Effect of Stress Singularity on Scaling of Quasibrittle Fracture

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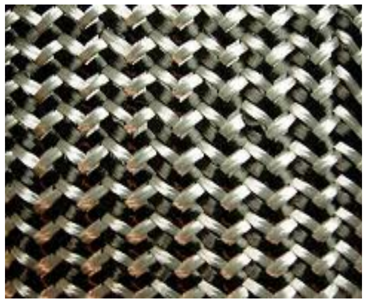
Workshop in honor of Prof. Zdeněk P. Bažant, Framcos 9, UC Berkeley.

May 29 2016

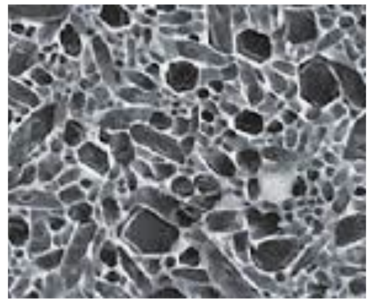
Overview of Quasibrittle Structures

Brittle heterogenous (quasibrittle) materials

concrete

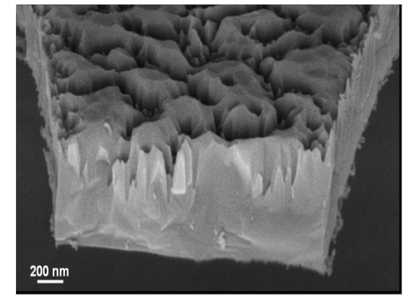


composites

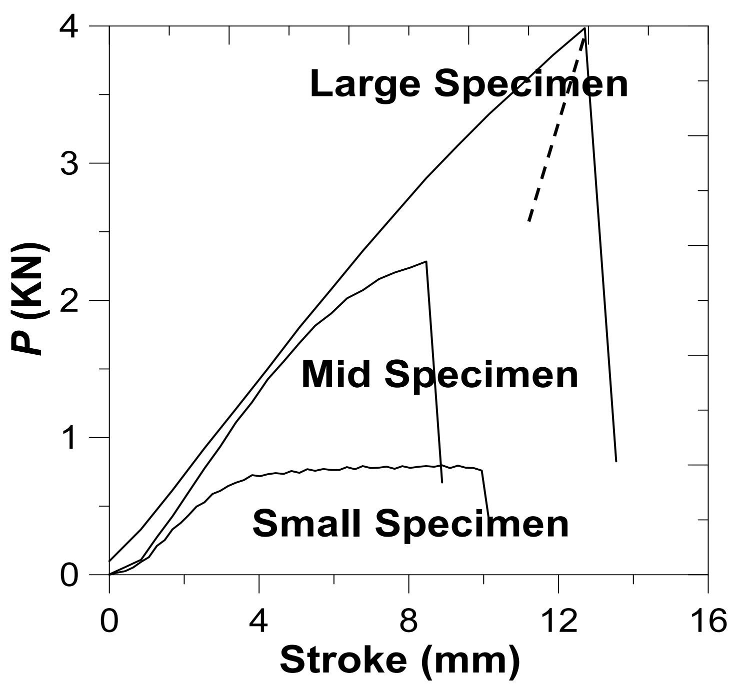


ceramics

rock



poly-Si

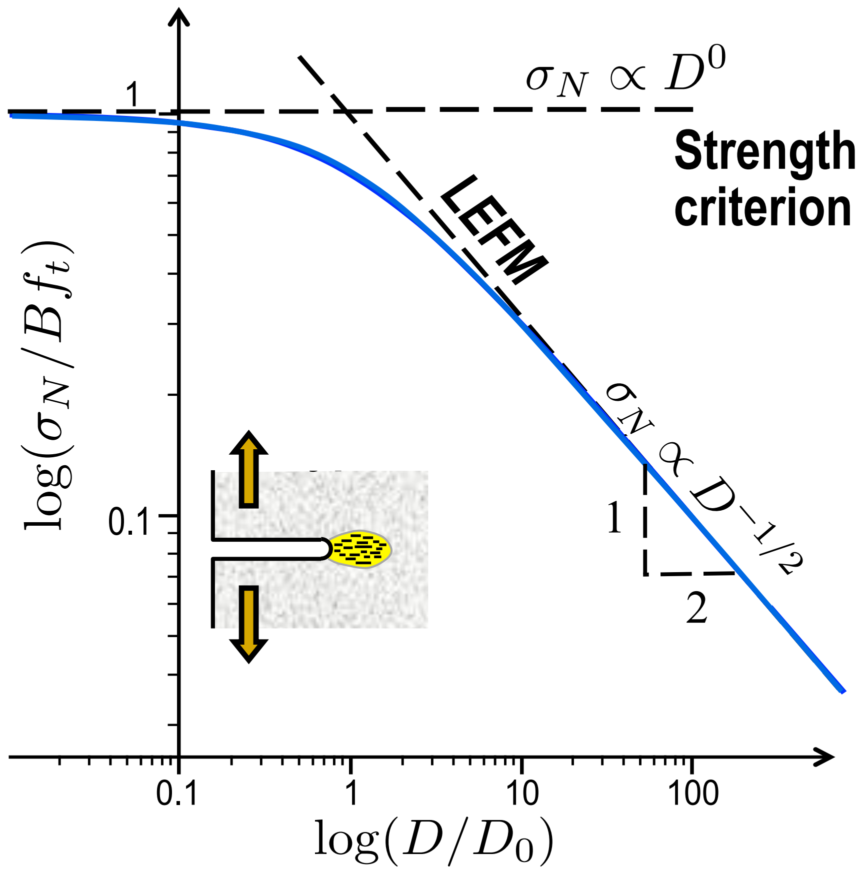


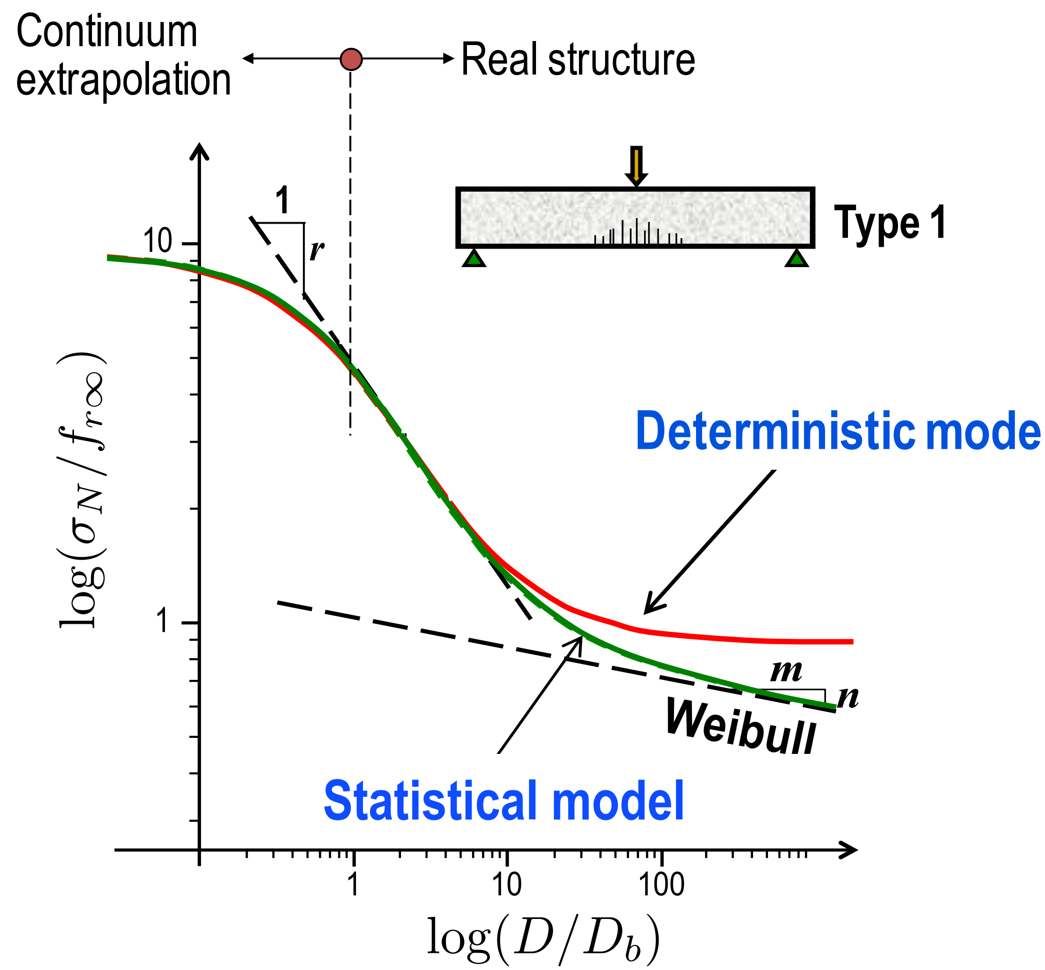
Size-dependent failure behavior:

