to show the two scaling parameters B_t and D_0 in Eq. (2) are not empirical, and different predictions can be made by varying those physical parameters.

However, the explanations provided in Eq. (3) are still confusing and contradict to the common experimental observations. It is well known that at the point of unstable fracture, the stable crack growth, Δa, varies with the initial notch/crack a_0 even for a fixed crack-bridging or tensile softening relation in FPZ. The assumption that $a = a_0 + \Delta a = a_0 + c_f$ = constant is clearly incorrect. $g(\alpha)$ contains the common geometry factor $Y = Y(\alpha)$ as $K = Y\sigma_N\sqrt{(\pi a)}$. It is not feasible to do the derivation of $Y(\alpha)$ even for simple three-point-bending specimens as required by $g'(\alpha_0)$. Furthermore, if the so-called transitional size D_0 is a scaling parameter for size, it should only be influenced by aggregate size, not by the initial notch/size ratio α_0, as indicated in Eq. (3). The fact that $G_{F\text{-}ini}$, only part of G_F, is used in Eq. (3) implies, in some way, a smaller local fracture energy is required, which is what has been suggested in Figs. 1 and 2.

Due to those confusing and inconsistent modifications in Eq. (3), few attempt the combination of Eqs. (2) and (3), and most still use Eq. (2) simply treating B_t and D_0 as two empirical curve-fitting parameters.

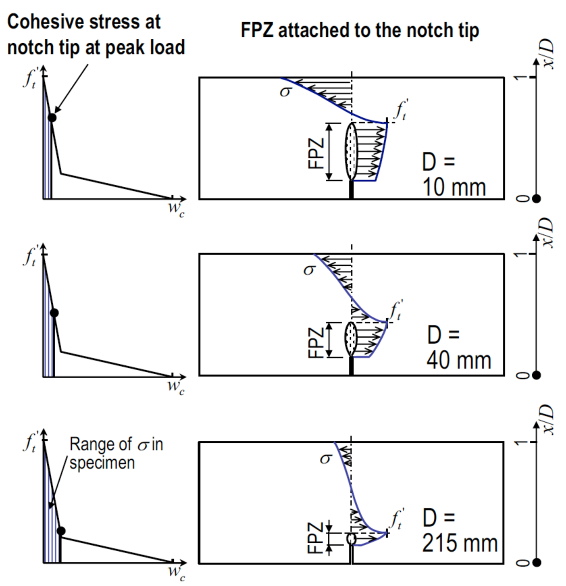

Figure 3: Simulated FPZ in different concrete specimens, using single tensile softening relation [12], while AE measurements suggest otherwise [26,27,30].

A single tensile softening relation has been adopted by Bazant's group [12] to simulate FPZs in different concrete specimens at the critical loads, as shown in Fig. 3. The aggregate size is 9.5 mm, and the initial notch/size ratio $\alpha_0 = a_0/D = 0.15$.

We have measured FPZ up to 40 mm in length in mortar specimens with 2 mm sand [32]. If size D is sufficiently large, the fully-developed FPZ for the concrete with 9.5 mm aggregate should be at least around 80 mm, larger than the specimen size, D = 10 and 40 mm. The RILEM fracture energy G_f for specimens with D = 10, 40 and 215 mm shown in Fig. 3 is definitely size-dependent, as suggested by numerous reports in literature [e.g. 1]. When a single tensile softening relation and thus a constant G_F has been assumed for those concrete specimens in Fig. 3, Bazant [12] already overlooked SE on the RILEM fracture energy G_f. Therefore, the simulations in Fig. 3 [12] do not represent the true SE on quasi-brittle fracture of those concrete specimens. Yet, the simulated results in Fig. 3 were used to conclude – This further invalidates the BE hypothesis about FPZ-boundary interaction [12].

Specimens with D = 10 and 40 mm are clearly not big enough to accommodate the fully-developed FPZ in the concrete with aggregate of 9.5 mm, which itself suggests FPZ-boundary interaction is inevitable and must be considered. The influence of the front boundary may still exist for specimens with D = 215 mm, as $a_0 = 0.15D$ = 32 mm, still shorter than the fully-developed FPZ. If FPZ cannot be fully developed in both length and width, its softening relation will be influenced by the front boundary as shown in Figs. 1 and 2. The FPZ width/height controls the local fracture energy g_f [10], which is illustrated by the wide and narrow FPZ boundaries and the two corresponding local fracture energy distributions in Fig. 1, which then leads to two different RILEM fracture energy G_f values.

On the other hand, there should be no SE and LEFM should apply, if D = 5 m or 50 m as $a_0 = 0.15D$ = 750 mm or 7.5 m so FPZ is sufficiently away from all boundaries.

