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EFFECTS OF FRACTURE PROCESS ZONE ON FATIGUE CRACK GROWTH IN CONCRETE

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Abstract. In the present work, an attempt has been made to propose an analytical model to predict crack propagation in plain concrete member under the action of repetitive loading. The analytical formulation incorporates the influence of fracture process zone, which is considered to be evolved at the micro/meso-scale level together with other crack growth characterising parameters. These parameter includes the size dependent fracture energy, change in energy release rate, frequency, initial crack length, structural size. Generalised Barenblatt and Botvina dimensional analysis approach for fatigue crack growth problems has been adopted in conjunction with the theory of intermediate asymptotic. The scaling behaviour of various parameters has been explored for the fatigue crack propagation problem in plain concrete specimens. The proposed model has been calibrated and validated with the available experimental results from the literature

1 INTRODUCTION

It is well-known that, all structural components are subjected to several fluctuating loadings in the real life conditions. Civil engineering structures such as airport pavements, railroad bridges, highway pavements, offshore structures etc. are subjected to repetitive loads from various sources. Under the action of such repeated loading, micro structural changes takes place by means of crack initiation due to the existence pre-existing of flaws. This leads to a subsequent grow of sizeable crack, which cause a final failure after a sufficient number of stress and strain fluctuations. In case of quasi-brittle material like concrete, a large and sizeable inelastic zone i.e. fracture process zone always exists ahead of the crack tip. The toughening mechanisms which are responsible for the formation of the fracture process zone are very much complicated in nature, due to the heterogeneous nature of concrete. The problem of fatigue initiates from micro-structural level and therefore, the crack initiation and propagation are essentially influenced by important parameters evolving at the micro/meso level. An accurate prediction of fatigue life and the crack propagation analysis of concrete members require the consideration of micro/meso-scale parameters in the formulation.

One of the most extensively used fracture mechanics based fatigue crack propagation model is known as Paris law which was proposed by Paris and Erdogen [1]. This Paris law was mainly developed for metallic structures. Straight forward application of this model to more heterogeneous material like concrete is restricted due to the existence of large sizeable fracture process zone ahead of crack tip and exhibition of size effect. In order to make these models applicable to concrete, many attempt have been made by many researchers by modifying this law. An earliest attempt was done by Bazant and Xu [2] who have incorporated size effect in Paris law by combining Paris law with size effect law. Slowik et al. [3] have proposed a model for low cycle fatigue under variable amplitude loading by performing experiments on wedge splitting test specimens. Paggi [4] proposed an fatigue crack growth model using the generalized Barenblatt and Botvina dimensional analysis approach in order to give prominence to the discrepancy of classical power law equations, which are being used to predict fatigue behaviour of quasi-brittle materials like concrete. Sain and Chandra Kishen [5] have proposed a model for plain concrete to incorporate the influence of loading frequency. Ray and Chandra Kishen [6] have proposed an analytical model for estimating fatigue crack propagation in plain concrete by using the concepts of dimensional analysis. Ray and Chandra Kishen [7] modified the previous model to include the effects of overloads. Le et al. [8] established a size adjusted Paris law for rocks and by incorporating a new physical quantity as fatigue energy U_c. The term U_c has been expressed as:

$$U_c = U_{c,\infty}\frac{D}{D+D_{0c}} \tag{1}$$

Where $U_{c,\infty}$ is the fatigue fracture energy when $D \longrightarrow \infty$. The final form of model proposed by Le et al. [8] considering size effect is provided below :

$$\begin{aligned}\frac{da}{dN} &= C\left(1+\frac{D_{0c}}{D}\right)^{m/2}(\Delta K)^m \\ &= C(\Delta K_D)^m \end{aligned} \tag{2}$$

Where, $\Delta K_D = \Delta K\left(1+D_{0c}/D\right)^{1/2}$ is a size adjusted ΔK. Kirane and Bazant [9] has been modified the proposed model by Le et al. [8] using the dimensional analysis approach based on self-similarity concept. The final expression has been expressed as:

$$\frac{da}{dN} = C_1(\Delta K)^m\left(\frac{D+D_{0m}}{D+D_0}\right)^{m/2} \tag{3}$$

Where, C_1 is a material constant, which is a function of fatigue fracture energy $U_{c,\infty}$. This coefficient is difficult to determine through experimental or analytical approach. In the present work, an attempt has been made to propose a fatigue crack propagation model considering the effect of FPZ through the use of energy based parameters.