small size limit: quasi-plastic

large size limit: perfectly brittle

Two Types Scaling Laws of Quasibrittle Fracture

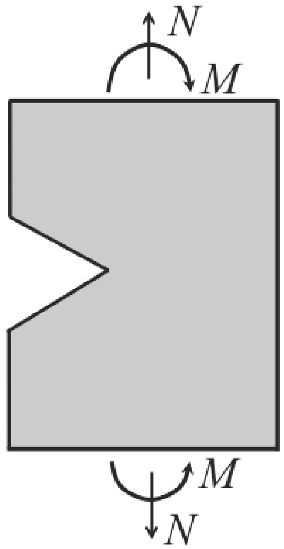


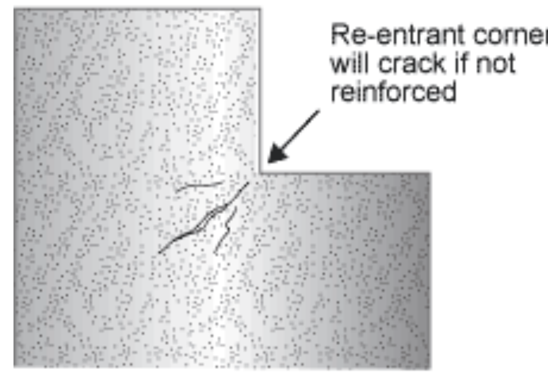


Bažant 2005

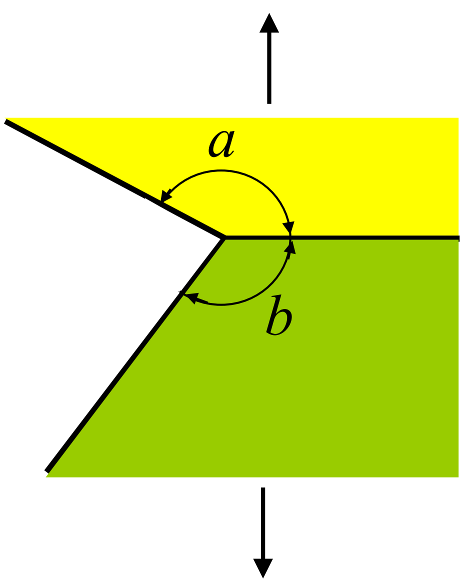
Engineering Structures with Weak Stress Singularities

1. Weak stress singularities caused by geometry

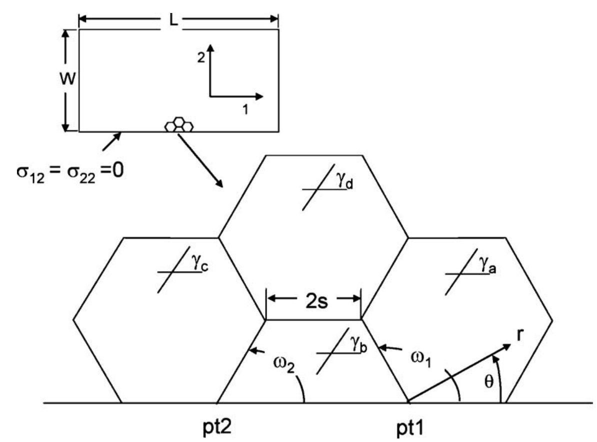




2. Weak stress singularities caused by material mismatch



bi-material lap joint

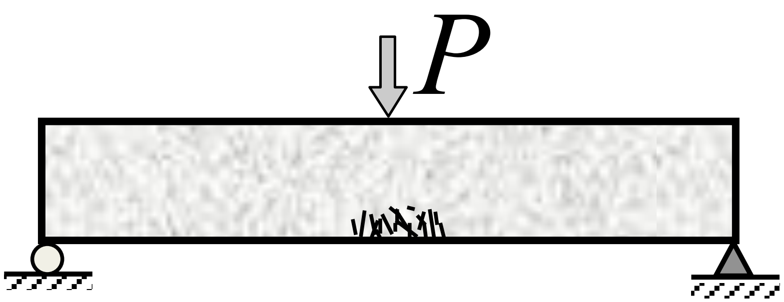


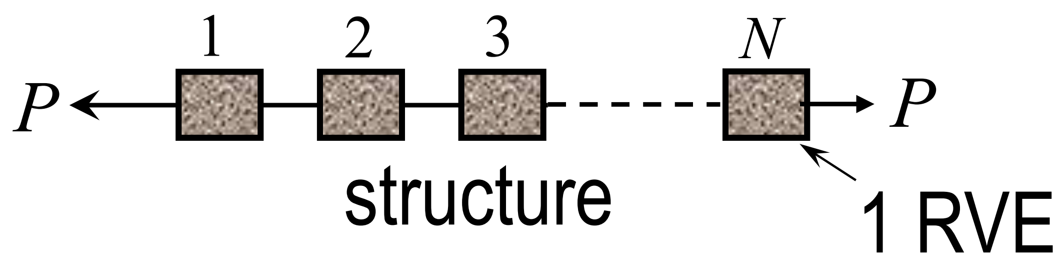
Poly-Si MEMS

Theoretical Framework

Statistical Size Effect — Structures without Stress Singularities

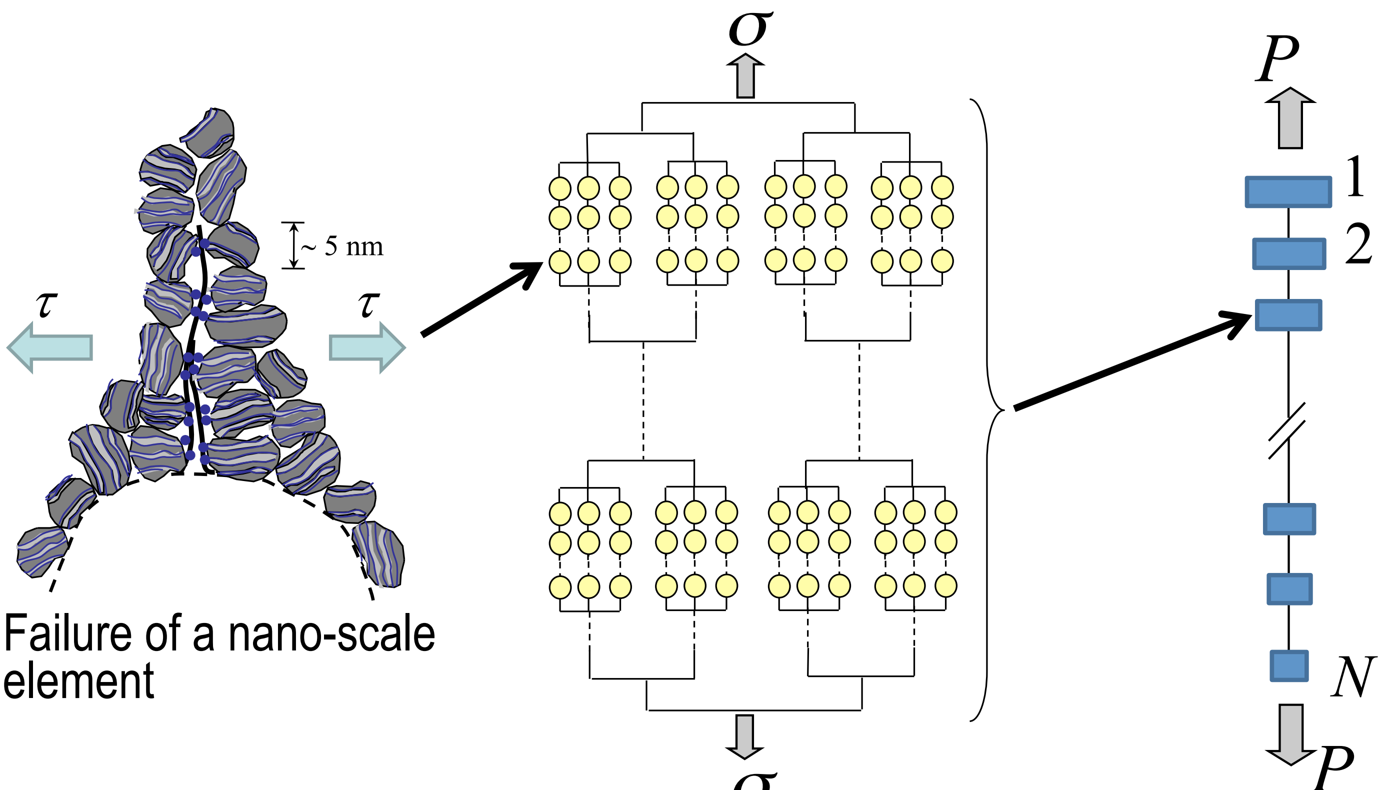
Structures of positive geometry: peak load is reached once one representative volume element (RVE) is fully damaged — Weakest link statistical model.





$$1 - P_f(\sigma_N) = \prod_{k=1}^{N} [1 - P_1(\sigma_N s(x_k))]$$

Multiscale Statistical Model for RVE Strength



Failure of a nano-scale element

Hierarchical model for RVE strength distribution

Finite weakest link model for macro-scale structures

$$P_1(\sigma) = 1 - \exp\left[-(\sigma/s_0)^m\right] \qquad (\sigma < \sigma_{gr})$$

$$P_1(\sigma) = P_{gr} + \frac{r_f}{\sqrt{2\pi}\delta_G}\int_{\sigma_{gr}}^{\sigma} e^{-(\sigma'-\mu_G)^2/2\delta_G^2}\mathrm{d}\sigma' \qquad (\sigma \geq \sigma_{gr})$$