SE on tensile softening relation exists even in direct tensile tests [33-35]. Again, it can be explained by the influence of free specimen boundaries. Within the inner zone away from the specimen boundaries, a single tensile softening relation exists as frictional crack-bridging can be fully established. A less-developed tensile softening exists around or close to the free specimen boundaries as frictional crack-bridging cannot be fully and effectively built up close to the free surface [35]. Therefore, the SE simulations in Fig. 3 are questionable for small specimens and for shallow notches because the fictitious crack tip is too close to the boundary to have a single tensile softening relation.

3.2 BE models

Following the notations in our recent work on BE [36,37], SE on quasi-brittle fracture of concrete can be expressed as follows:

$$\sigma_n(P_{cri}) = \frac{f_t}{\sqrt{1+\dfrac{a_e}{a_{FPZ}}}}$$

where (4)

$$a_{FPZ} = 0.25\left(\frac{K_{IC}}{f_t}\right)^2 = \text{constant} \propto \text{FPZ}$$

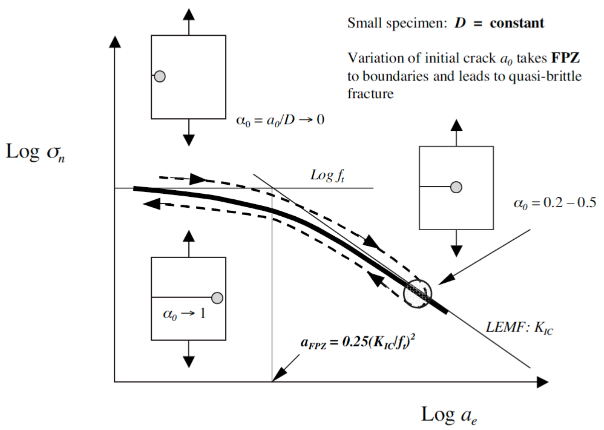

Figure 4: Interactions between FPZ and specimen boundaries for different crack/size ratios, $\alpha_0 = a_0/D \rightarrow 0$ is the same as $\alpha_0 \rightarrow 1$.

Eq. (4) has two material constants, f_t and a_{FPZ} (proportional to the fully-developed FPZ for a given concrete). The nominal strength σ_n can be easily linked to the nominal strength σ_N in Eq. (2) once the loading condition is specified [e.g. 36,37]. The equivalent crack a_e contains the normal geometry factor $Y = Y(\alpha)$ used in the stress intensity factor formula, $K = Y\sigma_N\sqrt{(\pi a)}$ [36,37]. This is because the real stress concentration is determined not only by the crack size, but also by the distance of the crack-tip and FPZ to boundaries, measured by a_0 and $(D - a_0)$.

The equivalent crack a_e measures the distance of FPZ to either the front or back boundary by mathematically combining the two measurements a_0 and $(D - a_0)$ together. Therefore, the conditions that $\alpha_0 = a_0/D \rightarrow 0$ and $\alpha_0 = a_0/D \rightarrow 1$ yield the same boundary influence, as shown in Fig. 4. The "turn-back" point is determined by size D. K_{IC} and then LEFM applies only if size D is big enough and both a_0 and $(D - a_0)$ are sufficiently large in comparison with a_{FPZ}. This conclusion based on what is shown in Fig. 4 is the exactly the same as the requirements of the ASTM standard for K_{IC} measurements [6,38].

Bazant's SE model, Eq. (2), also indicates LEFM or K_{IC} applies if $D/D_0 >> 1$. However, the transitional size D_0 cannot be linked to the ASTM standard [38], purely due to its ambiguous physical definition. Furthermore, the transitional size D_0 is not even a constant, but varies with $\alpha_0 = a_0/D$ as shown in Eq. (3). To use Eq. (1) with any certainty, one has to make the transitional size D_0 a constant scaling parameter. That is the reason why the condition, that $\alpha_0 = a_0/D =$ constant, or "geometry similarity" is commonly imposed for applications of Bazant's SE models.

Different to D_0, the scaling parameter a_{FPZ} used in the BE model, Eq. (4) is a well-defined material constant. The condition, $\alpha_0 = a_0/D =$ constant, is not required. In fact, α_0 $(= a_0/D)$ can take any value between 0 and 1, as shown in Fig. 4. As a special case, the condition $\alpha_0 = a_0/D =$ constant, can also be assumed for Eq. (4). Interestingly, under such a special condition, Bazant's SE model, Eq. (2), is obtained, which proves Bazant's SE model is just a special case of the BE model.

Nevertheless, obvious difference between Bazant's SE model and "Hu-Duan" BE model was shown in Bazant's recent work [12] for un-notched concrete specimens with aggregate of 9.5 mm, here shown again in Fig. 5. As discussed in this study, FCM or the cohesive crack model with due consideration of the front boundary influence, as shown in Figs. 1 and 2, will not give the same prediction as that of SE shown in Fig. 5, where FCM has been incorrectly applied.

