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INVESTIGATION OF MESH DEPENDENCE OF STOCHASTIC SIMULATIONS OF QUASIBRITTLE FRACTURE

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Abstract. This paper presents a new crack band model for stochastic finite element (FE) simulations of quasibrittle fracture. The model is anchored by a probabilistic treatment of damage initiation, localization and propagation. This study focuses on the case in which the finite element size is larger than the width of the localization band. A weakest link model is used to describe the probabilistic onset of damage localization inside the element, where the randomness of the location of the localization band is related to the random material strength. Meanwhile, the model also includes the regularization of fracture energy for the transition from damage initiation to localization. The proposed model is applied to analyze the probability distributions of nominal strength of quasibrittle structures of different geometries. The results show that, without a proper treatment of probability distributions of constitutive parameters, the direct application of the conventional crack band model for stochastic FE simulations would lead to mesh-sensitive results. The proposed model is able to effectively mitigate this issue of spurious mesh sensitivity.

1 INTRODUCTION

Quasibrittle materials are brittle heterogenous materials. Common examples include concrete, composites, tough ceramics, etc. These materials have widely been used for many modern engineering structures, such as civil infrastructure, aircraft, ships, military armors, biomedical implants, microscale devices, etc. It has been well known that the post-peak constitutive response of quasibrittle materials can be characterized by a gradual loss of load-carrying capacity. Such a strain-softening behavior gives rise to the strain localization phenomenon. It has been shown that the necessary condition of localization instability can be determined by the eigenvalue analysis of the acoustic tensor at a material point. Meanwhile, it has also been recognized that in addition to the constitutive material behavior the boundary condition and overall stress field may also affect the onset of localization [12].

Localization instability is known to cause spurious mesh sensitivity in the finite element (FE) simulations. This is because the strain softening behavior would cause damage to localize into a single layer of elements. Thus the energy needed to cause material damage would depend on the mesh discretization. One solution to this problem is to introduce a material length scale into the model. This class of models is usually referred to as the localization limiters. The simplest localization limiter is the crack band model developed by Bažant and Oh [5], in which the post-peak portion of the stress-strain curve of the material is adjusted such that the overall fracture energy is kept constant. While the crack band model is easy to implement, special care needs to be taken for the proper definition of the element size under a multi-axial stress state as well as for high-order elements [2, 13, 14]. Other localization limiter adopts the concept of the nonlocal continuum, where it is assumed that the constitutive behavior of a material point depends on both the local constitutive variables and the values of these constitutive variables in the surrounding material points. The class of nonlocal models can be further divided into the integral-type [7, 22] and the gradient-type models [1, 21].

All existing localization limiters were developed for deterministic analysis. Some recent research attempts have been directed towards the understanding of the effect of strain localization on the reliability analysis due to its paramount importance for structural design [4, 15]. So far, there is still a lack of understanding of the effect of the strain localization mechanism on the stochastic FE analysis of quasibrittle fracture [9, 23].

This contribution presents a probabilistic crack band model (PCBM) for stochastic FE simulations of quasibrittle structures. This model extends the conventional crack band model by a probabilistic treatment of damage evolution. It is limited to the case where the finite element size is larger than the crack band width.

2 PROBABILISTIC TREATMENT OF DAMAGE PROCESS

The essential idea of the conventional crack band model lies in the adjustment of the material's constitutive relationship in order to preserve fracture energy for localized damage. Following this concept, here we attempt to investigate how to adjust the probability distributions of the constitutive parameters to ensure the mesh objectivity of stochastic FE simulations.

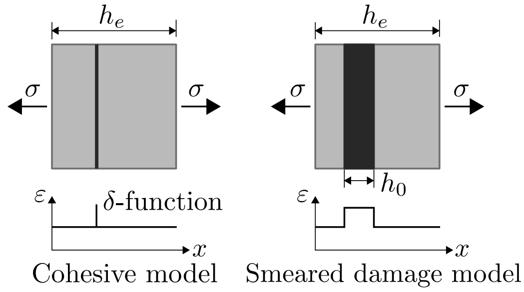

Figure 1: Localization of damage in one material element.

