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A STUDY ON FATIGUE CRACK GROWTH IN CONCRETE IN THE PRE-PARIS REGION

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Abstract. The present work addresses the behavior of propagation of short cracks in concrete. In particular, we focus on understanding the short crack growth phenomenon when the crack emanates from a notch. An analytical model has been developed to estimate the rate of crack extension in the short crack regime using the principles of dimensional analysis. Important short crack growth characterizing parameters such as threshold energy release rate, fracture energy, stress ratio, tensile strength, characteristic length, modulus of elasticity, crack length and ratio of maximum aggregate size to structural size have been taken into account for the model formulation. The model has been calibrated, validated and a sensitivity analysis has been performed. Out of the parameters considered, crack propagation rate in short crack region is found to be more sensitive to threshold stress intensity factor range and less sensitive to stress ratio.

1 INTRODUCTION

Fatigue is a common phenomenon that arises due to oscillatory loading. As a result, fatigue crack grows starting from a flaw or a point of high stress until it becomes unstable. The rate of crack growth is not uniform but depends on crack size, loading, material and geometric parameters. A crack can be considered as short or long based on various factors such as size of crack or rate of crack propagation.

The predictions based on short crack growth has poor agreement with the experimental results [1]. This can be attributed to short crack anomaly [2]. In the early stage of propagation of a short fatigue crack, there may be a competition of multiple micro-cracks. One of the micro-cracks become critical while the others cease to propagate [3]. The traditional methods which were used in the past for studying the behavior of long cracks, especially the linear elastic fracture mechanics (LEFM) approach will not yield proper results since crack growth is elasto-plastic in nature rather than linear-elastic. Very little literature is available which pertains to the quantification of the growth rate of short cracks in quasi-brittle materials like concrete.

Experimental results on concrete fracture due to application of cyclic loading [1] show a higher growth rate for small cracks in comparison to the propagation rate computed using the existing fatigue crack propagation models that are applicable in the Paris (linear) region of the crack growth curve. This implies that the crack growth models developed for concrete over-predicts the fatigue life keeping the structure in potentially dangerous state and should not be applied for the initial portion of the crack propagation curve.

An analytical model has been developed in this work to estimate the rate of crack extension in the short crack regime when the crack emanates from a notch. The model has been validated with available experimental data [1]. A sensitivity analysis has been performed to determine the role of each parameter in the rate of crack propagation in the pre-Paris regime.

2 ZONES IN CRACK PROPAGATION

The behavior of crack propagation can be visualized in three known zones:

- Zone 1 (Short crack regime): Short crack anomaly is observed in this zone.
- Zone 2 (Paris regime): Stable crack propagation is observed in this zone.
- Zone 3 (Failure region): Unstable crack propagation is observed in this zone.

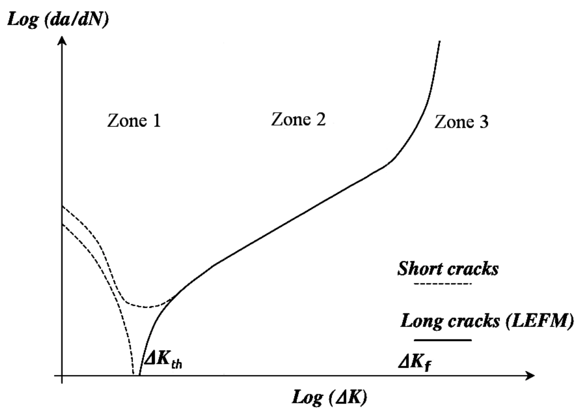

Figure 1: Zones in crack propagation [4].

Figure 1 shows the different zones in crack propagation. The short crack anomaly can be observed in Zone 1 of the figure.

3 DEFINITION OF SHORT CRACKS

The short cracks are defined based on several parameters such as size, rate of crack propagation and the driving force.