Bažant and Pang 2007, Bažant et al. 2009, Le et al. 2011

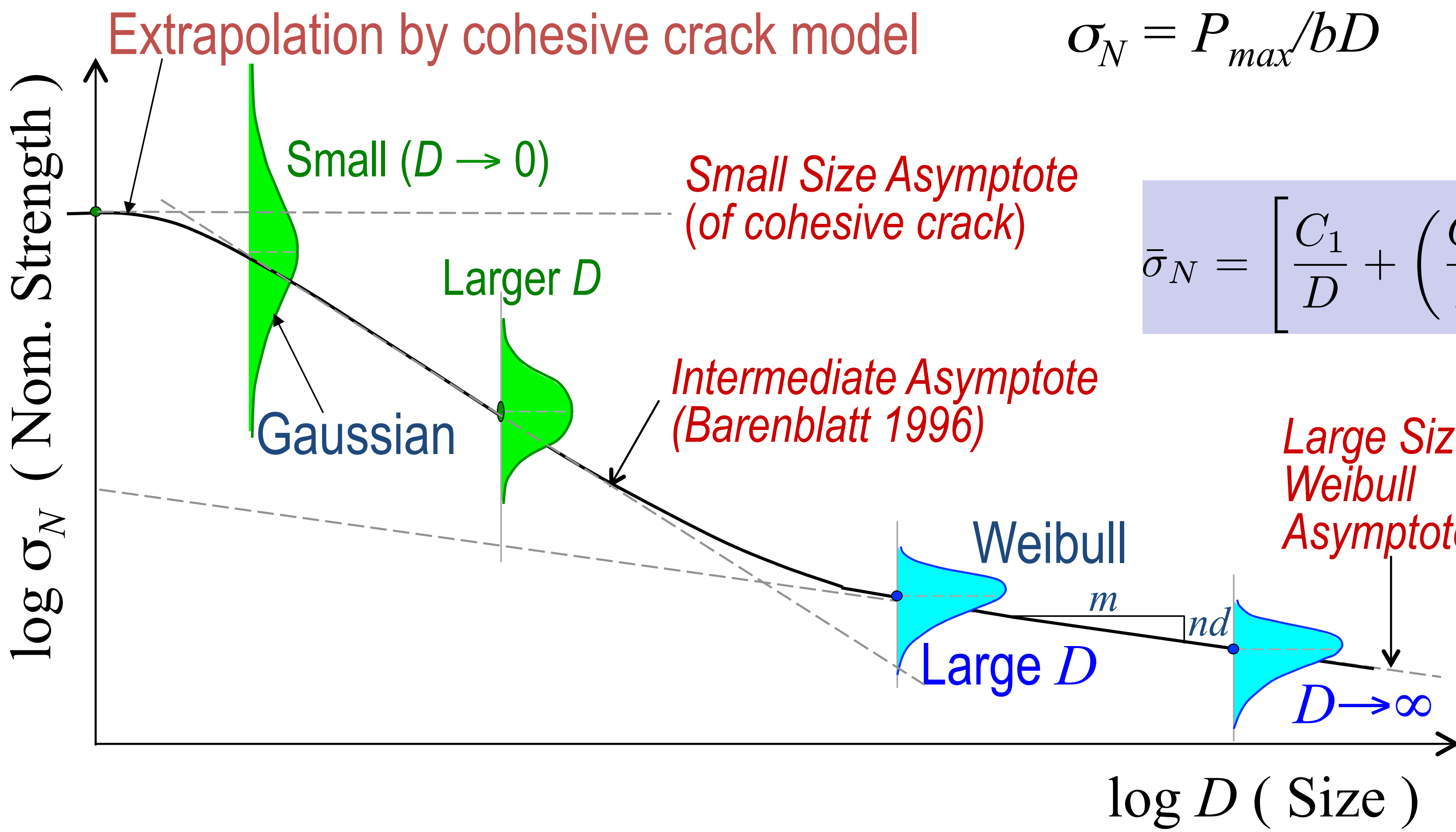
Mean Size Effect Behavior

Mean structural strength:

$$\bar{\sigma}_N = \int_0^1 \sigma_N \mathrm{d}P_f = \int_0^\infty [1 - P_f(\sigma_N)]\mathrm{d}\sigma_N$$

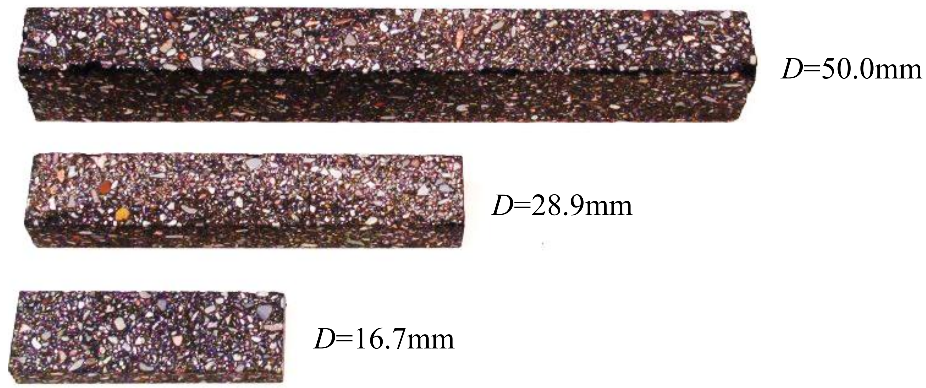
$$\sigma_N = P_{max}/bD$$

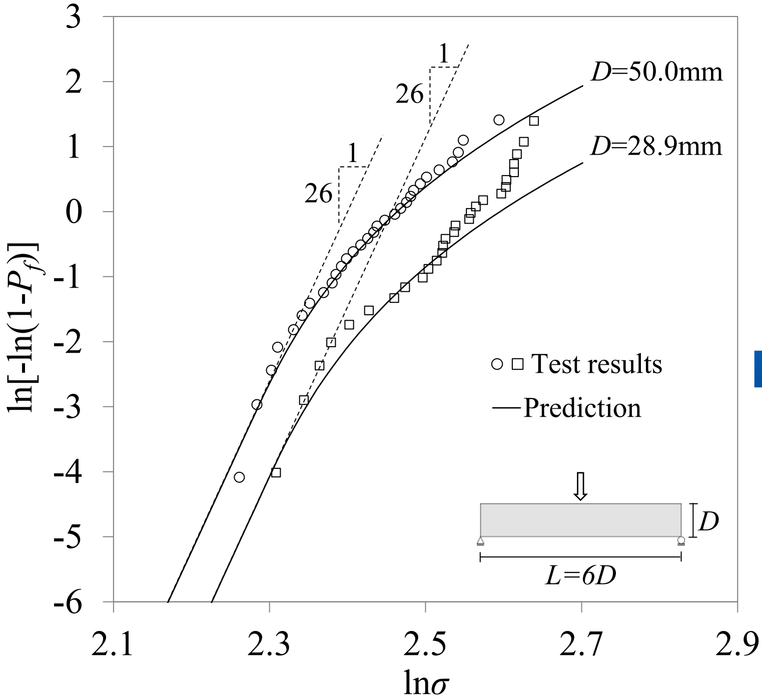
$$\bar{\sigma}_N = \left[\frac{C_1}{D} + \left(\frac{C_2}{D}\right)^{\frac{rn}{m}}\right]^{\frac{1}{r}}$$