We limit our attention to tensile damage, where the stress-strain response is characterized by three material parameters: the elastic modulus E, the tensile strength f_t and the energy dissipation density γ. In this study we consider the randomness in the tensile strength and the energy dissipation density and we assume that these two variables are independent. The same assumption has been adopted by some recent studies on probabilistic analysis of quasibrittle structures [10, 11]. Now, the goal is to determine the probability distribution of f_t and γ with taking into consideration the processes of damage initiation and localization.

The constitutive behavior of each Gauss point represents the mechanical behavior of a material element in the FE simulation. Consider this finite element of size h_e subjected to an tensile stress, as shown in Fig. 1. Upon loading, a damage band of fixed width h_0 (usually referred to as the crack band width) would form in this material element. The width h_0 is estimated by previous studies to be about two to three times the maximum size of the material inhomogeneities [3, 6, 8]. This crack band width represents a characteristic length scale of the damage localization process. In probabilistic analysis, another length scale, the correlation length, enters the problem as well. However, recent studies have shown that for material properties of finite elements of size equal to the crack band width can be considered as statistically independent [6, 11, 15]. Thus, the correlation length is expected to be considerably smaller than the crack band width. Assuming $h_e > h_0$, we can treat the material properties of each element as statistically independent.

2.1 Definition of localization level

The fracture process of quasibrittle materials can be considered to consist of three stages: damage initiation, damage localization, and damage propagation. During the damage initiation stage, a large distributed cracking zone is formed in the structure. As the loading continues, these distributed cracks start to localize into one macro-crack; and this macro-crack propagates with a fracture process zone attached at its tip. These three stages have very different implications on the regularization of energy dissipation as well as on the probabilistic treatment of localized damage, it is therefore essential to differentiate them for the probabilistic analysis of quasibrittle fracture since.

In the context of FE simulations, we propose a parameter for each Gauss point that measures the level of localization

$$\kappa_c = \begin{cases} \frac{1}{(n_i+n_o)}\left[\frac{(n_i+n_o+1)\max\limits_{k\leq n_o+n_i}(\phi_k)}{\sum_{k=0}^{n_i+n_o}\phi_k}-1\right] & \text{if } \phi_0>0 \\ 0 & \text{if } \phi_0=0 \end{cases} \tag{1}$$

where n_i = number of surrounding Gauss points within the element of interest (i.e. inner neighbors in Fig. 2), n_o = number of surrounding Gauss points within the neighboring elements (outer neighbors in Fig. 2), ϕ_0 = damage level of the Gauss point of interest, ϕ_k = damage level of the kth surrounding Gauss points, $k \in \{1, \ldots, n_i\}$ refers to inner neighbors, and $k \in \{n_i+1, \ldots, n_i+n_o\}$ refers to outer neighbors. The level of damage, ϕ, may simply be chosen to be equal to the damage parameter. It is noted that the detailed definition of ϕ_k is not of particular importance since Eq. 1 uses the damage levels in a relative sense. Eq. 1 implies that the value of κ_c would increase with the level of strain localization, and the maximum possible value of κ would be equal to 1, which corresponds to the case where only one Gauss point exhibits damage.

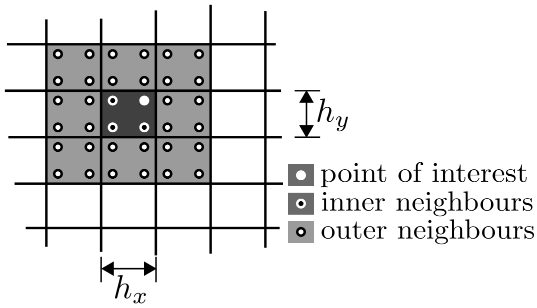

Figure 2: Determination of localization levels using information of neighboring Gauss points.