3.1 Based on size

Fatigue cracks can be defined as short cracks when the length of the crack is [5]:

- mechanically small (comparable with the extent of local plasticity in case of metals)
- micro-structurally small (comparable with the scale of micro-structure)
- physically small (typically less than 1 mm in size)
- chemically small (small with respect to the environmental conditions)

3.2 Based on rate of propagation

Short cracks usually propagate at higher rates than long cracks at the same nominal Stress Intensity Factor (SIF) [5]. This is known as short crack anomaly. The main reason for this behavior is attributed to breakdown in similitude. Linear Elastic Fracture Mechanics (LEFM) cannot be applied for short cracks for the aforementioned reasons.

3.3 Based on crack driving force

Let a_{th} be the crack length at $\Delta K = \Delta K_{th}$, where ΔK is the SIF range and ΔK_{th} is its threshold value. SIF should be high enough for the crack to propagate. For a given material & geometry, above ΔK_{th}, crack propagates by the virtue of crack length (a). Short cracks can be defined as cracks with $a < a_{th}$. Short cracks cannot propagate without an external force which is high enough to drive the crack.

4 PROPAGATION OF SHORT CRACKS IN CONCRETE

Flaws are present in any structural component. These flaws start to propagate initially as short cracks when subjected to fatigue type of loading. Cracks propagate through the weakest paths in the structural component. In metals, when the stress reaches the yield stress, a plastic zone will be formed ahead of the crack tip, whereas in concrete, a zone of reduced stiffness known as process zone will be formed in front of the crack tip. This zone can be characterized by the toughening mechanisms like micro-cracking and aggregate bridging. When the stress reaches tensile strength of the material, the micro-cracks in the process zone coalesce and a higher rate of crack growth can be observed.

The characteristic length (l_{ch}) of the material or Irwin's estimate for the length of the nonlinear zone [6] can be estimated using the equation:

$$l_{ch} = \frac{EG_f}{\sigma_t^2} \tag{1}$$

where E is the modulus of elasticity, G_f is the fracture energy and σ_t is the tensile strength.

5 ANALYTICAL MODEL USING DIMENSIONAL ANALYSIS

In this work, an analytical model has been developed for predicting short crack growth in concrete by performing dimensional analysis using the Buckingham Π method. The important physical parameters chosen for determining short crack growth in concrete are threshold energy release rate (ΔG_{th}), fracture energy (G_f), loading parameter (ΔG_I), stress ratio (R), tensile strength (σ_t), characteristic length of fracture process zone (l_{ch}), modulus of elasticity (E), crack length (a) and ratio of maximum aggregate size to structural size (d/D). We assume that the rate of crack propagation (da/dN) is a function of all these parameters and write,

$$\frac{da}{dN} = f(\Delta G_{th}, G_f, \Delta G_I, \sigma_t, l_{ch}, E, a, 1-R, \frac{d}{D}) \tag{2}$$

In the above equation, there are ten variables one dependent (da/dN) while the others independent. According to Force-Time-Length (FLT) system, there are two fundamental dimensions (Force and Length). Therefore the number of dimensionless groups is equal to eight. Table 1 gives the dimensions of parameters considered.

Table 1: Parameters characterizing short crack growth.

Parameter	Dimensions
da/dN	F^0L^1
ΔG_{th}	F^1L^{-1}
G_f	F^1L^{-1}
ΔG_I	F^1L^{-1}
E	F^1L^{-2}
σ_t	F^1L^{-2}
l_{ch}	F^0L^1
a	F^0L^1
d/D	F^0L^0
R	F^0L^0

Choosing ΔG_{th} and σ_t as repeating variables, we obtain,

$$\begin{aligned}
\Pi_1 &= \frac{\sigma_t}{G_f}\frac{da}{dN} \\
\Pi_2 &= \frac{\Delta G_I}{G_f} \\
\Pi_3 &= \frac{\Delta G_{th}}{G_f} \\
\Pi_4 &= \frac{\sigma_t}{G_f} l_{ch} \\
\Pi_5 &= \frac{E}{\sigma_t} \\
\Pi_6 &= \frac{\sigma_t}{G_f} a \\
\Pi_7 &= 1-R \\
\Pi_8 &= \frac{d}{D}
\end{aligned} \tag{3}$$