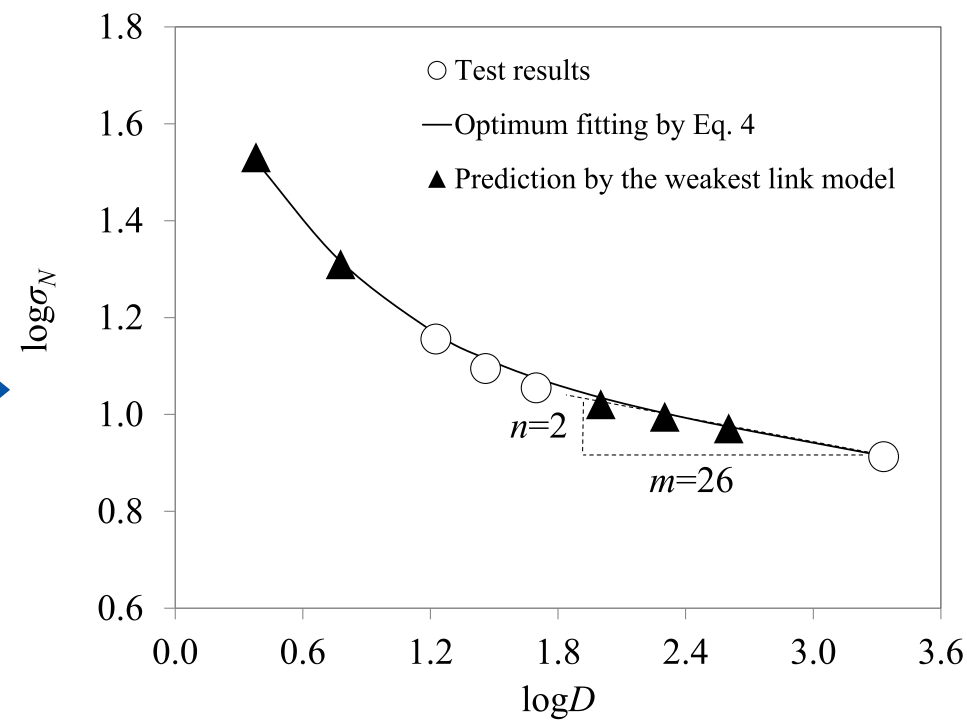


Statistical SE — Experimental Validation

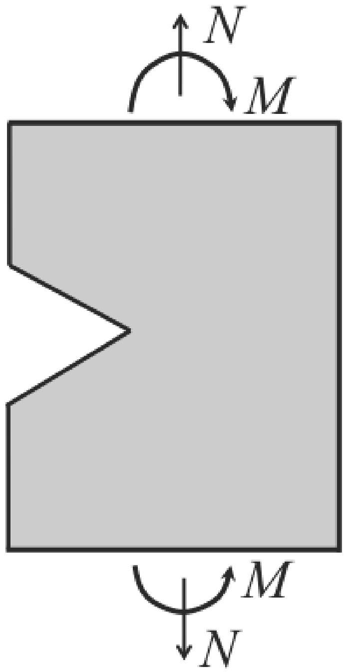
Size effect tests on both strength histograms and mean strength of asphalt mixture at low temperature (T = −24°C) (*Le et al., 2013*).







Energetic Size Effect — Structures with Strong Stress Singularities



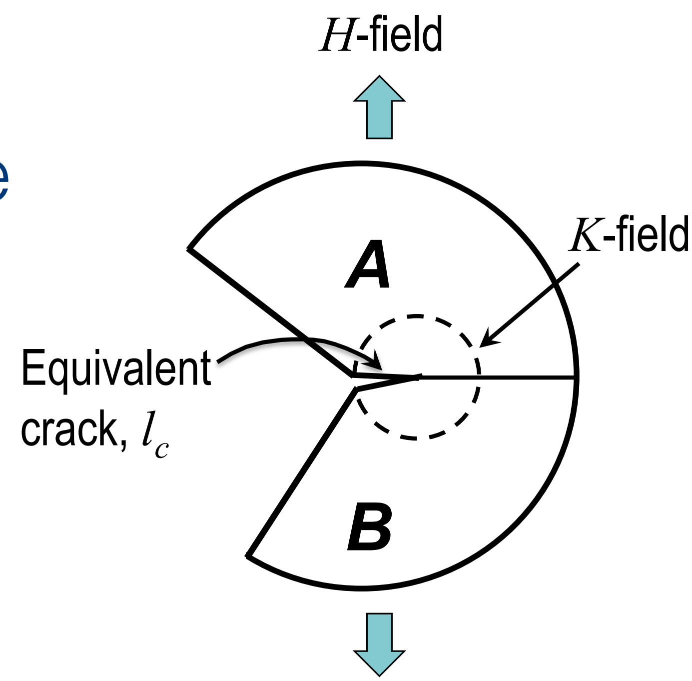
1) Large-size limit:

Near-tip stress field: $\sigma_{ij} = Hr^{\lambda} f_{ij}(\theta)$

Stress intensity factor: $H = \sigma D^{-\lambda} h$

At the large-size limit, the FPZ can be replaced by an LEFM crack (Equiv. LEFM) (*Liu and Fleck, IJF, 1997*):

$$K = aHl_c^{\lambda+1/2}$$



Energetic Size Effect — Structures with Strong Stress Singularities

At the peak load, the FPZ is fully developed, i.e. the equiv. crack starts to propagate. This leads to a power-law scaling relation:

$$\sigma_N \propto D^{\lambda}$$

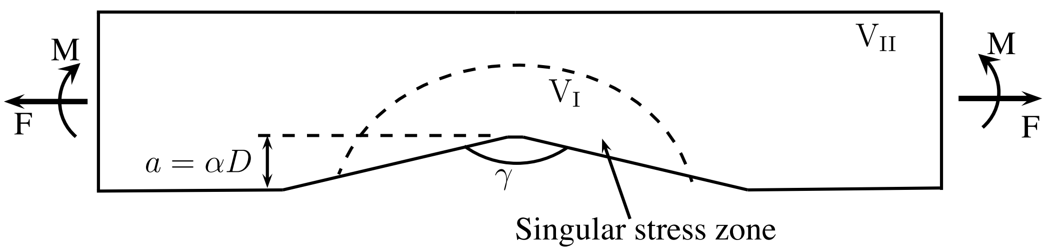
2) Small-size limit:

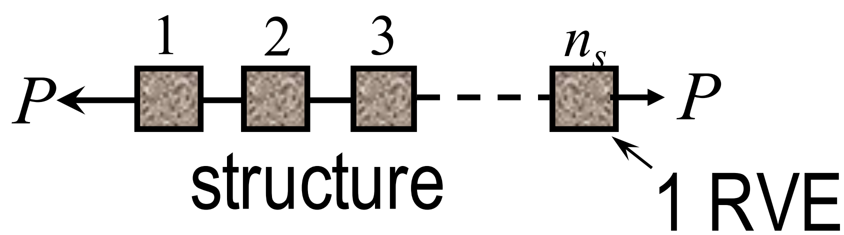
The entire ligament behaves as a crack filled by a plastic glue – No size effect

Asymptotic Matching:

$$\sigma_N = \sigma_s \left[1 + (D/D_{0\gamma})^{1/\beta_\gamma}\right]^{\lambda\beta_\gamma}$$

Scaling Equation for Structures with Weak Stress Singularities — Generalized Weakest Link Model





Weakest link model: $P_f = 1 - (1 - P_{f,V_I})\,(1 - P_{f,V_{II}})$

$$P_{f,V_I}(\sigma_N) \quad = \quad 1 - \prod_{i=1}^{N_1} \{1 - P_1[\mu(D)\sigma_N s(\boldsymbol{x}_i)]\}$$

where: $$\boxed{\mu(D) \quad = \quad \left[1 + (D/D_{0\gamma})^{1/\beta_\gamma}\right]^{-\lambda\beta_\gamma}}$$

$$P_{f,V_{II}}(\sigma_N) = 1 - \prod_{i=1}^{N_2} \{1 - P_1[\sigma_N s(\boldsymbol{x}_i)]\}$$

Le, IJF, 2012

Generalized Weakest Link Model

RVE strength distribution:

$$P_1(\sigma) = 1 - \exp\left[-(\sigma/s_0)^m\right] \qquad (\sigma < \sigma_{gr})$$

$$P_1(\sigma) = P_{gr} + \frac{r_f}{\sqrt{2\pi}\delta_G} \int_{\sigma_{gr}}^{\sigma} e^{-(\sigma' - \mu_G)^2/2\delta_G^2} \mathrm{d}\sigma' \qquad (\sigma \geq \sigma_{gr})$$

Mean structural strength:

$$\bar{\sigma}_N = \int_0^1 \sigma_N(P_f)\mathrm{d}P_f = \int_0^\infty (1 - P_f)\mathrm{d}\sigma_N$$

Closed-form expression of $\bar{\sigma}_N(D)$ is impossible –
Approximate equation via asymptotic matching

Generalized Weakest Link Model

Large size limit:

$$P_f(\sigma_N) = 1 - \exp\left\{-\int_{V_I} \frac{\mu^m(D)\sigma_N^m\langle s(\boldsymbol{x})\rangle^m}{s_0^m}\frac{\mathrm{d}V(\boldsymbol{x})}{l_0^2} - \int_{V_{II}} \frac{\sigma_N^m\langle s(\boldsymbol{x})\rangle^m}{s_0^m}\frac{\mathrm{d}V(\boldsymbol{x})}{l_0^2}\right\}$$

$$\bar{\sigma}_N = s_0[\mu^m(D)\Psi_1 + \Psi_2]^{-1/m}\Gamma\left(1+\frac{1}{m}\right)\left(\frac{l_0}{D}\right)^{2/m}$$

Small size limit:

For structures without stress singularities: $\bar{\sigma}_N \propto (D/D_b)^{-1/r}$