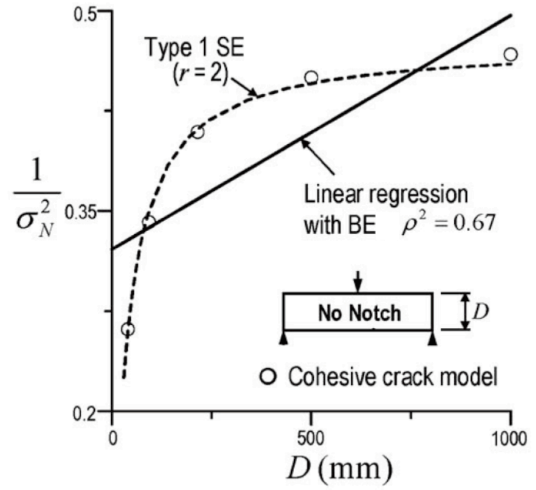

Figure 5: Predictions of Bazant's SE model and FCM based on constant G_f for un-notched specimens, and α_0 = 0.15 was selected for BE model [12].

It is incorrect to ignore SE on G_f as suggested in Figs. 3 and 5. For instance, SE on G_f definitely exists for D less than 500 mm, as widely reported in the available literature on concrete fracture [e.g. 1]. It is incorrect to assign α_0 = 0.15 for BE to model un-notched specimens for such a large size range from 10 mm to 1,000 mm. This inadequate assumption would yield the a_0 range from 1.5 mm to 150 mm for those un-notched concrete specimens with 9.5 mm aggregate. In general, it is correct to assume statistically that a larger concrete specimen contains larger micro- pre-existing defects or cracks, but it is incorrect to assume a linear function or constant α_0 –ratio for such a large size range. Therefore, the predictions of FCM and BE model have been incorrectly presented in Fig. 5 [12]. As a result, the comparisons between SE and BE models are invalid.

4 CONCLUSIONS

This study shows that all SE models, including the well-accepted Weibull strength distribution and FCM, have their limitations. Although Bazant's early SE in 1984 [2] can be used empirically to fit experimental results of geometrically-similar specimens, it fails to reveal the true mechanism behind the apparent SE, i.e. the interaction between FPZ and specimen boundaries as pointed out by the ASTM standard requirements for K_{IC} measurement [38]. In other words, both a_0 and (D – a_0) have to be sufficiently large in comparison with a_{FPZ} to have the K_{IC} controlled brittle fracture, or to avoid quasi-brittle fracture of concrete.

As pointed out at the beginning of this paper, study of SE on quasi-brittle fracture of concrete is simply driven by the need of finding a suitable failure criterion, as a constant fracture toughness K_{IC} does not exist or cannot be used to characterize cracking-induced unstable fracture of concrete specimens commonly tested in laboratories.

Therefore, study of quasi-brittle fracture of concrete and the associated SE cannot be complete without due consideration of a_0 and (D – a_0) in comparison to a_{FPZ}. The BE models are proposed to consider the influence of a_0 and (D – a_0), or specimen boundaries, which is clearly in line with the statement of the ASTM standard on non-LEFM fracture, or quasi-brittle fracture in the case of concrete.

For geometrically-similar specimens with a constant ratio $\alpha_0 = a_0/D$ = constant, the initial crack length a_0 and then the a_0/a_{FPZ} ratio is changed when size D is changed. Due to this reason, quasi-brittle fracture behavior of concrete has been wrongly attributed to the size variation, although in reality the interaction between FPZ and specimen boundary controls the quasi-brittle fracture behavior. Eq. (4) measures the influence of a_0/a_{FPZ} and $(D - a_0)/a_{FPZ}$ ratios using the a_e/a_{FPZ} ratio (to decide whether a crack is too close to either the front or back boundaries, as shown in Fig. 4) is thus more adequate than Bazant's SE models. Furthermore, Eq. (4) is not limited only to the geometrically-similar specimens, the condition always required by Bazant's SE model, Eq. (2).

However, for convenience and practical applications, it is acceptable that quasi-brittle fracture of concrete is due to the "size variation", if only geometrically-similar concrete specimens are considered.

It should also be addressed the more elaborate SE models proposed by Bazant after 1984, e.g. Eq. (3), are not user friendly as too many adjustable parameters, such as c_f and $G_{F\text{-}ini}$, have been introduced, and unnecessarily complex mathematics such as the derivative of $g(\alpha_0)$, or $g'(\alpha_0)$, is introduced. However, the conceptual mistakes of Bazant's SE models, such as ignoring SE on RILEM G_f as shown in Fig. 3 [12], are simply too obvious to ignore.

Finally, the research on FPZ detection by Mindess [39] shows that the extent of FPZ depends on specimen geometry, size and boundary conditions. Although the fracture energy G_F appears to be a material constant for very large specimens (e.g. > 1 m), SE on the nominal strength is still determined by the early stage of FPZ evolution and thus will be influenced by the boundary conditions, or the early stage of the assumed tri-linear g_f-distribution. To have a better understanding of SE and to avoid empirical curve-fitting models, further research on the evolution of FPZ for various specimen and boundary conditions are clearly warranted.

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