In addition to the strain localization level of each Gauss point, it is also necessary to determine the localization level of the surrounding Gauss points, which is described by

$$\kappa_w = \frac{1}{n_o-1}\left[\frac{n_o\cdot\max\limits_{k=n_i+1}^{n_i+n_o}\phi_k}{\sum_{k=n_i+1}^{n_i+n_o}\phi_k}-1\right] \tag{2}$$

In contrast to the previously defined parameter κ_c, the localization parameter κ_w only considers the surrounding Gauss points. In Section 2.3, we will show that κ_w provides useful information that determines the randomness of the onset of the localization band for that Gauss point. This is essential for constructing the probability distribution function of the strength. Furthermore it is noted that both localization parameters κ_c and κ_w are treated as non-decreasing, i.e. damage localization is an irreversible process.

2.2 Regularization of fracture energy

We first focus on the treatment of energy dissipation density, where the probabilistic formulation is based on the requirement of preservation of fracture energy for localized damage. Energy regularization is an essential concept of the conventional crack band model [5, 8], which can be demonstrated by approximating the localized damage band as a cohesive crack (Fig. 1). The fracturing strain of the finite element equals to the opening of the cohesive crack divided by the material element size. Therefore, we have

$$\gamma h_e = G_f \tag{3}$$

where G_f = fracture energy of the material. Eq. 3 leads to the fact that the stress-strain response must depend on the material element size. Here we define the reference stress-strain response of a material element that has the same size as the crack band width, h_0. γ_0 is the strain energy density corresponding to this reference stress-strain response. Clearly we have $\gamma_0 h_0 = G_f$.

Eq. 3 is derived under assumption that the localization has already occurred, which does not explicitly address the transition from damage initiation to localization. At the damage initiation stage, the total energy dissipation of the material should be proportional to the element size because the entire material element would suffer distributed damage. To account for such a transition, we propose a phenomenological energy regularization equation using the localization parameter κ_c

$$\gamma = \gamma_0 f(\kappa_c) \tag{4}$$

where: $$f(\kappa_c) = \frac{h_0}{h_e} + \left(1 - \frac{h_0}{h_e}\right) \exp\left(-\frac{\kappa_c}{\kappa_{0c}}\right) \tag{5}$$

Function $f(\kappa_c)$ leads to a smooth transition of the energy dissipation density from γ_0 to G_f/h_e as the damage localizes. The transition is governed by the parameter κ_{0c}.

We can determine the cumulative distribution function (cdf) of the energy dissipation density γ from the probability distribution of the fracture energy

$$F_\gamma(x) = \Pr(\gamma \leq x) \tag{6}$$

$$= F_{G_f}[x h_0 / f(\kappa_c)] \tag{7}$$

Eq. 7 implies that the probability distribution of the energy dissipation density is governed by the localization parameter as well as the ratio between the finite element size and the crack band width. The distribution function F_{G_f} is assumed to obey a Gaussian-Weibull grafted distribution [16, 17].

2.3 Probabilistic treatment of tensile strength

The foregoing analysis only considers the formation of a damage band in single material element. However, there is an inherent variability of the location of the damage band inside this material element. We may consider that the location of the damage band is determined by the tensile strength. In other words, the random onset of the damage band in the material element must be reflected by the statistics of the tensile strength.

Since we assume independent strength at possible locations of the damage band, we may use the classical weakest link model to describe the strength cdf for each Gauss point, i.e.

$$F_{f_t}(\sigma) = 1 - [1 - P_1(\sigma)]^{n_e} \tag{8}$$

where n_e = number of potential crack bands that could be formed in the material element represented by the Gauss point, and $P_1(x)$ = cdf of the tensile strength of the material element of a size equal to the crack band width. According to [4, 15], $P_1(x)$ is described by the Gaussian-Weibull grafted distribution function (Eq. 8) as well.

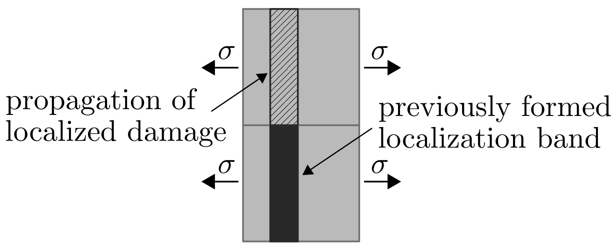

Figure 3: Propagation of localized damage.