2 DIMENSIONAL ANALYSIS

Dimensional analysis assembles various independent variables that governs the physical phenomenon under consideration and converts them into dimensionless numbers having total physical dimension equal to unity. The use of dimensionless numbers instead of independent variables is advantageous due to several reasons. Use of dimensionless numbers significantly reduce the number of variables needed for describing a physical problem hence reduce the amount of experimental data required. Secondly, dimensionless numbers also simplify the governing equations by making them dimensionless and also by neglecting the terms which are either too small or large. Further, properly formed dimensionless numbers gives the physical meaning of the parameter and hence helps in understanding the phenomenon better.

Consider a physical problem in which a is the quantity to be determined and is dependant on n independent variables. Then a can be expressed as a function of the independent variables as given below:

$$a = f(a_1, a_2, ..., a_k; a_{k+1}, a_{k+2}, ..., a_n) \tag{4}$$

where, $(a_1, a_2, ..., a_k)$ have independent physical dimensions. $(a_{k+2}, ..., a_n)$ can be expressed as the products of powers of the dimensions of the parameters. Using Buckingham Π theorem.

$$a = a_1^p, .., a_1^p\Phi\left(\frac{a_{k+1}}{a_1^{p_k+1}..a_k^{r_k+1}}..\frac{a_n}{a_1^{p_n}...a_k^{r_k+1}}\right) \tag{5}$$

Equation 5 can be written in terms of dimensionless quantities as

$$\Pi = \Phi \left(\Pi_1, ..., \Pi_{n-k} \right) \tag{6}$$

Where, Φ is a function containing dimensionless terms. On applying the Π theorem, Φ turns out to be a function of $(n-k)$ variables only. Now we want to see if the number of quantities involved in the relationship of the physical problem can be further reduced. This can be done using the concept of self similarity. There are two types of self-similarity, self-similarity of first kind and self-similarity of second kind.
A parameter a_1 can be considered as non-essential if the corresponding dimensionless parameter Π_1 is too large or too small (tend to infinity or zero), giving rise to a finite non-zero value of the function Φ with the other similarity parameters remaining constant. The number of arguments can now be reduced by one and Equation 6 can be written as:

$$\Pi = \Phi_1 \left(\Pi_2, ..., \Pi_{n-k} \right) \tag{7}$$

Where, Φ_1 is the limit of function Φ as $\Pi_1 \to 0$ or $\Pi_1 \to \infty$. This is called as complete self-similarity or self-similarity of first kind.
If Φ also tends to zero or infinity for $\Pi_1 \to 0$ or $\Pi_1 \to \infty$, then the quantity Π_1 becomes essential, no matter how large or small it becomes. However, in some cases, the limit of the function Φ tends to zero or infinity, but the function Φ has power type asymptotic representation which can be written as,

$$\Phi \cong \Pi_1^{\gamma} \Phi_1 \left(\Pi_2, ..., \Pi_{n-k} \right) \tag{8}$$

Where, constant γ and and the non-dimensional parameter $\Phi_1$cannot be obtained from the dimensional analysis alone.
This is the case of incomplete self-similarity or self-similarity of second kind. It can be noted here that, the parameter γ can only be obtained either from a best fitting procedure on experimental results or according to numerical simulations.