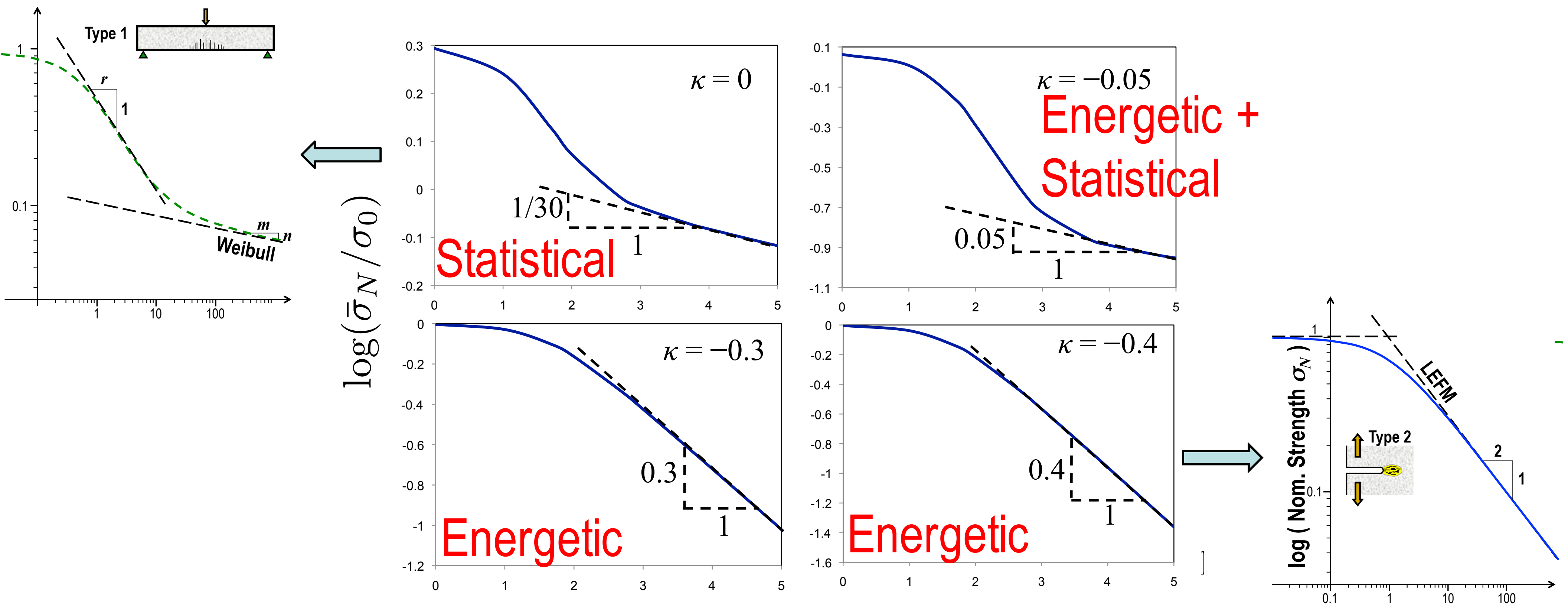
For structures with weak stress singularities:

$$\bar{\sigma}_N \propto [\mu(D)]^{-1}(D/D_b)^{-1/r}$$

Generalized Weakest Link Model

Asymptotic matching:

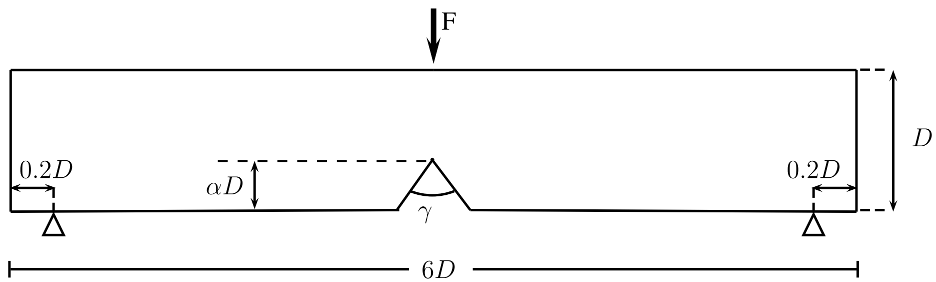
$$\bar{\sigma}_N = \sigma_0 \left\{ C_1 \left[\mu^m(D)\Psi_1 + \Psi_2\right]^{-r/m} \left(\frac{D+l_s}{l_0}\right)^{-2r/m} \exp[-(\lambda/\lambda_1)^2] + \frac{\mu^{-r}(D)D_b}{\exp[-(\lambda/\lambda_2)^2]D + l_p} \right\}^{1/r}$$

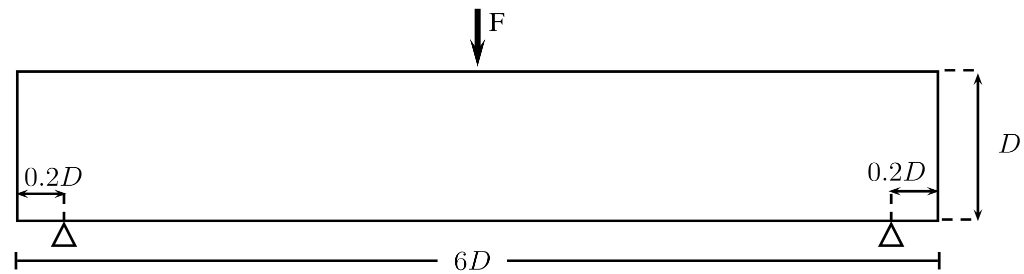


Le, IJF, 2012

Computational Studies

Case 1 — Concrete Beams with a V-Notch





$$\gamma = 0°, 90°, 120°, 135°, 170°$$

$$\lambda = -0.5, -0.4555, -0.3843, -0.3264, -0.0916.$$

Deterministic calculations:

— Size effect for structures with strong stress singularities

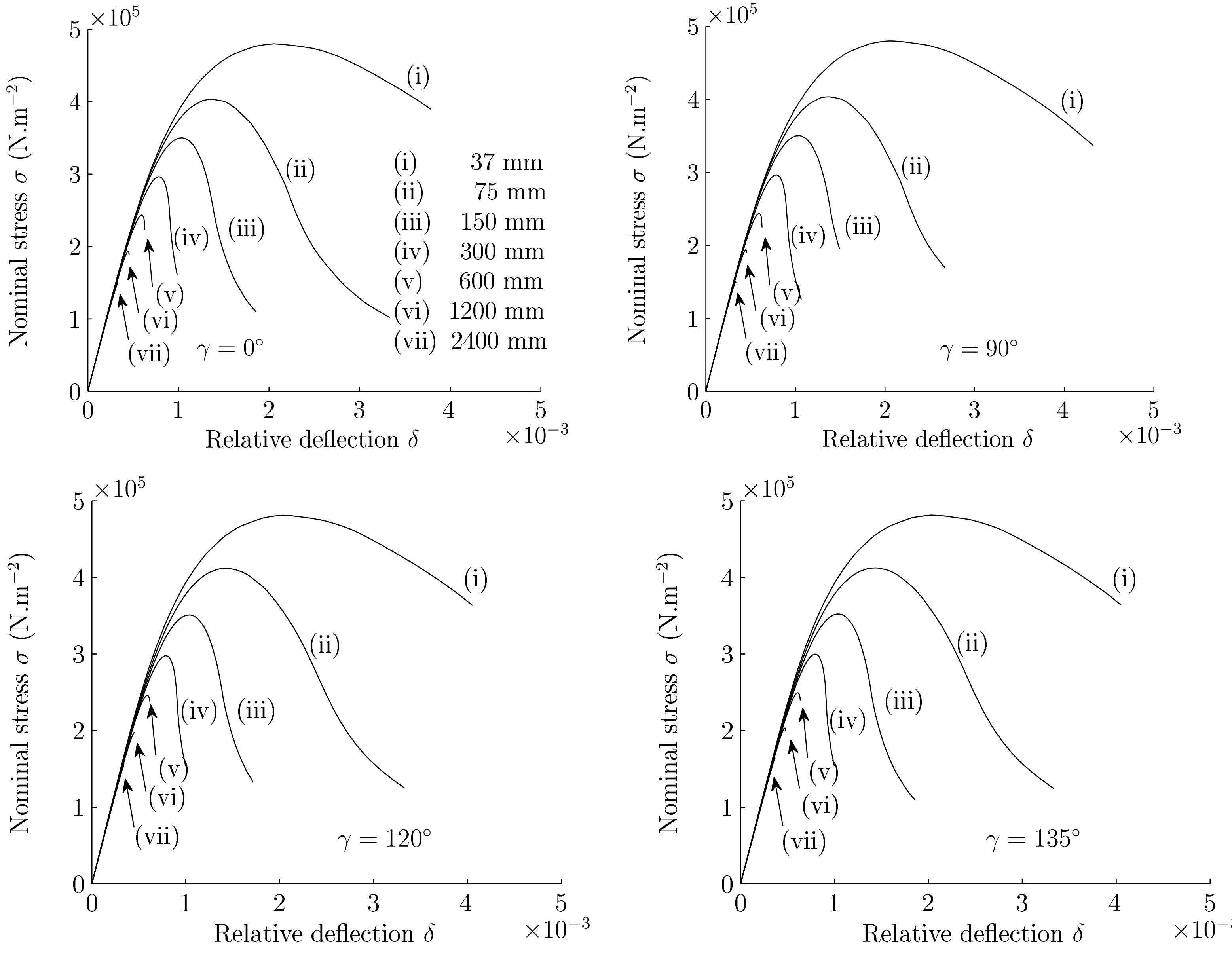
— Size effect in the small and intermediate size ranges for structures with weak/zero stress singularities