The number of potential crack bands n_e is largely governed by the strain localization level in the surrounding material elements. Consider that one surrounding material element has experienced damage localization as shown in Fig. 3. In such a case, the localized damage in the element would lead to stress concentration and it would dictate the location of the onset of localization band in the element of interest. Therefore, there is only one potential crack band inside the element, $n_e = 1$. The effect of the damage localization of the surrounding Gauss points on the weakest link model is described through an empirical function

$$n_e = 1 + \left(\frac{h_e}{h_0} - 1\right) \exp\left(-\frac{\kappa_w}{\kappa_{0w}}\right) \tag{9}$$

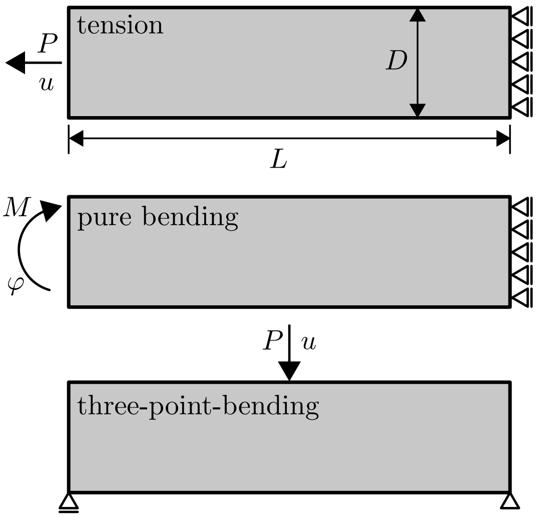

Figure 4: Three loading configurations: a) uniaxial tension, b) pure bending, and c) three-point bending.

3 NUMERICAL EXAMPLES

The proposed PCBM is applied to simulate the cdf of the maximal nominal stress, $\sigma_{N,\max}$, of three concrete specimens under different loading configurations (Fig. 4). The nominal stress for these three specimens are expressed as the maximum principal stress based on the elastic analysis

$$\sigma_N = \begin{cases} P/bD & \text{uniaxial tension} \\ 6M/bD^2 & \text{pure bending} \\ 3PL/2bD^2 & \text{three-point bending} \end{cases} \tag{10}$$

where P, M = the applied load and moment, D = specimen depth, L = specimen length, and b = width of the specimen in the transverse direction. Besides PCBM, two other models are also used to perform these simulations for comparison purpose. These models are 1) the crack band model (Eq. 7) without adjusting the probability distribution of tensile strength (i.e. $n_e = 1$ for Eq. 9), which is denoted by CBM, and 2) the crack band model (Eq. 7) with considering the weakest link model of tensile strength regardless of the localization level (i.e. $n_e = h_e/h_0$ for Eq. 9), which is denoted by WLM.

A simple isotropic damage model is used in the simulations. Its constitutive relationship can be written as

$$\boldsymbol{\sigma} = (1-\omega)\boldsymbol{D} : \boldsymbol{\epsilon} \tag{11}$$

where $\boldsymbol{D}$ =elastic stiffness tensor and ω = damage parameter describing the damage level of the material point. ω can be expressed as a function of the equivalent strain, $\bar{\boldsymbol{\epsilon}}$, defined by [18]

$$\bar{\boldsymbol{\epsilon}} = \sqrt{\sum_{I=1}^{3} \langle \boldsymbol{\epsilon}_I \rangle^2} \tag{12}$$

where $\boldsymbol{\epsilon}_{1-3}$ are principal strain values. The damage parameter is then calculated by assuming a linear softening behavior:

$$\omega = \begin{cases} 0 & \bar{\boldsymbol{\epsilon}}_m \le f_t/E \\ 1 - \dfrac{f_t\left(2\gamma - f_t\bar{\boldsymbol{\epsilon}}_m\right)}{\bar{\boldsymbol{\epsilon}}_m\left(2\gamma E - f_t^2\right)} & f_t/E < \bar{\boldsymbol{\epsilon}}_m \le 2\gamma/f_t \\ 1 & \text{otherwise} \end{cases} \tag{13}$$

where $\bar{\boldsymbol{\epsilon}}_m$ is the maximum value of $\bar{\boldsymbol{\epsilon}}$ that has ever been attained during the past loading history.