3 ANALYTICAL FORMULATION

In this part, a fatigue crack propagation model has been proposed using the concept of dimensional analysis and intermediate asymptote for the linear region of crack growth curve. The rate of crack growth in a fatigue crack propagation phenomenon is mostly represented in the stable regime of crack growth curve and also manifests intermediate asymptotic behaviour.
Crack growth rate (da/dN) in a concrete member subjected to repetitive loading depends on various factors such as loading characteristics, geometric properties and material properties. The governing loading parameters which influences the fatigue crack growth rate phenomenon are the change in energy release rate (ΔG_I), energy release corresponding to the upper fatigue load (ΔG_{Imax}), characteristic dimensions of the structure (D) and maximum size of the aggregate (d_{max}). The material properties which governs the fatigue crack growth rate are tensile strength (σ_t) and critical energy dissipation (G_f). In addition, rate of fatigue crack growth is influenced by thermal diffusion coefficient (χ), loading frequency (ω) and crack length (a). The governing variables with their physical dimensions are listed in Table (1).

Table 1: Governing parameters with their dimensions

ΔG_I	Energy release rate range	FL^{-1}
ΔG_{Imax}	Maximum value of energy release rate	FL^{-1}
D	Structural size	L
d_{max}	Maximum size of the aggregate	L
σ_t	Tensile strength	FL^{-2}
G_f	Critical energy dissipation	FL^{-1}
χ	Thermal diffusion coefficient	L^2T^{-1}
ω	Loading Frequency	T^{-1}
a	Crack length	L

Use of (G_f) is more useful than (K_{Ic}) for concrete due to the existence of large sizeable fracture process zone and heterogeneous behaviour of concrete. Writing the dependance of governing parameter in fatigue crack growth rate (da/dN) as follows :

$$\frac{da}{dN} = (\Delta G_I, \Delta G_{Imax}, D, d_{max}, \sigma_t, G_f, \chi, \omega, a) \quad (9)$$

Considering a state of no explicit time dependence and considering G_f, σ_t and ω as physically independent dimensions and applying Buckingham Π theorem to the Equation 9, crack growth rate can be expressed in terms of non-dimensional parameters as given below:

$$\frac{da}{dN} = \frac{G_f}{\sigma_t}\Phi\left(\frac{\Delta G_I}{G_f}, \frac{\Delta G_{Imax}}{G_f}, \frac{\chi}{\omega}\left(\frac{\sigma_t}{G_f}\right)^2, d_{max}\frac{\sigma_t}{G_f}, D\frac{\sigma_t}{G_f}, a\frac{\sigma_t}{G_f}\right) \quad (10)$$

Where non dimensional quantities are:

$$\Pi_1 = \frac{\Delta G_I}{G_f}, \quad \Pi_2 = \frac{\Delta G_{Imax}}{G_f}, \Pi_3 = \frac{\chi}{\omega}\left(\frac{\sigma_t}{G_f}\right)^2$$
$$\Pi_4 = d_{max}\frac{\sigma_t}{G_f}, \Pi_5 = D\frac{\sigma_t}{G_f}, \Pi_6 = a\frac{\sigma_t}{G\,f} \quad (11)$$

As discussed earlier the number of argument can be further reduced in order to simplify the model by using the concept of self-similarity. The dimensionless parameter Π_1, is a function of energy release rate range and the fracture energy. The assumption of complete self-similarity to this parameter would imply that the crack growth rate is independent of energy release rate range ΔG_I. It is well known that crack growth rate strongly depends on the energy release rate range ΔG_I and cannot be eliminated from the crack propagation model. Hence, assuming incomplete self-similarity in Π_1, we can write:

$$\frac{da}{dN} = \left(\frac{G_f}{\sigma_t}\right)\left(\frac{\Delta G}{G_f}\right)^{\gamma_1}\Phi_1\left(\Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6\right) \quad (12)$$

The dimensionless number Π_2 is a function of ΔG_{Imax}. Since the parameter ΔG_{Imax} corresponds to the upper limit of a load cycle it shows a strong dependence on crack growth rate. An assumption of incomplete self similarity is made for the non-dimensional parameter Π_2. Further, when $\Pi_2 \rightarrow 0$ indicates a non-propagating crack and $\Pi_2 \rightarrow 1$ is the case of unstable crack propagation. This provides an indication towards the consideration of intermediate asymptotic nature of Π_2.