— Damage-plasticity model with crack band model

Le et al., JEM, 2014

Simulation Results — I. Load-Deflection Curves

$$\sigma = P/bD$$

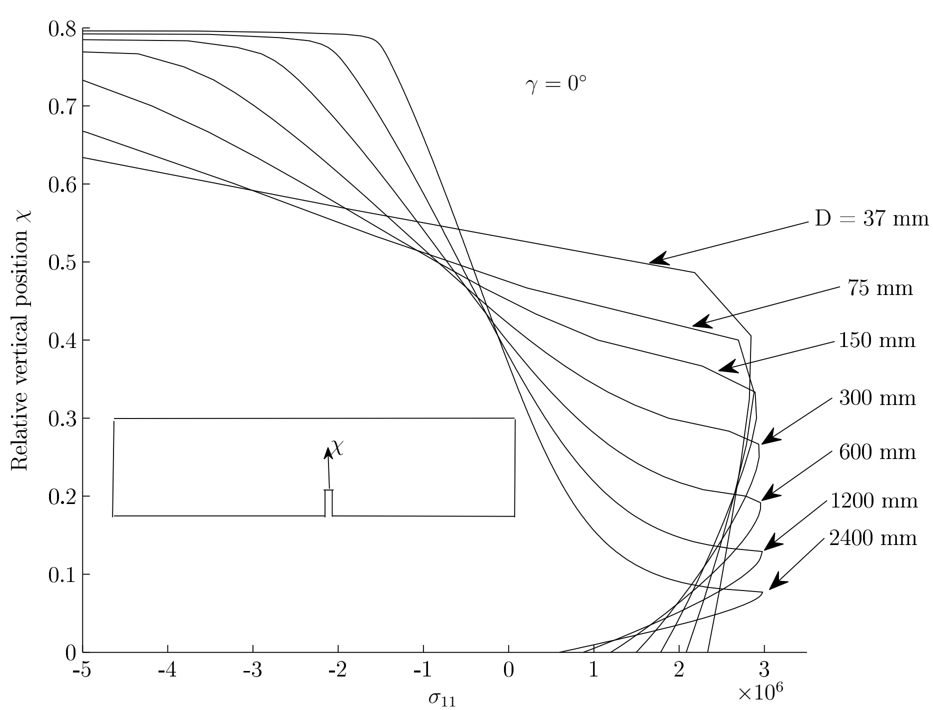


$$\delta = \Delta/D$$

Le et al., JEM, 2014

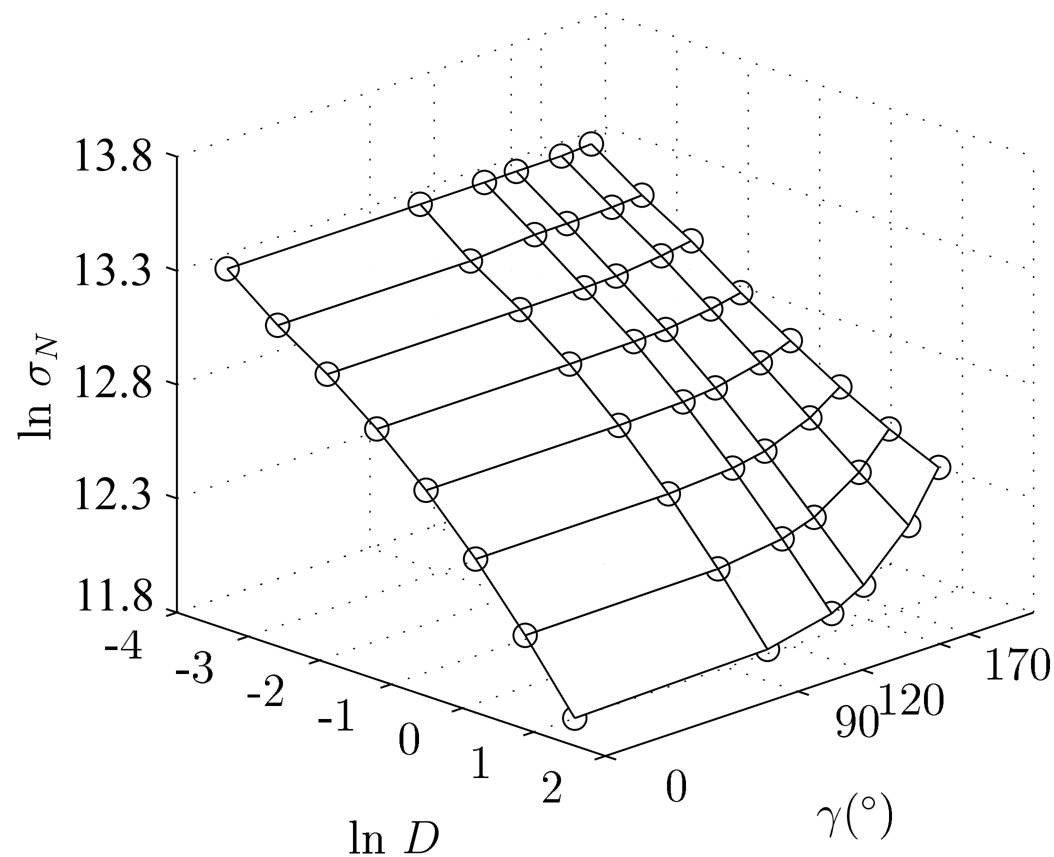
Simulation Results —II. Stress Profile and Size Effect

Stress profile along the ligament



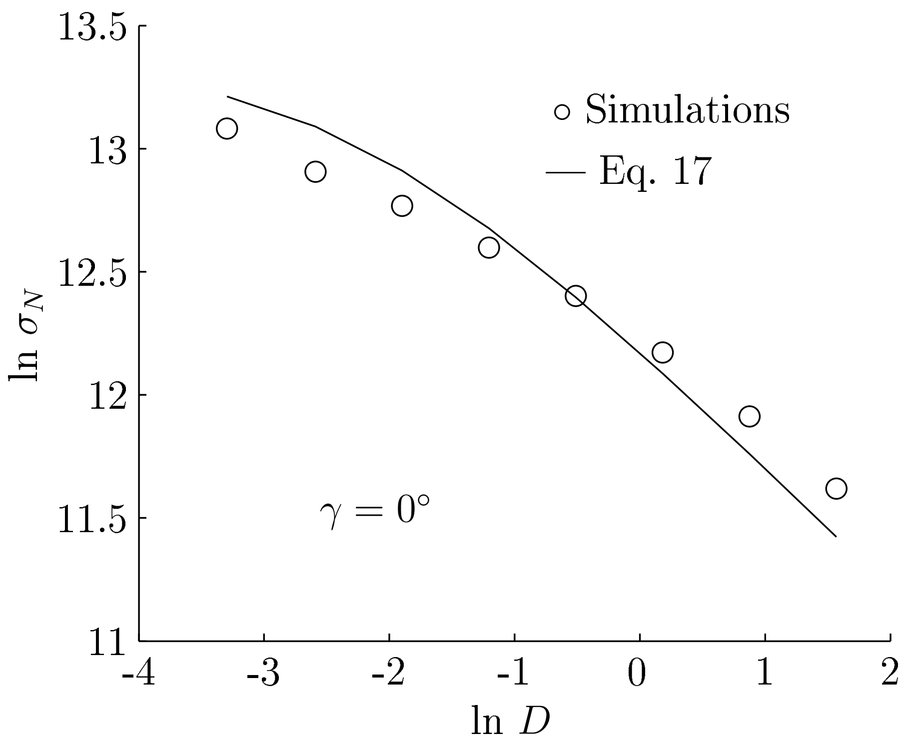
Transition from ductile failure to brittle failure