To prevent a snap-back stress-strain behavior, $\gamma \le f_t^2/2E$, and from Eq. (3) we have $h_e \le 2G_f E/f_t^2$. This represents an upper limit of the element size h_e. Both f_t and G_f are random and there is always some probability of having a snap-back stress-strain curve. However, this probability is extremely low for the mesh sizes used in the present study. Specimens are discretized using linear quadrilateral elements with four integration points assuming a 2D plane stress condition, the solver we modified and used is the OOFEM software [19, 20].

All specimens have a depth of $D = 0.5\,\text{m}$ and a length of $L = 4\,\text{m}$. The beams are loaded by a prescribed deformation as shown in Fig. 4. The following material parameters are used in the analysis: elastic modulus E =

30 GPa, Poisson's ratio $\nu = 0.2$, mean tensile strength $\overline{f_t} = 3$ MPa, mean fracture energy $\overline{G_f} = 80$ J/m^2. For the probability distributions of tensile strength and fracture energy, we set the coefficient variation CoV= 0.15, the grafting distribution $P_{gr} = 10^{-3}$, and the Weibull modulus $m = 26$. Three different mesh sizes are considered: $(h_x, h_y) = (50, 50)$ mm, $(100, 50)$ mm, and $(200, 50)$ mm, where h_x, h_y denote the width and depth of the element, respectively. The crack band width h_0 is set to be 50 mm.

The model parameters κ_{0c} and κ_{0w} are found by minimization of the difference between results computed on different mesh sizes. This minimization yields $\kappa_{0c} = 0.190$ and $\kappa_{0w} = 0.283$.

4 RESULTS AND DISCUSSION

Fig. 5 presents the cdf of the nominal strength of the beams simulated by the aforementioned three methods. Each of these strength distributions are obtained from 1000 realizations. For the reference size, $h_x = h_0$, all three methods yield the same result. As we use a larger element size, the CBM overestimates the structural strength because it does not take into account the potential randomness of the onset of localization band. On contrary, the WLM underestimates the strength distribution of the beam as the element size increases because it does not account for the crack path predetermined by the previous damage in the surrounding elements.

The results show that the PCBM can effectively mitigate the mesh dependence of probabilistic simulation of quasibrittle structure. It is interesting to note that for the uniaxial tension the results of PCBM and WLM exhibit a large difference compared to what is seen in the other two loading scenarios. This can be attributed to the spatial distribution of the parameter κ_w at the peak load shown as Fig. 6. In both three-point bending and pure bending cases, the specimen exhibits relatively high values of κ_w, which indicates that the PCBM is close to the CBM. In the uniaxial tension case, the values of κ_w are more spread and the difference between the PCBM and CBM is more pronounced.