Assuming incomplete self-similarity in other dimensionless parameters Π_2, Π_3 and Π_6. Equation 12 can be re-written as:

$$\frac{da}{dN} = \left(\frac{G_f}{\sigma_t}\right)\left(\frac{\Delta G_I}{G_f}\right)^{\gamma_1}\left(\frac{\Delta G_{Imax}}{G_f}\right)^{\gamma_2}\left[\frac{\chi}{\omega}\left(\frac{\sigma_t}{G_f}\right)^2\right]^{\gamma_3}\left(a\frac{\sigma_t}{G\,f}\right)^{\gamma_4} \quad \Phi_2\left(\Pi_4, \Pi_5\right) \quad (13)$$

Where coefficients γ_1, γ_2, γ_3 and γ_4 and the function Φ_2 can be obtained through an error minimisation technique using available experimental results.

The closed form expression for the proposed fatigue crack propagation model expressed in Equation 13 is described as below:

$$\frac{da}{dN} = G_f^{1-\gamma_1-\gamma_2-2\gamma_3-\gamma_4}\Delta G_I{}^{\gamma_1}\Delta G_{Imax}^{\gamma_2}\sigma_t^{2\gamma_3+\gamma_4-1}\omega^{\gamma_3}\chi^{-\gamma_3}a^{\gamma_4}\Phi_2\left(\Pi_4, \Pi_5\right) \quad (14)$$

Comparing the proposed model described in Equation 14 with the well known Paris law and using the linear elastic fracture mechanics relation $\Delta G_I = (\Delta K_I)^2/E$ and $\Delta G_{Imax} = \Delta G_I/(1-R)$, The Paris law constants can be written as:

$m = 2(\gamma_1 + \gamma_2)$

$C = G_f^{1-\gamma_1-\gamma_2-2\gamma_3-\gamma_4}E^{-\gamma_1-\gamma_2}\sigma_t^{2\gamma_3+\gamma_4-1}(1-R)^{-\gamma_2}\omega^{\gamma_3}\chi^{-\gamma_3}a^{\gamma_4}\Phi_2$

Where, E is the modulus of elasticity, R is the loading ratio and ΔK_I is the stress intensity factor range.

4 CALIBRATION OF DEVELOPED MODEL

In this section, the coefficients introduced in the proposed model described in Equation 13 are determined by using the experimental results of Shah [10]. Shah has conducted three point bending tests on three geometrically similar specimen of varying sizes under varying amplitude sinusoidal loading. The details of geometrical properties are given in Table (2). Critical energy dissipation [8] has been calculated by using the Equation 1. Size of the cyclic fracture process zone, (D_{0c}) calculated as 16.667 mm by taking R_f value as 0.12 [9], where, $R_f = D_{0C}/D_{0m}$.

Table 2: Details of dimensions of beams [10]

Dimensions	Small	Medium	Large
Depth$(D)(mm)$	76	152	304
Span $(S)(mm)$	190	380	760
Length (L) (mm)	241	331	810
Thickness $(b)(mm)$	50	50	50
Notch size (a_0) (mm)	15.2	30.4	60.8

The value of D_{0m} and d_{max} has been taken to be 138.89 mm and 12.5 mm respectively [10]. G_f has been determined for small, medium and large beam specimens as 0.001263 N/mm, 0.002847 N/mm and 0.01229 N/mm respectively. The value of ω and χ are 1 Hz [10] and 6.33E-06 m^2/s respectively.

For determining the unknown coefficients γ_1, γ_2, γ_3 and γ_4 and the function Φ_2 the input parameters which required are (da/dN), ΔG_{max}, ΔG, χ, ω and a. These coefficients are determined through an optimisation process using the least squares error minimisation technique. The best suited values of γ_1, γ_2, γ_3, γ_4 and Φ_2 are found to be 3.2576, 0.0567, 0.0232, 1.1743 and 0.1345, respectively and are treated as material constants. The expression for Φ_2 as a function of Π_5 has been proposed by a best fit equation and is given by :

$$\log\left(\Phi_2\right) = -37.34\log\left\{\log\left(D\frac{\sigma_t}{G_f}\right)\right\} + 61.285 \quad (15)$$

The value of the parameter Φ_2 for any size of specimen can be obtained using the Equation 15.