Size effect surface

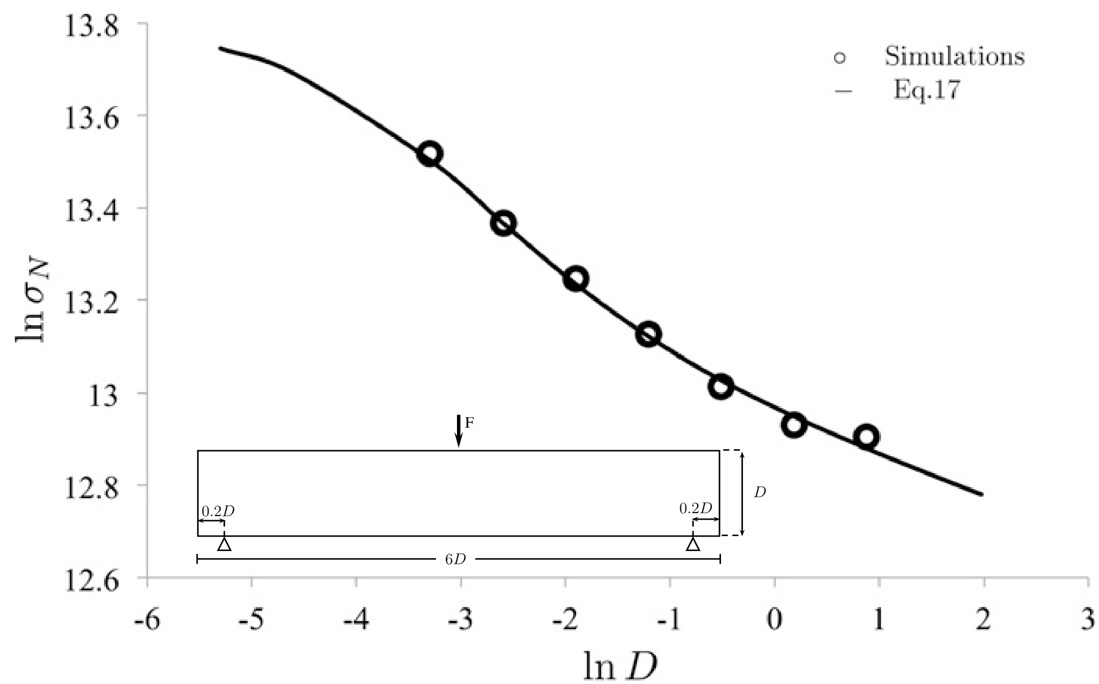


Le et al., JEM, 2014

Comparison Between Analytical Model and Numerical Simulation

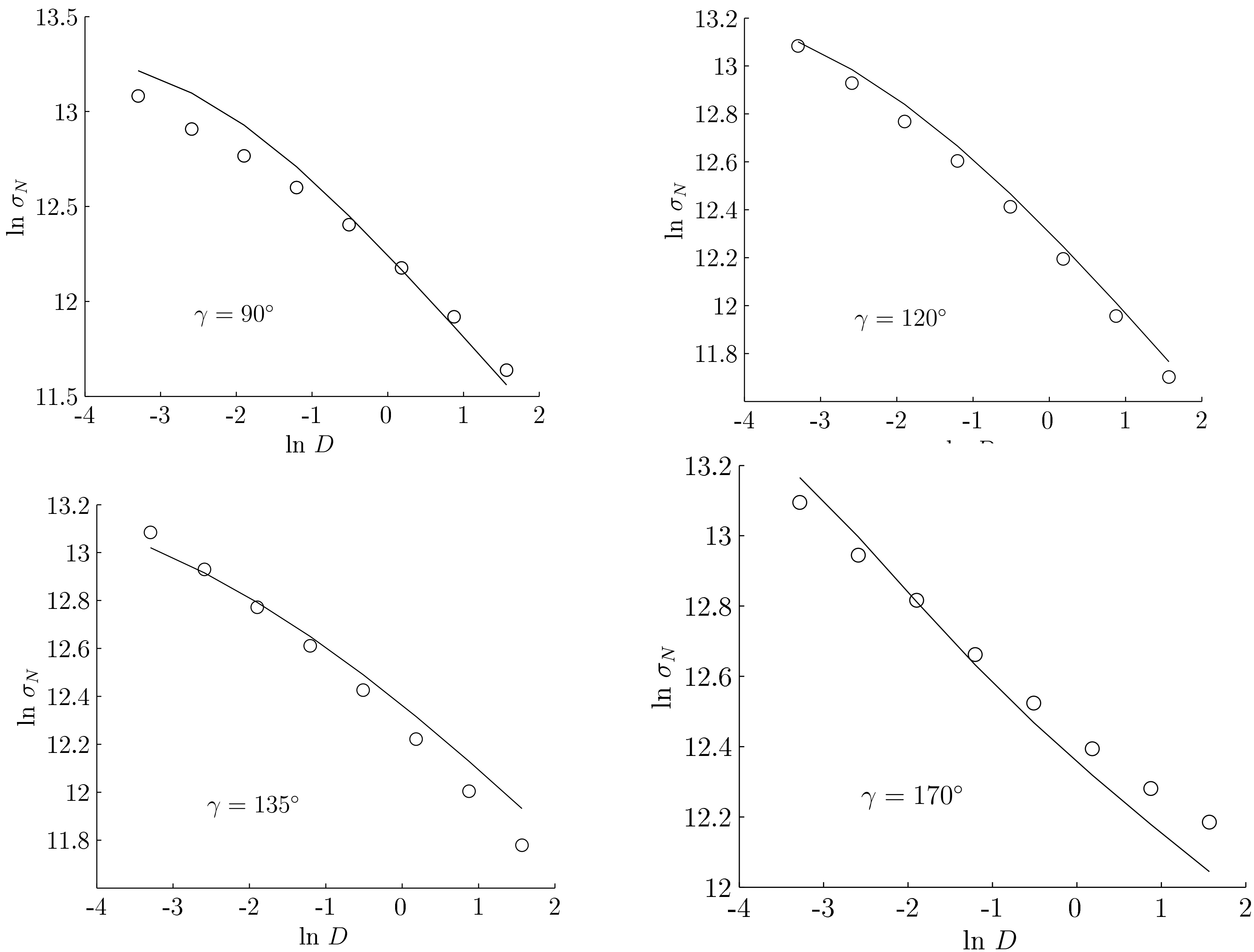


Type-2 SE

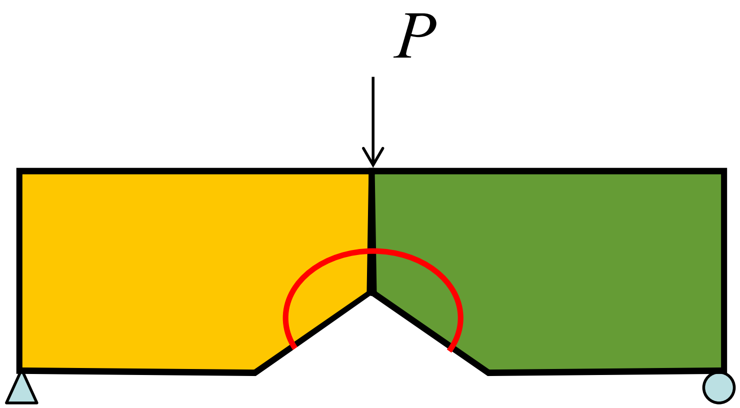


Type-1 SE

Comparison Between Analytical Model and Numerical Simulation



Case 2 — Bimaterial Hybrid Beams



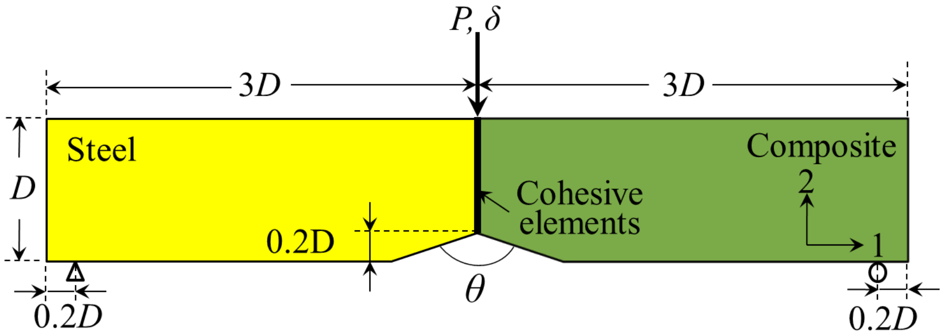
A bimaterial corner may exhibit
1) complex stress singularities;
2) two real stress singularities;
3) one real stress singularities

Assuming weak interface, we consider failure always initiates from the interface though the failure location is random for the case of weak stress singularities.

$$\bar{\sigma}_N = \sigma_0 \left\{ C_1[\mu^m(D)\Psi_1 + \Psi_2]^{-r/m} \left(\frac{D + l_s}{l_0} \right)^{-r/m} \exp[-(\lambda/\lambda_1)^2] + \frac{\mu^{-r}(D) D_b}{\exp[-(\lambda/\lambda_2)^2] D + l_p} \right\}^{1/r}$$

$$\mu(D) = \left\{ 1 + [(D/D_1)^{-2\lambda_1} + (D/D_2)^{-2\lambda_2} + (D/D_3)^{-\lambda_1 - \lambda_2}]^{\beta} \right\}^{1/2\beta}$$

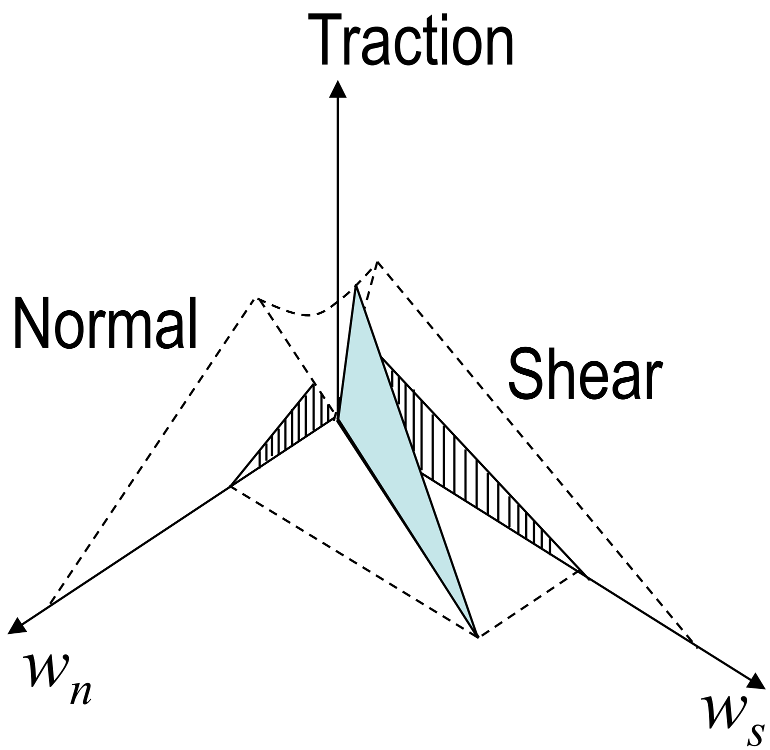
Stochastic Simulation of Fracture of Bimaterial Hybrid Beam