From dimensional analysis using Buckingham Π theorem,

$$\Phi(\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6, \Pi_7, \Pi_8) = 0 \tag{4}$$

The rate of crack propagation can be written as:

$$\frac{da}{dN} = \frac{G_f}{\sigma_t}\Phi_1(\Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6, \Pi_7, \Pi_8) \tag{5}$$

According to complete Self-similarity or Self-similarity of the first kind, a parameter can be considered as non-essential when, for very large or very small values of corresponding dimensionless parameter Π_a, a finite non-zero limit of the function Φ exists [7, 8].

If the limit of the function Φ tends to zero or infinity, the quantity Π_a remains essential no matter how small or large it becomes. However, in some cases, the limit of the function Φ tends to zero or infinity, but the function Φ has a power-type asymptotic representation. This is classified as Incomplete Self-similarity or Self-similarity of the second kind [7, 8].

We consider incomplete self-similarity in $\Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6$ and Π_7. Thus the expression for da/dN takes the form:

$$\frac{da}{dN} = \frac{G_f}{\sigma_t}\left(\frac{\Delta G_I}{G_f}\right)^{\beta_1}\left(\frac{\Delta G_{th}}{G_f}\right)^{\beta_2} \left(\frac{\sigma_t}{G_f}l_{ch}\right)^{\beta_3}\left(\frac{E}{\sigma_t}\right)^{\beta_4}\left(\frac{\sigma_t}{G_f}a\right)^{\beta_5} (1-R)^{\beta_6}\,\Phi_2\left(\frac{d}{D}\right) \quad (6)$$

The exponents β_i in Equation 6 cannot be obtained from the principles of dimensional analysis alone. We use experimental data to determine these β_i values through a calibration process.

6 CALIBRATION OF THE PROPOSED MODEL

The model proposed is calibrated using available experimental data [1], by minimizing the error and making use of the principle of least squares [9].

Shah [1] had conducted experiments with geometrically similar three-point-bend beams made of plain concrete. Fatigue loading was applied with a minimum load (P_{min}) of 0.2 kN. The maximum load (P_{max}) was incremented by 0.5 kN after every 500 cycles, starting with an initial P_{max} of 0.5 kN. The specifications of the specimens used are given in Table 2.

Table 2: Specifications of the specimens.

Size	Depth (mm)	Notch Size (mm)	σ_t (MPa)	E (MPa)
Small	76	15.2	4.32	30,000
Medium	152	30.4	3.62	30,000
Large	304	60.8	3.98	30,000

The calibration of the model is carried out using small sized specimen. Figure 2 shows the calibration graph.

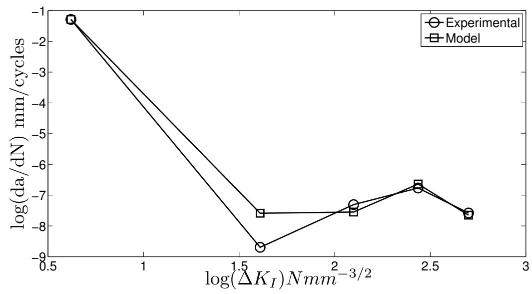

Figure 2: Calibration using small sized specimen.

The values of exponents (β_i) obtained are given in Table 3.

Table 3: Exponent values obtained.

β_i	Value
β_1	-3.9927
β_2	7.7962
β_3	-1.5434
β_4	-1.3593
β_5	5.9693
β_6	0.0894

The function Φ_2 is assumed to be quadratic in d/D and takes the form:

$$\Phi_2 = \gamma_1 + \gamma_2\left(\frac{d}{D}\right) + \gamma_3\left(\frac{d}{D}\right)^2 \quad (7)$$

The values of coefficients (γ_i) obtained are given in Table 4.

Table 4: Coefficient values obtained.