5 VALIDATION OF THE PROPOSED MODEL

To verify the applicability of the proposed model with other specimens, validation study has been done. Another set of experiment results of Shah [10] has been used for validating the model.

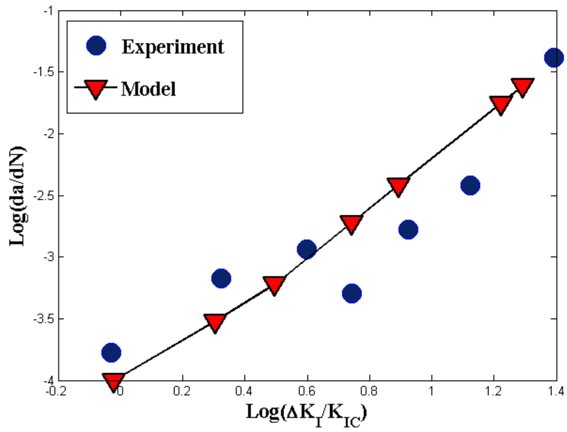

Figure 1: Validation of proposed model for small specimen [10].

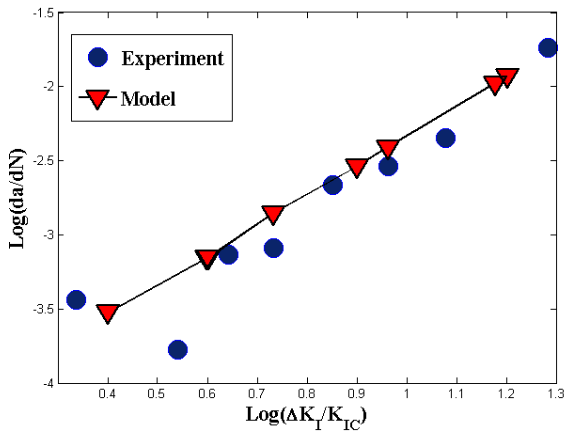

Figure 2: Validation of proposed model for medium specimen [10].

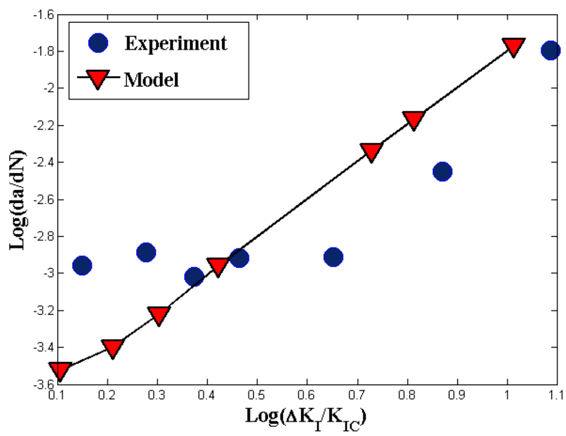

Figure 3: Validation of proposed model for large specimen [10].

In Figures $(1) - (3)$ crack growth rate has been plotted as a function of variation in the stress intensity factor for small, medium and large specimens respectively. Both the model predictions have been plotted along with the experimental results.
A reasonably good agreement is seen between the two plots in case of small and medium specimen. However, for large specimen there is a sudden increase in the rate of fatigue crack growth observed after a constant growth rate in case of experimental results which does not agree with the results generated by the proposed model. Further, it can be seen that the model is able to predict the crack growth rate fairly for the three specimens of varying size and hence, the model is able to capture the effect of fracture process zone (FPZ) and size effect.

6 CONCLUSIONS

In this present work, an analytical model has been proposed to predict crack propagation in plain concrete member under the action of repetitive loading. This model has been developed using the concept of dimensional analysis in conjunction with intermediate asymptotic and self-similarity. The model incorporates the influence of fracture process zone along with other fatigue crack growth parameters such as the size dependent fracture energy, change in energy release rate, frequency, initial crack length, structural size, maximum size of the aggregate. Further this model has been validated with the experimental results available in the literature.

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