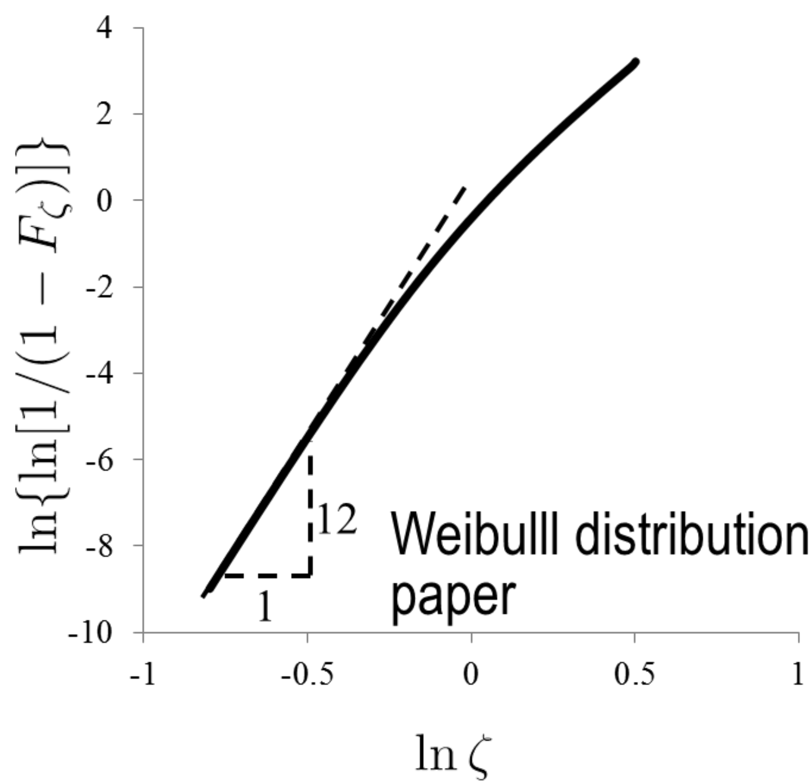
Steel/unidirectional carbon-epoxy composite

Weak interface — stochastic mixed-mode cohesive model

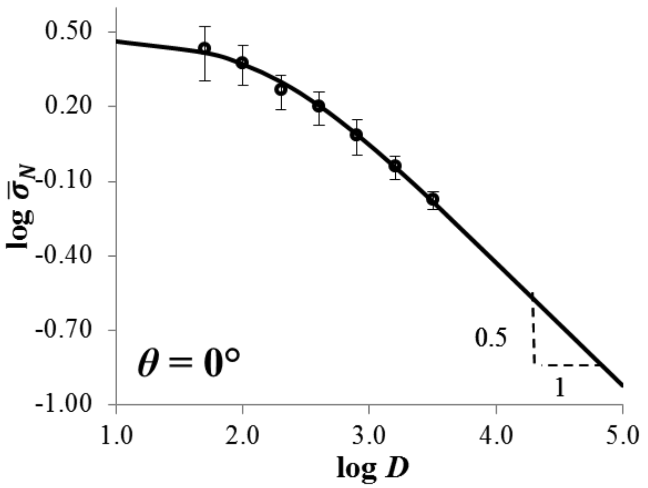
$$B_i(x) = \zeta(x)\bar{B}_i$$

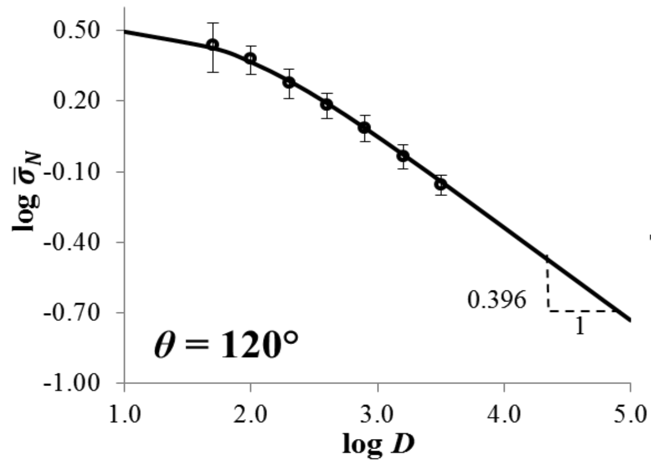


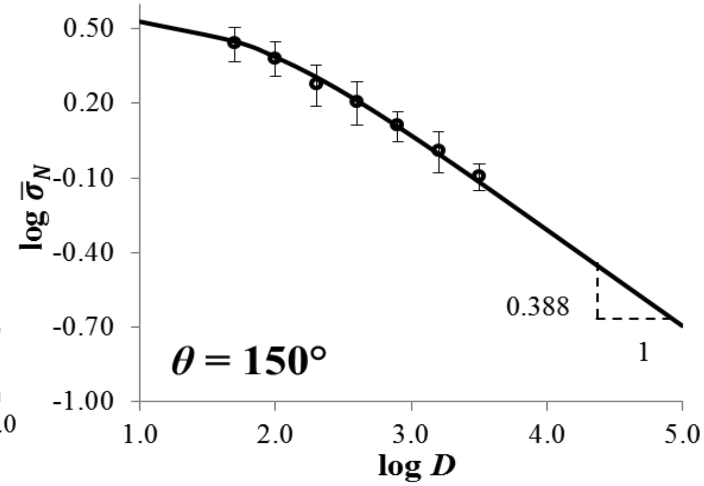
Mixed-mode cohesive element

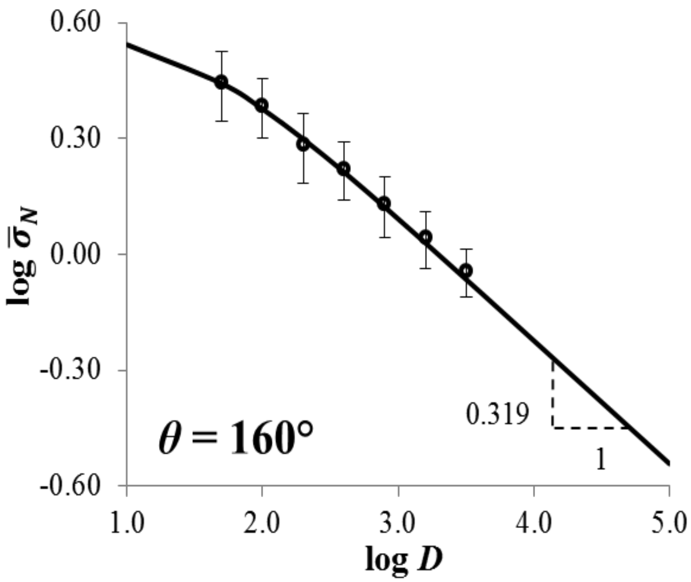


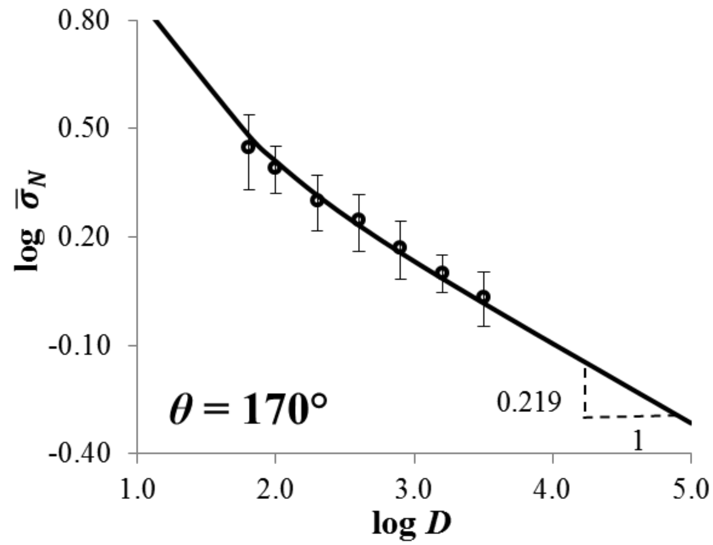
Comparison Between Analytical Model and Numerical Simulation

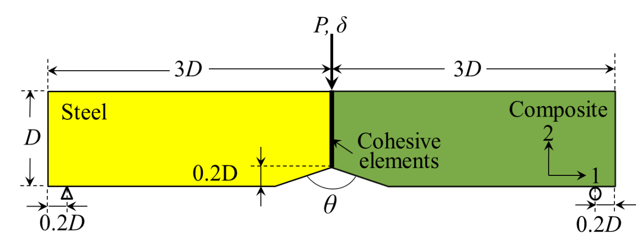












Le and Xue., EFM, 2013

Conclusions

1. For structures with strong stress singularities, the size effect is energetic (deterministic). The large-size asymptote is governed by the order of the dominant stress singularity.

2. For structures without stress singularities, the size effect can be explained by the weakest link statistical model.

3. For structures with weak stress singularities, the size effect consists of both energetic and statistical components.

4. Some open problems: influence of stress singularities on scaling of reliability indices? general scaling law of fatigue lifetime?…

Thank You