γ_i	Value
γ_1	11.46034101
γ_2	-421.2418745
γ_3	3466.315744

7 VALIDATION OF THE PROPOSED MODEL

The proposed model is validated using medium sized and large sized specimens. The graphs comparing the experimentally obtained and predicted trend of short crack propagation are given in Figures 3 & 4.

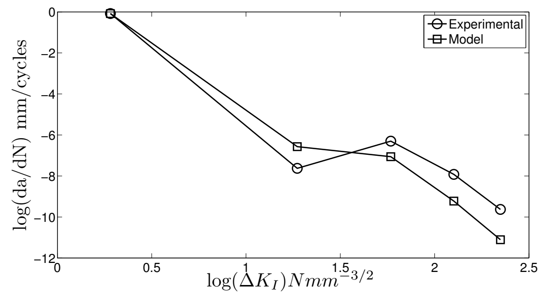

Figure 3: Validation using medium sized specimen.

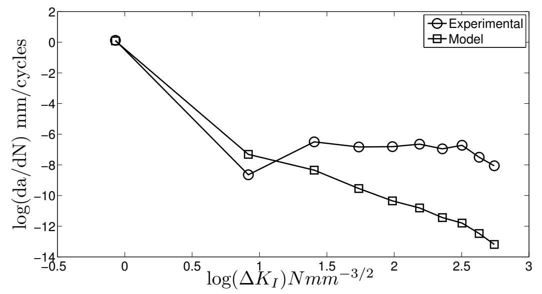

Figure 4: Validation using large sized specimen.

The plots obtained using the proposed model agrees fairly well with the experimental data plots.

8 SENSITIVITY ANALYSIS

A sensitivity analysis has been done to determine the sensitivity of crack propagation rate towards various parameters considered. The results of the sensitivity analysis is graphically represented through a Tornado diagram.

In the sensitivity analysis, the dependent variable da/dN is assumed to be a known deterministic function of a set of input random variables whose probability distributions are assumed. For each input variable (RV), two extreme values (mean and mean + 2 $\times$ standard deviation) are computed and using these two extreme values the deterministic function is evaluated twice, keeping the other input RVs at their means. The statistical parameters used in the study are listed in Table 5.

Table 5: Statistical parameters used for sensitivity analysis.

Parameters	Units	Mean	Coefficient of variation
G_f	N/mm	0.07	0.30
ΔK_{th}	Nmm$^{-3/2}$	14.88	0.12
σ_t	MPa	4.32	0.15
E	MPa	30,000	0.15
D	mm	76	0.05
d	mm	12.5	0.15
a	mm	1.27	0.30
R	-	0.1	0.10

The absolute difference of the two values of da/dN normalized with respect to the mean value of da/dN is the swing of the output corresponding to the selected input RV. This process is repeated for all other input RVs to compute the swings of the output. Finally, tornado diagram is obtained by arranging the obtained swings in a descending order [10]. The generated Tornado diagram is given in Figure 5.

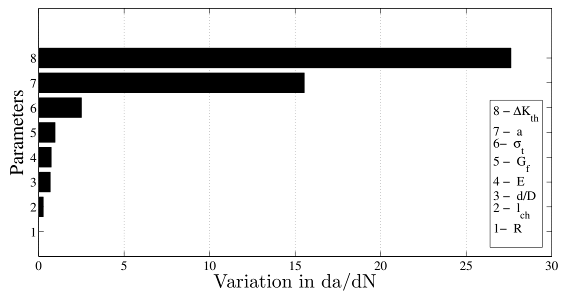

Figure 5: Tornado Diagram for Sensitivity Analysis.

It is observed that crack propagation rate in short crack region is more sensitive to threshold stress intensity factor range and less sensitive to stress ratio.

9 CONCLUSIONS

A model has been proposed to understand the behavior of short crack propagation in concrete. The proposed law has been calibrated and validated with available experimental data. The plots obtained using the proposed model agrees fairly well with the experimental data plots. A sensitivity analysis has been performed. Out of the parameters considered, crack propagation rate in short crack region is found to be more sensitive to threshold stress intensity factor range and less sensitive to stress ratio.

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