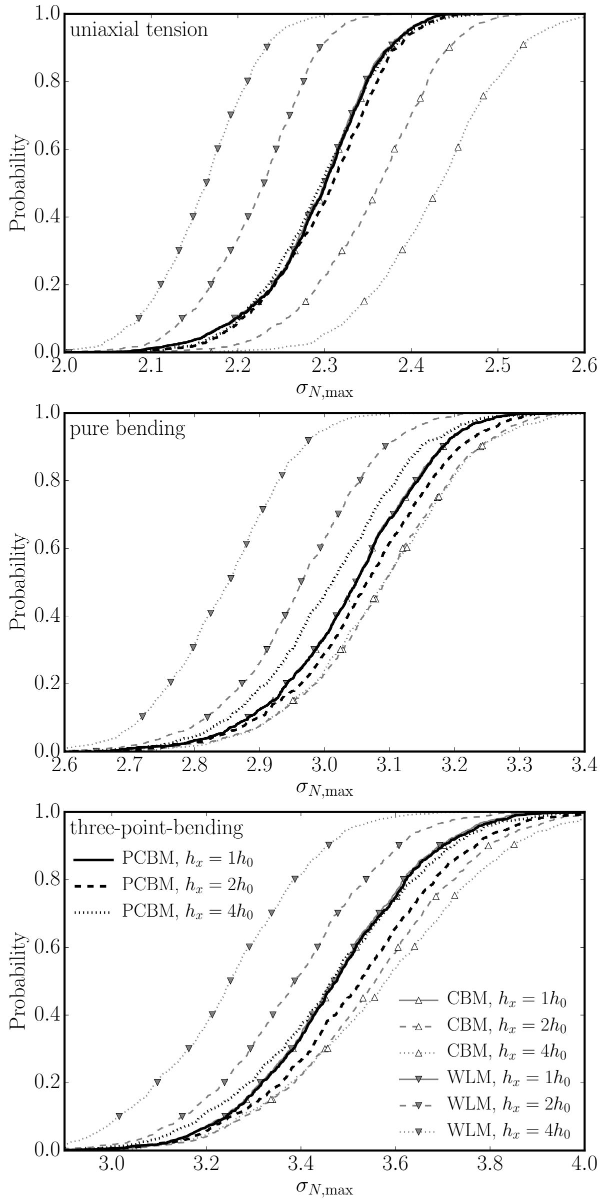

Figure 5: Simulated cdfs of nominal strength of three specimens with different mesh sizes.

5 CONCLUSIONS

This study shows that stochastic simulations by using the conventional crack band model could suffer the issue of mesh dependence. The underlying reason for such mesh dependence lies in the lack of consideration of the random onset of localization band inside the material element. The issue can effectively be mitigated by using a finite weakest link model for the

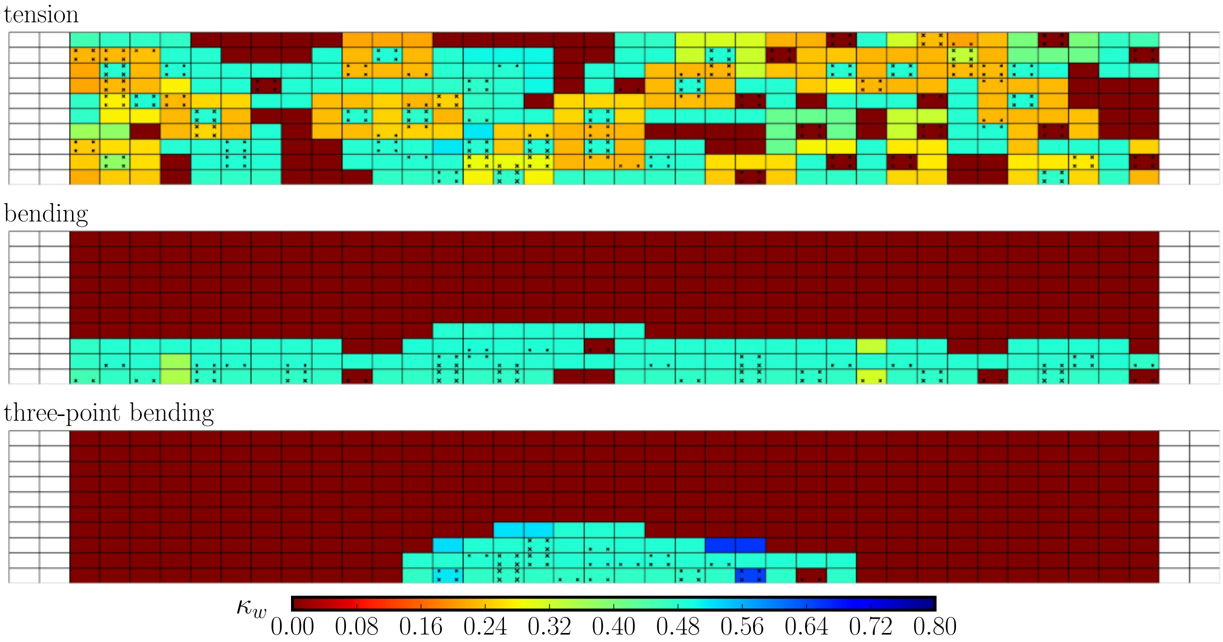

Figure 6: Localization parameter κ_w at the peak load.

randomness of the damage localization band in each material element, where the statistics of the random onset of localization is governed by the localization level of its neighboring elements. In addition to the consideration of random onset of localization band in the material element, it is essential to take into account the regularization of fracture energy for the transition between damage initiation and localization.

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